(1) $i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^{125} = i^{124} \cdot i = i$

$i^{-3} = \frac{1}{i^3} = \frac{1}{-i} = \frac{i}{-i^2} = i$

$i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$

$i^{-78} = \frac{1}{i^{-78}} = \frac{1}{i^{40} \cdot i^{36} \cdot i^2} = \frac{1}{-1} = -1$

(2) najdi $\bar{z}$. Komplexně sdružené k

(b) $\overline{\frac{3+4i}{1-2i}} = \frac{(3+4i)(1+2i)}{1-4i^2} = \frac{3+6i+4i+8i^2}{5} =$

$= \frac{1}{5}(-5+10i) = \underline{\underline{-1-2i}}$

$\overline{\frac{1+i}{2-i}} = \frac{1-i}{2+i} = \frac{(1-i)(2-i)}{2-i^2} = \frac{1}{5}(2+i^2-3i) =$

$= \frac{1}{5} - \frac{3}{5}i$

(2)

(a) $\frac{2+i}{i} + \frac{i}{i+1} - \frac{2i+1}{i-1} = \frac{(2+i)i}{-1} + \frac{i(i-1)}{i^2-1} - \frac{(2i+1)(i+1)}{i^2-1}$

$= -2i+1 + \frac{1}{-2}(-1-i) - \frac{1}{-2}(-2+1+2i+i) =$

$= -2i + 1 + \frac{1}{2} + \frac{i}{2} - \frac{1}{2} + \frac{3}{2}i = 1$

(2)(c) Dokaž, že sobě rovná čísla.

$z_1 = \frac{1}{2}(\sqrt{2}+\sqrt{6}) + i\sqrt{2-\sqrt{3}}$

$z_2 = \sqrt{2+\sqrt{3}} + \frac{i}{2}(\sqrt{6}-\sqrt{2})$

$\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{6} \overset{?}{=} \sqrt{2+\sqrt{3}}$

$\sqrt{2}+\sqrt{6} = 2\sqrt{2+3^{\frac{1}{2}}} \quad /^2$

$\sqrt{2-\sqrt{3}} \overset{?}{=} \frac{1}{2}\sqrt{6} - \frac{1}{2}\sqrt{2} \quad /^2$

$2-\sqrt{3} = \frac{1}{4}6 + \frac{1}{4}2 - \frac{\sqrt{12}}{2}\sqrt{6}$

$\sqrt{3} = \frac{\sqrt{2}\sqrt{2}}{2}\sqrt{3}$ ✓

$2+6+2\sqrt{2\cdot 6} = 4(2+\sqrt{3})$

$8 + 2\sqrt{12\cdot 3} = 8+4\sqrt{3}$ ✓

(3) ABS

$$\left|\frac{4-2i}{3+i}\right| = \left|\frac{(4-2i)(3-i)}{9+1}\right| = \left|\frac{1}{10}\left(12-4i-6i+2i^2\right)\right| =$$

$$= \frac{1}{10}\left|10-10i\right| = \left|1-i\right| = \sqrt{1+1} = \underline{\underline{\sqrt{2}}}$$

$$\left|\frac{|4-3i|+i}{3-2i}\right| = \left|\frac{5+i}{3-2i}\right| = \left|\frac{(5+i)(3+2i)}{13}\right| =$$

$$= \frac{1}{13}\left|15+10i+3i-2\right| = \left|1+i\right| = \sqrt{2}$$

(4) Γραπ. μορφή

$$3 = 3 \cdot 1 = 3(1 + 0i) = 3(\cos 0 + i \sin 0)$$

$$10 - 10i = 10(1 - i) = 10 \cdot \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\left(\frac{-1}{\sqrt{2}}\right)\right) =$$

$$= 10\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right)$$

$$\sin 30^\circ + i \cos 30^\circ = \sin\frac{\pi}{6} + i\cos\frac{\pi}{6}$$

$$= \frac{1}{2} + i\frac{\sqrt{3}}{2} = 1\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)$$

(5) Α σραστή

$$5(\cos 11\pi + i \sin 11\pi) = 5(\cos\pi + i\sin\pi) =$$

$$= 5(-1 + i \cdot 0) = -5$$

$$\sqrt{2}\left(\cos\frac{105}{4}\pi + i\sin\frac{105}{4}\pi\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) =$$

$$\frac{105}{4}\pi \rightsquigarrow 10\frac{1}{4}\pi$$

$$= \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 1 + i$$

(7) trivial $x, y \in \mathbb{R}$

$2x + iy = 4 - 3i \qquad x = 2 \qquad y = -3$

$x(1+i) + y(1-i) = 4 + 2i$

$(x+y) = 4 \qquad x - y = 2 \qquad 2x = 6 \quad x = 3 \quad y = 1$

$x(y+i) + y(x-i) = 2x + 2yi$

$xy + xi + yx - iy = 2x + 2yi$

$2xy = 2x \quad \Rightarrow \quad x = 0 \quad y = 0$

$(x-y) = 2y \qquad y = 1 \quad x = 2$

(8) kwadr. tw $\quad x \in \mathbb{C}$

$ix^2 - 3x + 4i = 0$

$z_{1,2} = \dfrac{3 \pm \sqrt{9 - 4i \cdot 4i}}{2i}$

$= \dfrac{3 \pm \sqrt{25}}{2i}$

$z_1 = \dfrac{4}{i} = -4i$

$z_2 = -\dfrac{1}{i} = 1i$

$z^2 - 6iz - 12 = 0$

$z_{1,2} = \dfrac{6i \pm \sqrt{-36 + 48}}{2} = 3i \pm \dfrac{\sqrt{12}}{2} = 3i \pm \sqrt{3}$

$z^2 - 6iz - 9 = 0$

$z_{1,2} = \dfrac{6i \pm \sqrt{-36 + 36}}{2} = 3i$

$3z^2 - 2z + 1 = 0$

$z_{1,2} = \dfrac{2 \pm \sqrt{4 - 12}}{6} = \dfrac{2 \pm \sqrt{-8}}{6} = \dfrac{2 \pm 2\sqrt{2}i}{6}$ $\quad 8i^2$

(6) Určete čísla, pro něž

- $|z+3-5i| = 3$

  $|z - (3+5i)| = 3$ kružnice
  střed r

- $|z-i| = 1$

  $x_0 = i$

  $r = 1$ kružnice

- $1 < |z+i| < 2$

  $x_0 = -i$ mezikruží mezi $r_2 = 1$ a $r_1 = 2$

- $|z| = 1 - 2i$

  ABS = R, nelze

- $Re\ z = Im\ z$

  vzdálenost od obou os je stejná (a navíc obě záporné)

  → os 1. a 3. kvadrantu

- $Im\ z = -i$ h.r.

  $\cup$

  $\times \mathbb{R}$

- $|z+i| + |z+1-i| = 4$

  $|z-(-i)| + |z-(-1+i)| = 4$

  elipsa - obrázek