$$\boxed{\int_0^{2\pi} \frac{\cos^2\varphi}{5+3\cos\varphi}\,d\varphi}$$

$$= \int_{|z|=1} \frac{\frac{(z^2+1)^2}{4z^2}}{5+3\frac{z^2+1}{2z}} \cdot \frac{dz}{iz} = \int \frac{(z^2+1)^2\,dz}{2i\, z^2(3z^2+10z+3)}$$

$(3z+1)(z+3)$

$z_0=0 \quad z_1=-\frac{1}{3} \quad z_2=-3$

$$\operatorname{res}_0 = \lim_0 \left(\frac{(z^2+1)^2}{3z^2+10z+3}\right)' = \lim_{z\to 0} \frac{4z(z^2+1)(3z^2+10z+3)-(z^2+1)^2(6z+10)}{(3z^2+10z+3)^2}$$

$$= -\frac{10}{9}$$

$$\operatorname{res}_{-1/3} = \lim \frac{(z^2+1)^2}{3z^2(z+3)} = \left(\frac{10}{9}\right)^2 \frac{9}{8} = \frac{10}{9}\cdot\frac{5}{4}$$

$$I = \frac{2\pi i}{2i}\,\frac{10}{9}\left(-1+\frac{5}{4}\right) = \underline{\underline{\frac{5\pi}{18}}}$$

$$\boxed{\int_{-\pi}^{\pi} \frac{d\varphi}{\sqrt{5}+2\sin\varphi}}$$

$$= \int_{|z|=1} \frac{dz}{iz\left(\sqrt{5}+2\frac{z^2-1}{2zi}\right)} = \int \frac{dz}{z^2+i\sqrt{5}\,z-1}$$

$z_1 = \frac{-i(\sqrt{5}-1)}{2}$ → unutr

$z_2 = \frac{-i(\sqrt{5}+1)}{2}$

$$\operatorname{res}_{z_1} = \lim_{z_1} \frac{1}{z-z_2} = \frac{1}{i}$$

[illegible] $I = 2\pi i\,\frac{1}{i} = \underline{\underline{2\pi}}$

$$\boxed{\int_0^{2\pi} \frac{d\varphi}{1 - 2a\cos\varphi + a^2} \qquad 0 < a < 1}$$

$$= \int_{|z|=1} \frac{-dz}{i\left(az^2 - (a^2+1)z + a\right)}$$

póly $z_1 = a$ uvnitř

$$\operatorname{res}_a = \lim_{z \to a} \frac{-(z-a)}{ia(z-a)\left(z - \frac{1}{a}\right)} = \frac{-1}{i(a^2-1)}$$

takže

$$I = 2\pi i \frac{(-1)}{i(a^2-1)} = \frac{-2\pi}{a^2-1}$$

VZOR

$$\int_0^{2\pi} \frac{d\varphi}{5+4\cos\varphi}$$

$\cos\varphi = \frac{z^2+1}{2z}$ $\quad d\varphi = \frac{dz}{iz}$ $\quad z = e^{i\varphi}$

$$I = \oint_{|z|=1} \frac{dz}{iz\left(5+4\frac{z^2+1}{2z}\right)} = \oint_{|z|=1} \frac{dz}{i(2z^2+5z+2)}$$

$$z_1 = \frac{-5+\sqrt{25-16}}{4} = -\frac{1}{2} \quad \rightarrow \text{vnitř. pól 1. řádu}$$

$$z_2 = \frac{-5-\sqrt{25-16}}{4} = -2$$

$$\operatorname{res}_{z_1} = \lim_{z_1} \frac{z+\frac{1}{2}}{2i\left(z+\frac{1}{2}\right)(z+2)} = \frac{1}{3i}$$

celkem $\quad I = 2\pi i \cdot \frac{1}{3i} = \frac{2}{3}\pi$

$$\int_0^{2\pi} \frac{d\varphi}{(2+\cos\varphi)^2}$$

$$\left(2+\frac{z^2+1}{2z}\right)^2 iz^2 = i\left(2z+\frac{z^2}{2}+\frac{1}{2}\right) = i\frac{1}{2}\left(4z+z^2+1\right)$$

$$z_{1,2} = \frac{-2\pm\sqrt{4-1}}{1} = -2\pm\sqrt{3}$$

res $\quad \frac{4z}{i(z^2+4z+1)^2} = \frac{2}{3i\sqrt{3}}$

$$I = 2\pi i \cdot \frac{2}{3i\sqrt{3}} = \frac{4\pi}{3\sqrt{3}}$$

Najděte inverzní fci

$\boxed{f(z) = \frac{2z+1}{z-1}}$, f na $D = \mathbb{C} \setminus \{1\}$

$$\frac{2w+1}{w-1} = z$$

$$2w + 1 = z(w-1)$$

$$z + 1 = zw - 2w$$

$$\frac{z+1}{z-2} = w$$

$$f^{-1}(z) = \frac{z+1}{z-2} \quad \text{pro } z \neq 2$$

$\boxed{f(z) = iz + 1}$ $\mathbb{C}$

$$w = iz + 1$$

$$\frac{w-1}{i} = z \qquad f^{-1}(z) = i(1-w) \text{ na } \mathbb{C}$$

$\boxed{f(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)}$, $z \neq 0$ $\rightarrow$ mnohoznačné fce

$$\frac{1}{2}\left(w + \frac{1}{w}\right) = z \quad \rightarrow \quad w^2 + 1 = 2zw$$

$$w^2 - 2zw + 1 = 0 \qquad w_{1,2} = \frac{2z \pm \sqrt{4z^2 - 4}}{2}$$

$$w = z \pm \sqrt{z^2 - 1}$$

$\boxed{f(z) = \cos z}$

$$\cos w = z \qquad z = \frac{1}{2}\left(e^{iw} + e^{-iw}\right)$$

$$2ze^{iw} = e^{2iw} + 1 \qquad y^2 - 2zy + 1 \qquad \text{mnohoznačné}$$

$$y = e^{iw}$$

$$\Rightarrow \quad e^{iw} = z \pm \sqrt{z^2 - 1} \qquad iw = \mathrm{Ln}\left(z \pm \sqrt{z^2-1}\right)$$

$$\boxed{\begin{array}{l} f(z) = \text{lin. lom.} \\ f(0) = i \\ f(-1) = \frac{1}{2}(1-i) \\ f(-i) = 1 + \frac{i}{2} \end{array}}$$

z DP. vyjde $\qquad f(z) = w$

$$\frac{2(w-i)}{2w-1+i} : \frac{2-i}{1+2i} = \frac{z}{z+1} : \frac{-2i}{1-i}$$

vyjádříme w:

$$w = \frac{2z+1}{z-i}$$

$$\boxed{\lim_{n\to\infty} \frac{n}{1-ni}}$$

$$\frac{n}{1-ni} = \frac{n(1+ni)}{1+n^2} = \underbrace{\frac{n}{1+n^2}}_{\to\, 0} + \underbrace{\frac{n^2}{1+n^2}}_{\to\, 1} \cdot i$$

$$\lim \quad = i$$

$$\boxed{\lim_{n\to\infty} e^{ni}}$$

$$e^{ni} = \cos n + i \sin n$$

$\Rightarrow$ nemá limitu (proč?)