Fourierova transformace distribucí

(1): $f \in L^1(\mathbb{R}^N)$
$\mathcal{F}(f) = \int_{\mathbb{R}^N} f(x)\exp\{-2\pi i(x,\xi)\}\,\mathrm{d}x$

(2): $A \in \mathbb{R}^{N\times N}$, poz. definitní, symetrická
$\mathcal{F}(\exp\{-(Ax,x)\}) = \frac{(\sqrt{\pi})^N}{\sqrt{|\mathrm{detA}|}}\exp\{-\pi^2(A^{-1}\xi,\xi)\}$

(3): $\delta \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}(\delta) = 1$

(4): $1 \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}(1) = \delta$

(5): $x^n \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}(x^n) = \frac{1}{(-2\pi i)^n}\delta^{(n)}(\xi)$

(6): $\delta^{(n)} \in \mathcal{S}'(\mathbb{R}),\ n \in \mathbb{N}$
$\mathcal{F}\left(\delta^{(n)}\right) = (2\pi i)^n\xi^n$

(7): $b \in \mathbb{C}$
$\mathcal{F}(\exp(2\pi i bx)) = \delta_b$

(8): $b \in \mathbb{C}$
$\mathcal{F}(\sin(2\pi bx)) = \frac{1}{2i}(\delta_b - \delta_{-b})$

(9): $b \in \mathbb{C}$
$\mathcal{F}(\cos(2\pi bx)) = \frac{1}{2}(\delta_b + \delta_{-b})$

(10): $b \in \mathbb{C}$
$\mathcal{F}(\sinh(2\pi bx)) = \frac{1}{2}(\delta_{-ib} - \delta_{ib})$

(11): $b \in \mathbb{C}$
$\mathcal{F}(\cosh(2\pi bx)) = \frac{1}{2}(\delta_{-ib} + \delta_{ib})$

(12): $x_+^\lambda \in \mathcal{S}'(\mathbb{R}),\ \lambda \in \mathbb{C}$
$\mathcal{F}\left(\frac{x_+^\lambda}{\Gamma(\lambda+1)}\right) = \mathrm{e}^{-i(\lambda+1)\frac{\pi}{2}}(2\pi)^{-\lambda-1}(\xi - i0)^{-\lambda-1}$

(13): $x_+^n \in \mathcal{S}'(\mathbb{R}),\ n \in \mathbb{N}$
$\mathcal{F}\left(x_+^n\right) = (2\pi i)^{-n-1}n!\xi^{-n-1} + \frac{1}{2}(2\pi i)^{-n}(-1)^{-n}\delta^{(n)}$

(14): $H(x) \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}(H(x)) = \mathcal{F}\left(x_+^0\right) = \frac{1}{2\pi i}\xi^{-1} + \frac{1}{2}\delta$

(15): $x_-^\lambda \in \mathcal{S}'(\mathbb{R}),\ \lambda \in \mathbb{C}$
$\mathcal{F}\left(\frac{x_-^\lambda}{\Gamma(\lambda+1)}\right) = \mathrm{e}^{i(\lambda+1)\frac{\pi}{2}}(2\pi)^{-\lambda-1}(\xi + i0)^{-\lambda-1}$

(16): $|x|^\lambda = x_+^\lambda + x_-^\lambda \in \mathcal{S}'(\mathbb{R}),\ \lambda \in \mathbb{C},\ \lambda \neq -1,-2,-3,\ldots$
$\mathcal{F}\left(|x|^\lambda\right) = -2\Gamma(\lambda+1)(2\pi)^{-\lambda-1}\sin\left(\frac{\pi}{2}\lambda\right)|\xi|^{-\lambda-1}$

(17): $|x|^\lambda \operatorname{sign} x \in \mathcal{S}'(\mathbb{R}),\ \lambda \in \mathbb{C};\quad \mathcal{F}\left(|x|^\lambda \operatorname{sign} x\right)$
$= -2i(2\pi)^{-\lambda-1}\Gamma(\lambda+1)\cos\left(\frac{\pi}{2}\lambda\right)|\xi|^{-\lambda-1}\operatorname{sign}\xi$

(18): $x^{-m} \in \mathcal{S}'(\mathbb{R}),\ \lambda \in \mathbb{C},\ m \in \mathbb{N};\quad \mathcal{F}(x^{-m})$
$= \begin{cases} (-1)^{\frac{m+1}{2}} i\pi(2\pi)^{m-1}|\xi|^{m-1}\frac{\operatorname{sign}\xi}{(m-1)!} & m \text{ liché} \\ (-1)^{\frac{m}{2}}\frac{|\xi|^{m-1}\pi(2\pi)^{m-1}}{(m-1)!} & m \text{ sudé} \end{cases}$

(19): $x^{-1} \in \mathcal{S}'(\mathbb{R})$
$\mathcal{F}\left(x^{-1}\right) = -i\pi\operatorname{sign}\xi$

(20): $x^{-2} \in \mathcal{S}'(\mathbb{R})$,
$\mathcal{F}\left(x^{-2}\right) = -|\xi|2\pi^2$

(21): $(x+i0)^\lambda \in \mathcal{S}'(\mathbb{R}),\ \lambda \in \mathbb{C}$
$\mathcal{F}\left((x+i0)^\lambda\right) = \frac{\xi_+^{-\lambda-1}}{\Gamma(-\lambda)}\exp\{i\lambda\frac{\pi}{2}\}(2\pi)^{-\lambda}$

(22): $(x-i0)^\lambda \in \mathcal{S}'(\mathbb{R}),\ \lambda \in \mathbb{C}$
$\mathcal{F}\left((x-i0)^\lambda\right) = \frac{\xi_-^{-\lambda-1}}{\Gamma(-\lambda)}\exp\{-i\lambda\frac{\pi}{2}\}(2\pi)^{-\lambda}$

(23): $r = |x|, x \in \mathbb{R}^N,\ \lambda \in \mathbb{C},\quad \rho = |\xi|, \xi \in \mathbb{R}^N$
$\mathcal{F}\left(\frac{r^\lambda}{\Gamma\left(\frac{\lambda+N}{2}\right)}\right) = \frac{\rho^{-\lambda-N}}{\Gamma(-\lambda/2)\pi^{\lambda+N/2}}$