Sketching graphs of a function $f : \mathbb{R} \to \mathbb{R}$

Algorithm

1. **Find the domain**. Be careful especially of: fractions, radices, logarithm, arc functions.
2. **Intercepts with axes**. What happens, if $x = 0$? When $y = 0$?
3. **Even, odd or periodic?** Check the domain, if it is symmetric - then you can investigate $f(-x)$. Check the domain, if it is periodic, and the function, if it contains $\sin x$ or cos (it can be hidden in $\tan x$ or $\cot x$) - then investigate periodicity.
4. **Limits** at the endpoints of the domain. Do not forget about $\pm\infty$ (if they are endpoints of the domain). Compute limits at some special points. (Rmk: Periodic functions can not have limit at $\pm\infty$.)
5. **Continuity**. Is the function sum, product, composition of continuous functions?
6. **The first derivative** f'. Derive mechanically. Find the domain of f'.
7. f' **in special points**. Can you find points, where the function f is defined, but the mechanical derivative is not? Compute the left and right derivative as a limit.
8. **Monotonicity**. Find the zero points for the derivative. Sketch a diagram with the domain of f, domain of f' and the zero points - it gives intervals. Find sign of f' on these intervals and find monotonicity.
9. **Extrema**. From the previous step you have suspects of extremas.
10. **The second derivative** f''. Find the 2nd derivative mechanically. (Do not care about one sided 2n derivatives.)
11. **Convexity/Concavity**. Find zero points of f''. Make similar diagram as in the previous steps and find sign of f''. Then decide about concavity and convexity of f. Find points of inflection (warning: an inflection point needs to have the 1st derivative).
12. **Asymptotes**. Find the limits $k = \lim_{x\to\pm\infty} f(x)/x$ and $q = \lim_{x\to\pm\infty} f(x) - kx$. If they exist, you have asymptotes at $\pm\infty$ - $y = kx + q$. Rmk: for the first limit you can apply L'Hospital rule.
13. **Sketch the graph**.
14. **Extremas**. Check the extremas - points with zero derivative, points without derivative, endpoints of intervals. Decide, if they are local or global. Rmk: there can be extremum only if the point is in the domain (= no infinities).
15. **Range**. Find the range.