### MATHEMATICS I Exam, January 15, 2013

1. Compute the limit of the sequence or prove it does not exist:

$$\lim_{n \to \infty} \frac{n \cdot \left( \log \left( n^2 + \frac{1}{3} \right) - \log \left( n^2 \right) \right)}{1 - \cos \left( \frac{1}{\sqrt{n}} \right)} \cdot \operatorname{arctg} n.$$

## 10 points

2. Compute the limit of the function at  $+\infty$  or prove it does not exist:

$$\lim_{x \to +\infty} \sin\left(\frac{\pi}{4x}\right) \cdot \frac{\sqrt{e^{\frac{3}{x}} - 1}}{\sqrt{x^3 + 2} - \sqrt{x^3 + 1}}.$$

# 10 points

3. Find domain of the given function, determine the set of all points in which it is continuous (continuous from the left/from the right). Compute its derivative (oneside derivatives) at each point where it exists.

$$f(x) = \begin{cases} \log \left(2x^2 - 2x + 1\right) + \arctan \left(\frac{1}{2x-1}\right) & \text{for } x \neq \frac{1}{2}, \\ \frac{\pi}{2} - \log 2 & \text{for } x = \frac{1}{2}. \end{cases}$$

#### 15 points

4. Investigate properties of the function and draw its graph:

$$f(x) = x \cdot \sqrt{x^2 - 3}.$$

### 15 points

Write detailed arguments for each nontrivial step and verify assumptions of theorems you use.