

1. Řešit pomocí konvoluce $\frac{1}{(s-1)(s-2)}$.

$$L_{-1} \left(\frac{1}{s-1} \right) = e^t$$

$$L_{-2} \left(\frac{1}{s-2} \right) = e^{2t}$$

$$(f * g)(t) = \int_0^t e^{t-y} e^{2y} dy = \int_0^t e^{t-y+2y} dy$$

$$= \int_0^t e^{t+y} dy = \int_0^t e^t e^y dy$$

$$= [e^t e^y]_0^t = e^t e^t - e^t e^0$$

$$= e^{2t} - e^t = e^t(e^t - 1)$$

2. Řešit pomocí residu $\frac{2s+3}{s^2+4s+3}$.

$$\frac{2s+3}{s^2+4s+3} = \frac{2s+3}{(s+1)(s+3)}$$

Póly:

$$z_1 = -1$$

$$z_2 = -3$$

Residua:

$$res_{-1} \frac{2s+3}{s^2+4s+3} e^{st} = \lim_{s \rightarrow -1} \frac{2s+3}{s+3} e^{st} = \frac{1}{2} e^{-t}$$

$$res_{-3} \frac{2s+3}{s^2+4s+3} e^{st} = \lim_{s \rightarrow -3} \frac{2s+3}{s+1} e^{st} = \frac{-3}{-2} e^{-3t} = \frac{3}{2} e^{-3t}$$

$$L_{-1} \left(\frac{2s+3}{s^2+4s+3} \right) (t) = \frac{1}{2} e^{-t} + \frac{3}{2} e^{-3t}$$