## MATHEMATICS I Exam, January 15, 2013

1. Compute the limit of the sequence or prove it does not exist:

$$
\lim _{n \rightarrow \infty} \frac{n \cdot\left(\log \left(n^{2}+\frac{1}{3}\right)-\log \left(n^{2}\right)\right)}{1-\cos \left(\frac{1}{\sqrt{n}}\right)} \cdot \operatorname{arctg} n
$$

## 10 points

2. Compute the limit of the function at $+\infty$ or prove it does not exist:

$$
\lim _{x \rightarrow+\infty} \sin \left(\frac{\pi}{4 x}\right) \cdot \frac{\sqrt{e^{\frac{3}{x}}-1}}{\sqrt{x^{3}+2}-\sqrt{x^{3}+1}}
$$

## 10 points

3. Find domain of the given function, determine the set of all points in which it is continuous (continuous from the left/from the right). Compute its derivative (oneside derivatives) at each point where it exists.

$$
f(x)=\left\{\begin{array}{l}
\log \left(2 x^{2}-2 x+1\right)+\operatorname{arctg}\left(\frac{1}{2 x-1}\right) \quad \text { for } x \neq \frac{1}{2} \\
\frac{\pi}{2}-\log 2 \quad \text { for } x=\frac{1}{2}
\end{array}\right.
$$

## 15 points

4. Investigate properties of the function and draw its graph:

$$
f(x)=x \cdot \sqrt{x^{2}-3}
$$

## 15 points

Write detailed arguments for each nontrivial step and verify assumptions of theorems you use.

