Mathematics I - Introduction

23/24

Sets

Exercise (True or false)

A - set of all animals living in Australia.

A $a \in A$ B $b \in A$ C $c \in A$ D $d \in A$ E $e \in A$

a

c

b

d

e

Sets

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c


b


d


e

True: A, B, C, E

Sets

Exercise (True or false)

A - set of all animals living in Australia.

A $a \notin A$ B $b \notin A$ C $c \notin A$ D $d \notin A$ E $e \notin A$

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Sets

Exercise (True or false)

A - set of all animals living in Australia.

A $a \notin A$ B $b \notin A$ C $c \notin A$ D $d \notin A$ E $e \notin A$

a

c

b

d

e

True: D

Exercise

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{1, 2, 3, 4, 5\}$. Find

1. $A \cup B$
2. $A \cap B$
3. A^c
4. $(B^c)^c$
5. $A \setminus B$
6. $B \setminus A$

Exercise

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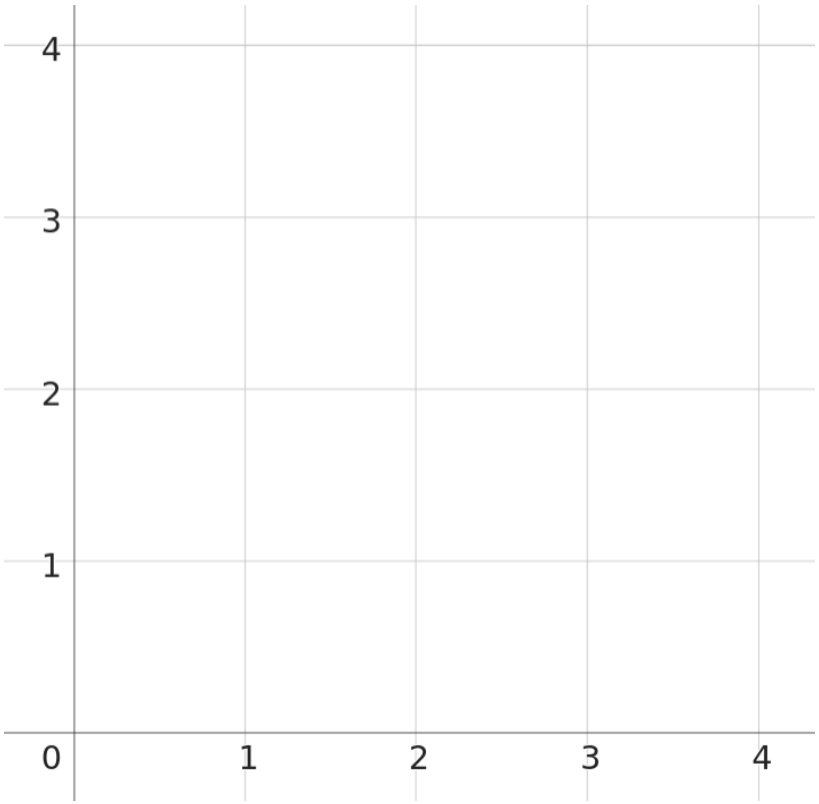
1. $A \cup B$
2. $A \cap B$
3. A^c
4. $(B^c)^c$
5. $A \setminus B$
6. $B \setminus A$

1. $\{1, 2, 3, 4, 5, 7, 9\}$
2. $\{1, 3, 5\}$
3. $\{2, 4, 6, 8\}$
4. B
5. $\{7, 9\}$
6. $\{2, 4\}$

Sets

Exercise

Let $A = \{1, 2, 3\}$, $B = \{2, 4\}$. Find $A \times B$, $B \times B$ and sketch them.



Sets

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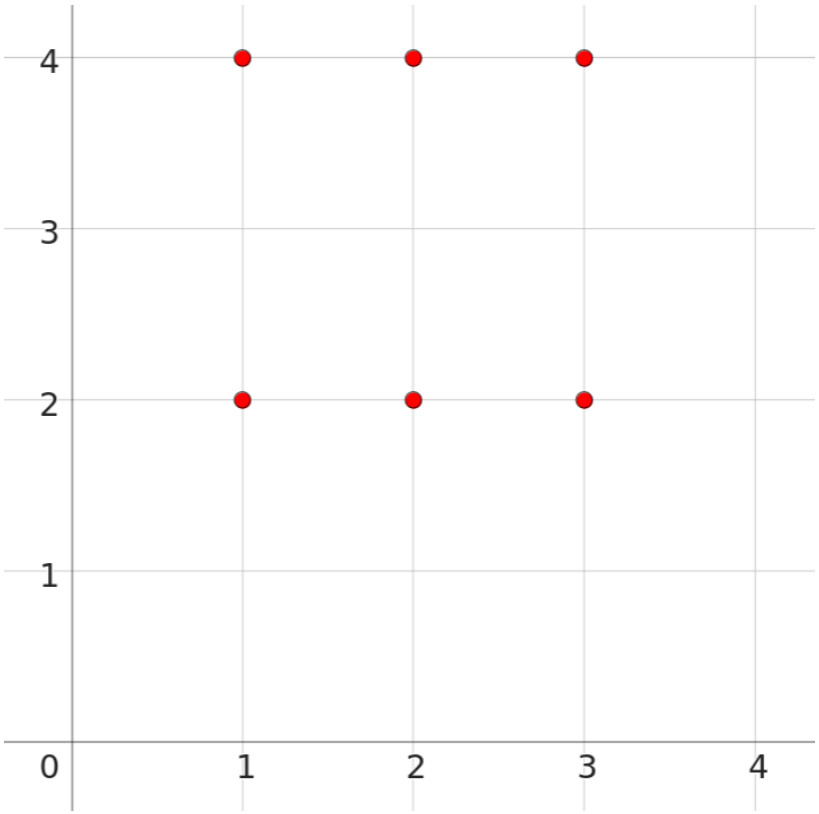

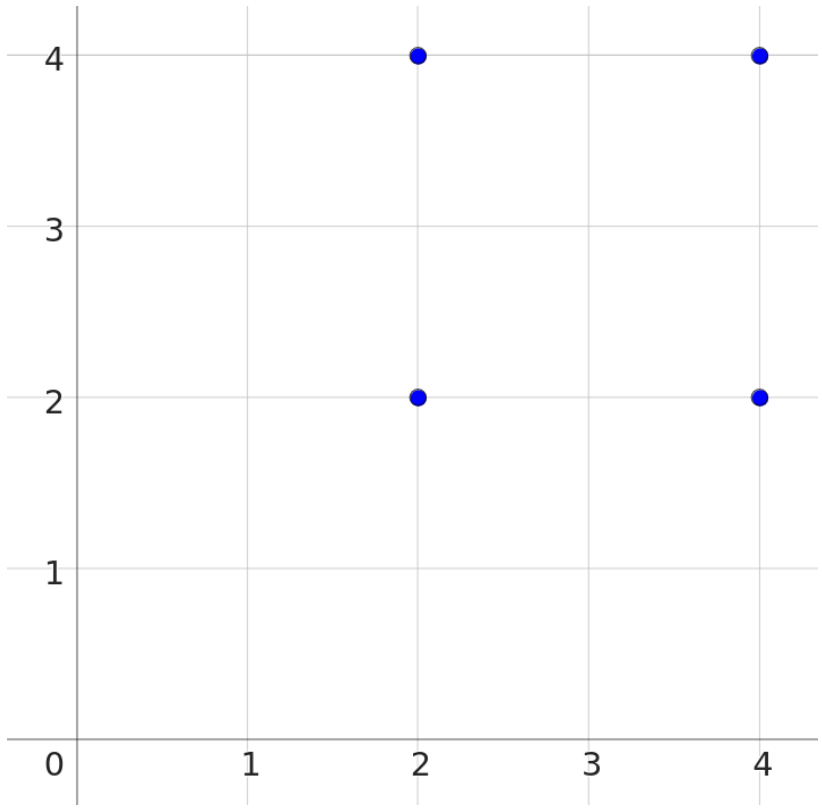

Sets

Exercise

Let $A_1 = \{0, 1\}, A_2 = \{0, 2\}, A_3 = \{0, 3\}$. Find

1. $\bigcup_{i=1}^{3} A_i$
2. $\bigcap_{i \in \{1,2,3\}} A_i$

Sets

Exercise

Let $A_1 = \{0, 1\}, A_2 = \{0, 2\}, A_3 = \{0, 3\}$. Find

1. $\bigcup_{i=1}^{3} A_i$
2. $\bigcap_{i \in \{1,2,3\}} A_i$

$\{0, 1, 2, 3\}, \{0\}$

Exercise

Which sets are bounded from below? Bounded from above? Bounded?

A $\mathbb{N}$

B $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\}$

C $\mathbb{R} \setminus \mathbb{Q} \cap (-3, 2]$

D $\{x \in \mathbb{R} : x < \pi\}$

E $(-\infty, -1) \cup \{0\} \cup [1, \infty)$

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below: A, B, C; above: B, C, D; bounded: B, C

Exercise

Find minimum and maximum:

1. $\{1, 2, 3, 4\}$
2. $[-2, 3]$
3. $(-2, 3]$
4. $[-2, -1) \cup (0, 25]$
5. $[0, \infty)$
6. $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$
7. $\mathbb{N}$
8. $(\mathbb{R} \setminus \mathbb{Q}) \cap [0, \pi]$

Exercise

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1. $\min = 1$, $\max = 4$
2. $-2, 3$
3. $\nexists, 3$
4. $-2, 25$
5. $0, \nexists$
6. $\nexists, 1$
7. $1, \nexists$
8. $\nexists, \pi$

Exercise

Find infimum, minimum, maximum and supremum:

1. $\{1, 2, 3, 4\}$
2. $\{-1, -2, -3, -4\}$
3. $[-2, 3]$
4. $(-2, 3)$
5. $(-2, 3]$
6. $[-2, -1) \cup (0, 25]$
7. $(-7, -0) \cup (1, 2)$
8. $[0, \infty)$
9. $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$
10. $\mathbb{N}$

Exercise

Find infimum, minimum, maximum and supremum:

1. $\{1, 2, 3, 4\}$
2. $\{-1, -2, -3, -4\}$
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5. $(-2, 3]$
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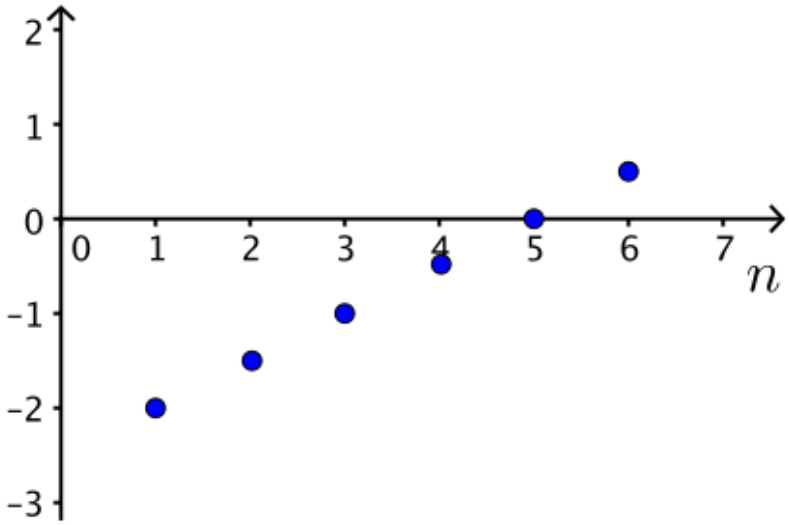
7. $(-7, -0) \cup (1, 2)$
8. $[0, \infty)$
9. $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$
10. $\mathbb{N}$

1. $1, 1, 4, 4$
2. $-4, -4, -1, -1$
3. $-2, -2, 3, 3$
4. $-2, \nexists, \nexists, 3$
5. $-2, \nexists, 3, 3$
6. $-2, -2, 25, 25$
7. $-7, \nexists, \nexists, 2$
8. $0, 0, \nexists, \infty$
9. $0, \nexists, 1, 1$
10. $1, 1, \nexists, \infty$

Exercise

Find the formula for a_n.



A. $a_n = (-\frac{1}{2})^n - \frac{3}{2}$

B. $a_n = \frac{1}{2}n + 5$

C. $a_n = \frac{1}{2}n - 2$

D. $a_n = -\frac{1}{2}n + \frac{5}{2}$

E. $a_n = \frac{1}{2}n - \frac{5}{2}$

Figure:
https://www.cpp.edu/conceptests/question-library/mat116.shtml

Exercise

Find the formula for a_n.

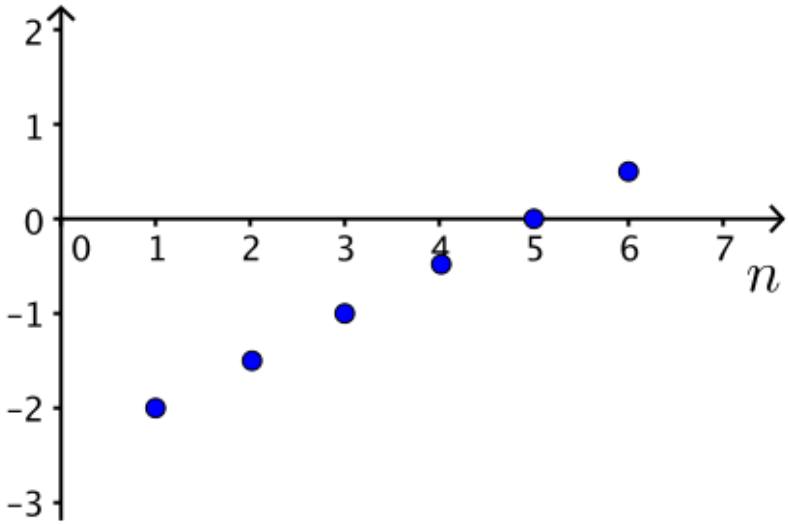


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Figure:
https://www.cpp.edu/conceptests/question-library/mat116.shtml

E

Exercise

Find the first 4 terms of the sequences

A $a_n = \dfrac{(-1)^n}{n}$

B $a_n = \dfrac{n+1}{n}$

Exercise

Find the first 4 terms of the sequences

A $a_n = \dfrac{(-1)^n}{n}$

$-1, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}$

B $a_n = \dfrac{n+1}{n}$

$2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$

Exercise

Find the first 4 terms of the sequences

A $a_n = \dfrac{(-1)^n}{n}$

$-1, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}$

B $a_n = \dfrac{n+1}{n}$

$2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$

Exercise

Find the formula for the following sequences

A $1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \cdots$

B $-1, \dfrac{1}{2}, \dfrac{-1}{3}, \dfrac{1}{4}, \dfrac{-1}{5} \cdots$

Exercise

Find the first 4 terms of the sequences

A $a_n = \dfrac{(-1)^n}{n}$

$-1, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}$

B $a_n = \dfrac{n+1}{n}$

$2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$

Exercise

Find the formula for the following sequences

A $1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \cdots$

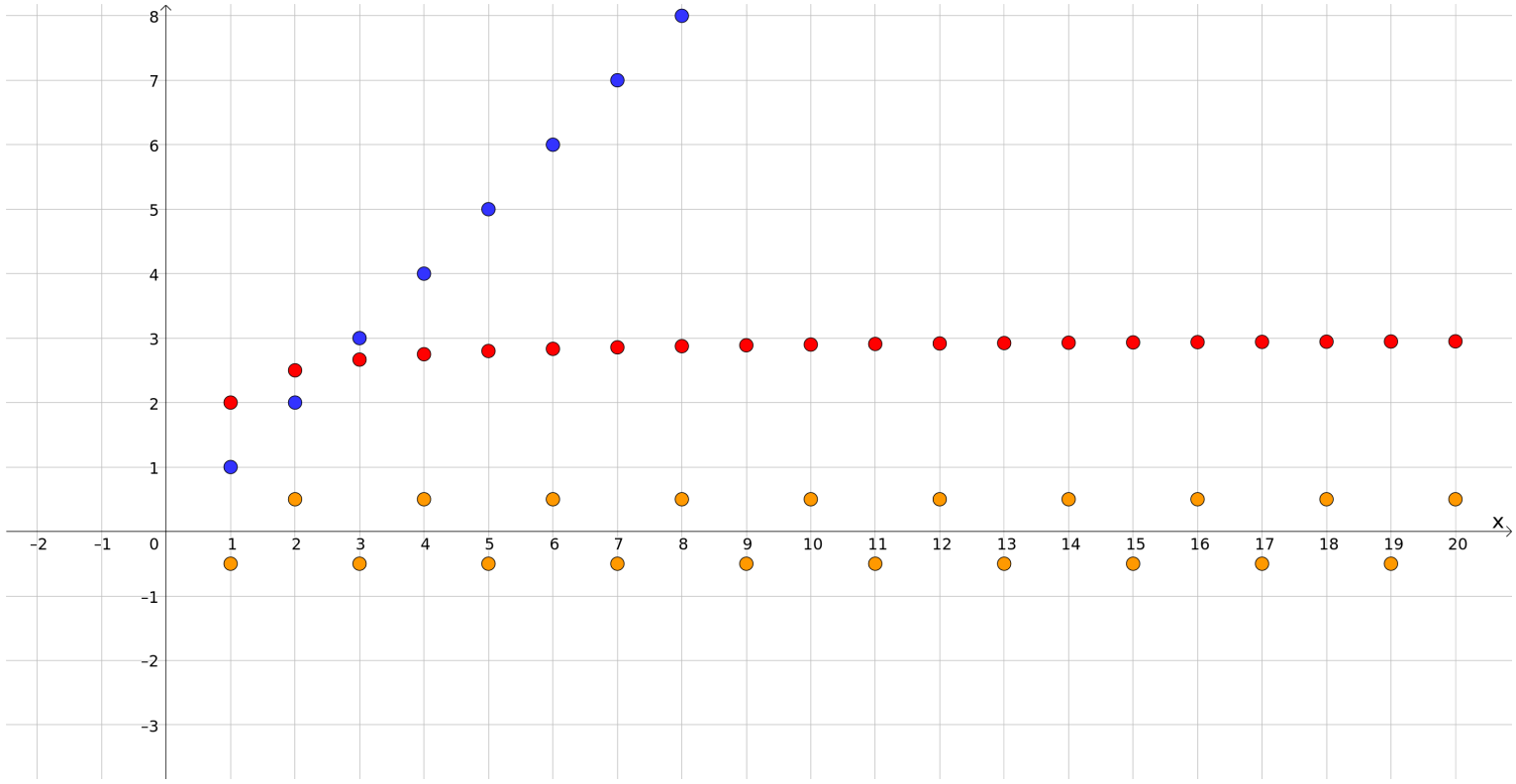
$\frac{1}{2^{n-1}}$

B $-1, \dfrac{1}{2}, \dfrac{-1}{3}, \dfrac{1}{4}, \dfrac{-1}{5} \cdots$

$\frac{(-1)^n}{n}$

Exercise

Which of these sequences are bounded?



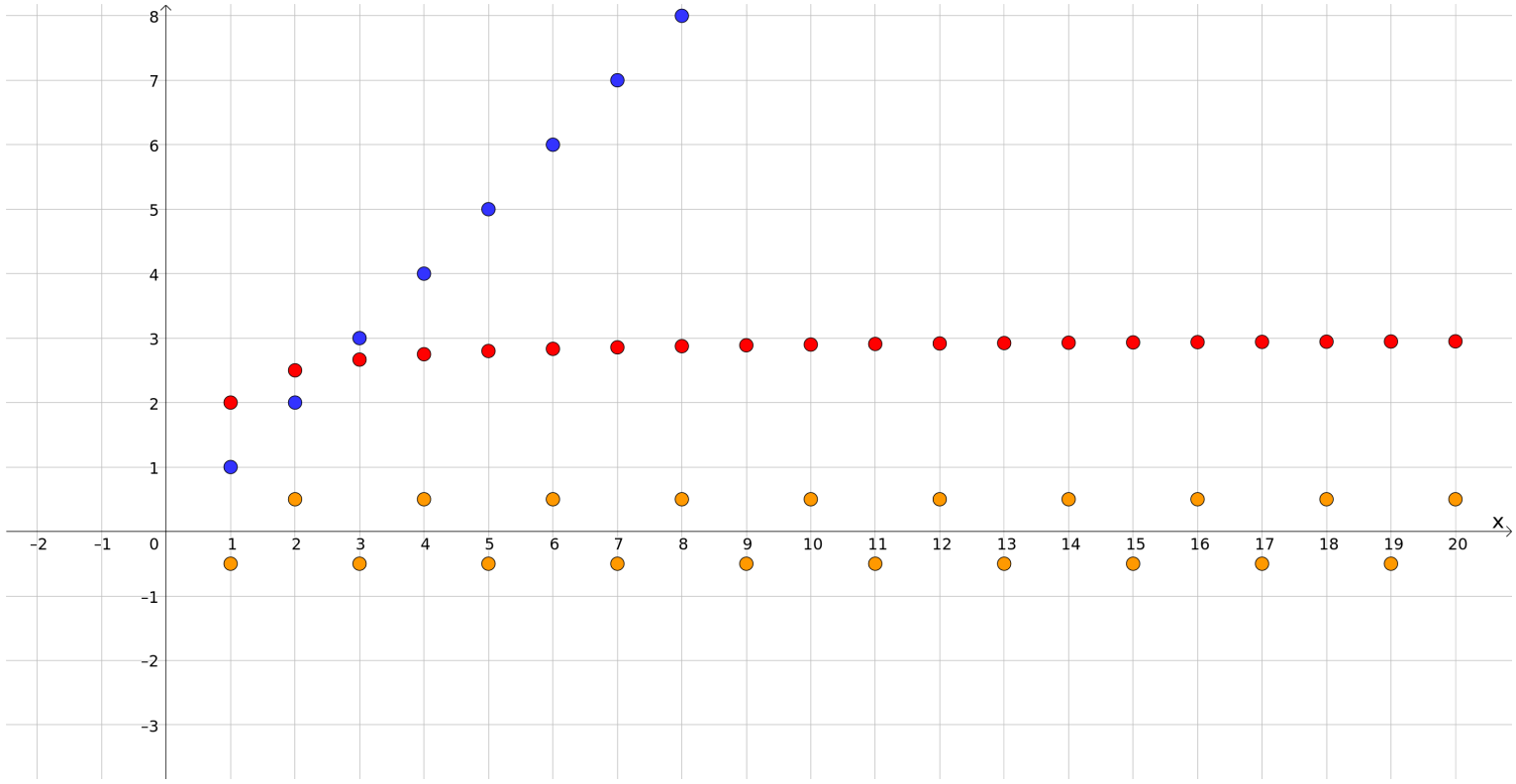
A blue

B red

C yellow

Exercise

Which of these sequences are bounded?



A blue

B red

C yellow

B, C

Exercise

Find non-decreasing sequences.

A $a_n = -4$

B $a_n = (-2)^n$

C $a_n = \dfrac{(-1)^n}{3^n}$

D $a_n = \log n$

E $a_n = e^{-n}$

Exercise

Find non-decreasing sequences.

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A, D

Exercise

Use the definition and check, if the sequence is monotone:

1. $a_n = \dfrac{n}{n+1}$

2. $a_n = \dfrac{n}{4+n^2}$

Exercise

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$$
\begin{aligned}
? \; a_n &\leq a_{n+1} \\
\frac{n}{n+1} &\leq \frac{n+1}{n+2} \\
n(n+2) &\leq (n+1)(n+1) \\
n^2 + 2n &\leq n^2 + 2n + 1 \\
0 &\leq 1
\end{aligned}
$$

https://www.geogebra.org/calculator/w4twpbu2

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2. $a_n = \dfrac{n}{4+n^2}$

$$
\begin{aligned}
? \; a_n &\geq a_{n+1} \\
\frac{n}{4+n^2} &\geq \frac{n+1}{4+(n+1)^2} \\
n(4+n^2+2n+1) &\geq (n+1)(4+n^2) \\
4n+n^3+2n^2+n &\geq 4n+n^3+4+n^2 \\
n^2+n &\geq 4
\end{aligned}
$$

true for $n \geq 2$.

https://www.geogebra.org/calculator/w4twpbu2

Exercise

Let $a_n = 1, 2, 3, 4, 5, \ldots$, $b_n = (-1)^n$. Find

A $a_n + b_n$ B a_n / b_n C $3a_n$

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$a_n = 1, 2, 3, 4, 5 \ldots$
$b_n = -1, 1, -1, 1, -1 \ldots$

A: $0, 3, 2, 5, 4 \ldots$
B: $-1, 2, -3, 4, -5 \ldots$
C: $3, 6, 9, 12, 15 \ldots$

Exercise

Find a sequence, which is

1. bounded and covergent
2. bounded and divergent
3. unbounded and covergent
4. unbounded and divergent

Exercise

Find a sequence, which is

1. bounded and covergent
2. bounded and divergent
3. unbounded and covergent
4. unbounded and divergent

1. $\frac{1}{n}$, $a_n = 42$
2. $a_n = (-1)^n$, $a_n = \sin n$
3. impossible
4. $a_n = n$, $a_n = (-1)^n n^2$

Exercise

Let $a_n = 3, 7, 4, 1/2, \pi, -1$. Find $b_n = a_{2n}$:

A $6, 14, 8 \ldots$

B $5, 9, 6 \ldots$

C $7, 1/2, -1 \ldots$

D $4, 1/2, \pi \ldots$

By:https://www.cpp.edu/conceptests/question-library/mat116.shtm

Exercise (True or false)

Let $\lim a_n = A \in \mathbb{R}$ and $\lim b_n = B \in \mathbb{R}$.
If $a_n < b_n$, then $A < B$.

Exercise (True or false)

Let $\lim a_n = A \in \mathbb{R}$ and $\lim b_n = B \in \mathbb{R}$.

If $a_n < b_n$, then $A < B$. False. Consider $a_n = \frac{1}{n}$, $b_n = -\frac{1}{n}$.

Exercise

Find the sandwich for the sequence $a_n = \dfrac{\cos n}{n}$.

Exercise

Give an example of $a_n \to \infty$ and find its lower bound.

Exercise

Give an example of $a_n \to \infty$ and find its lower bound.

$a_n = \log n$, $b = 0$.

Exercise

1. $2+\infty$
2. $-\infty+3$
3. $\pi\infty$
4. $-4(-\infty)$
5. -7∞
6. $\frac{\infty}{-3}$
7. $\frac{5}{\infty}$

Exercise

Find a sequence $\{x_n\}$ for a set $M = [2, 5)$.

Exercise

Find a sequence $\{x_n\}$ for a set $M = [2, 5)$.

$x_n = 4, 4.5, 4\frac{2}{3}, 4.75 \ldots, x_n = 5 - \frac{1}{n}$

Exercise

Find a convergent subsequence:

A $a_n = (-1)^n$

B $a_n = \{0, 2, 0, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 2, \ldots\}$

Exercise

Find a convergent subsequence:

A $a_n = (-1)^n$

B $a_n = \{0, 2, 0, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 2, \ldots\}$

1. $1, 1, 1, \ldots$
2. $0, 0, 0, \ldots$

Exercise

Find the domain and range for the following mappings:

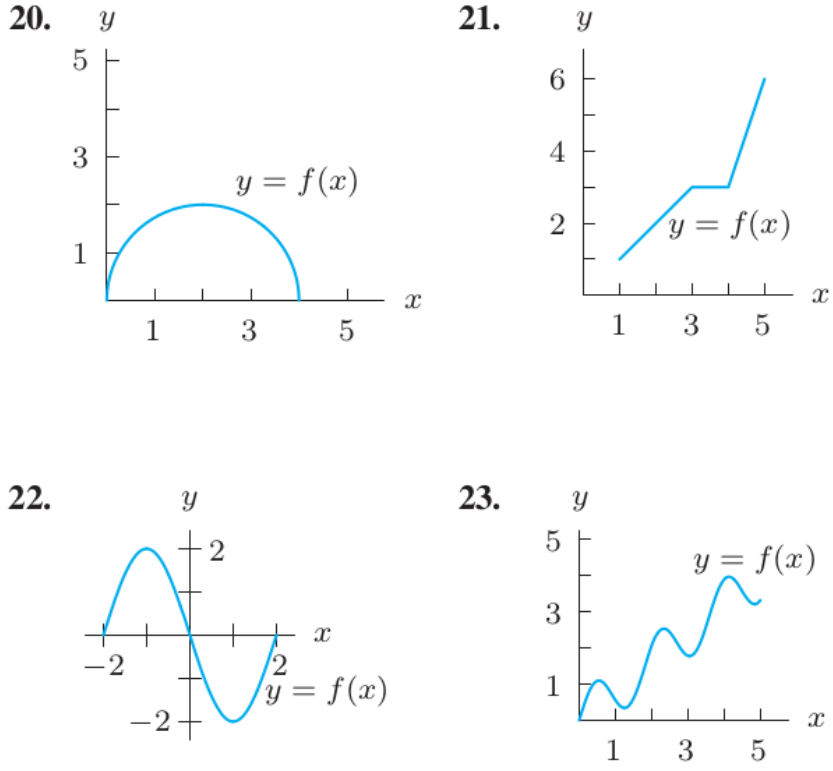

Figure: Calculus: Single and Multivariable, 6th Edition, Hughes-Hallett, col.

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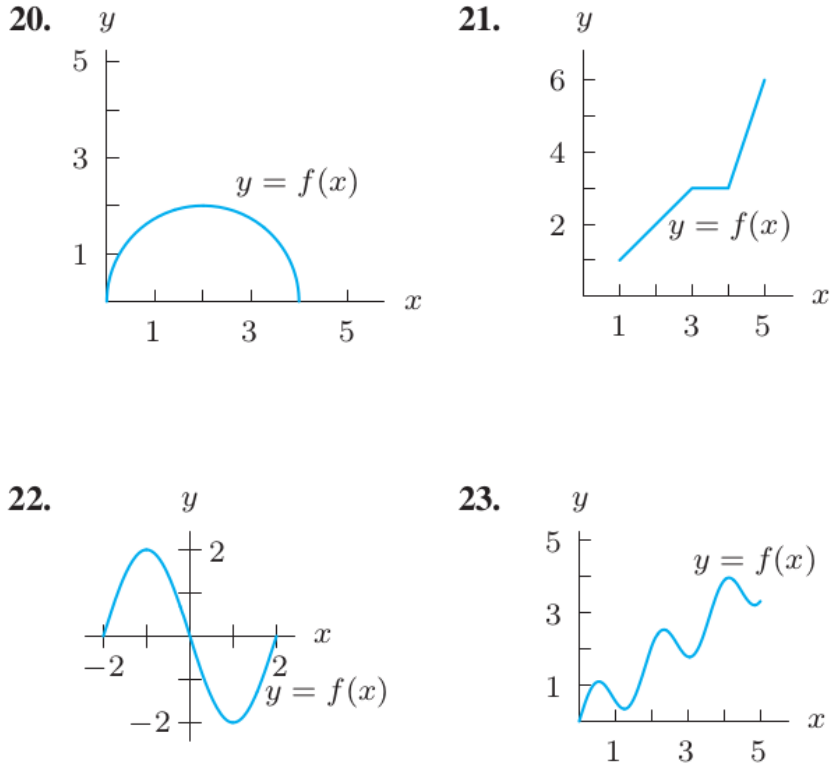

Figure: Calculus: Single and Multivariable, 6th Edition, Hughes-Hallett, col.

20. $[0, 4]$, $[0, 2]$

21. $[1, 5]$, $[1, 6]$

22. $[-2, 2]$, $[-2, 2]$

23. $[0, 5]$, $[0, 4]$

Exercise

Which of the following functions has its domain the same as its range?

A x^2 B $\sqrt{x}$ C x^3 D $|x|$ E $2x-3$

(Inspired by: Active Calculus & Mathematical Modeling, Carroll College Mathematics Department)

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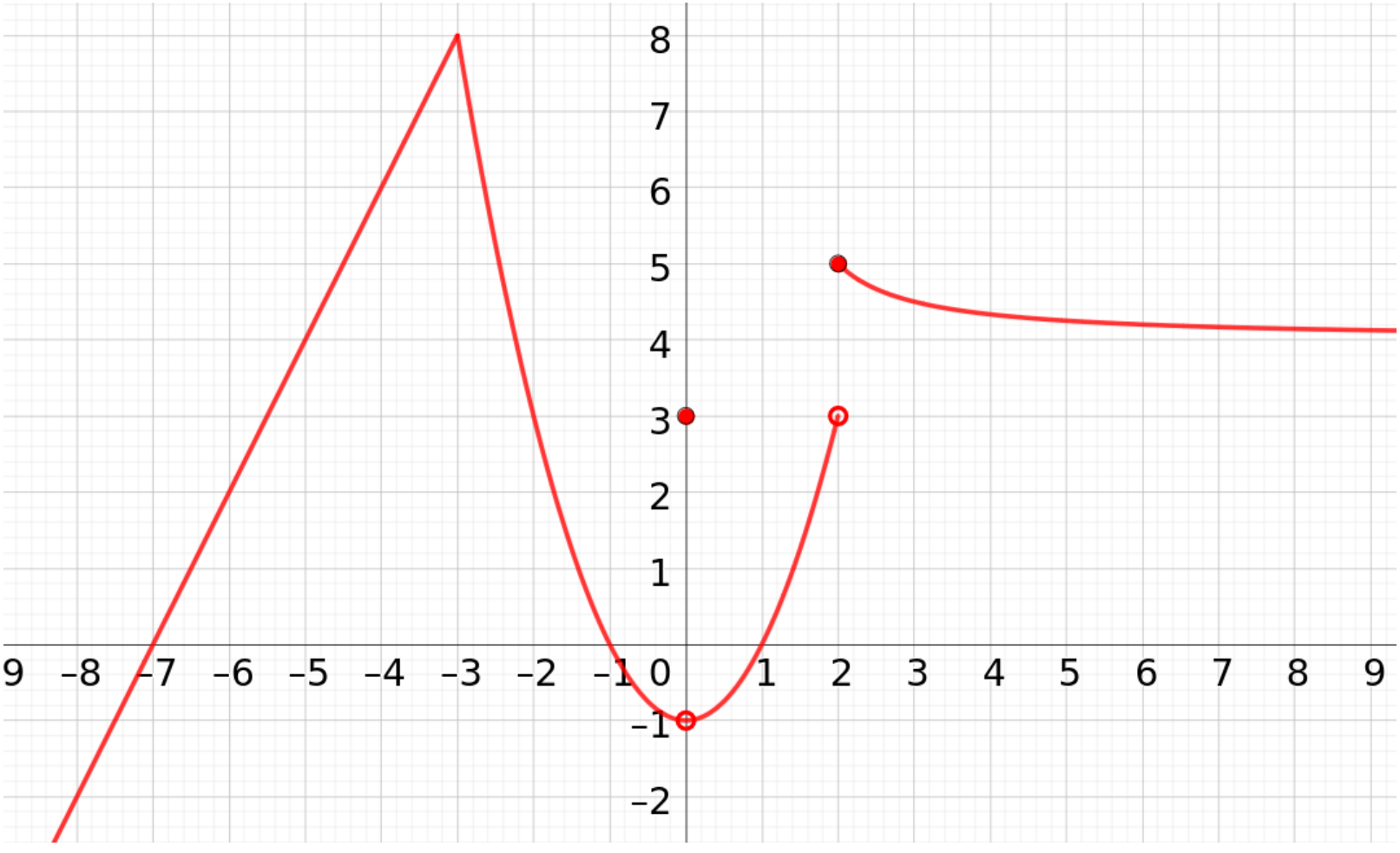
(Inspired by: Active Calculus & Mathematical Modeling, Carroll College Mathematics Department)

B, C, E

Exercise

Find the image:

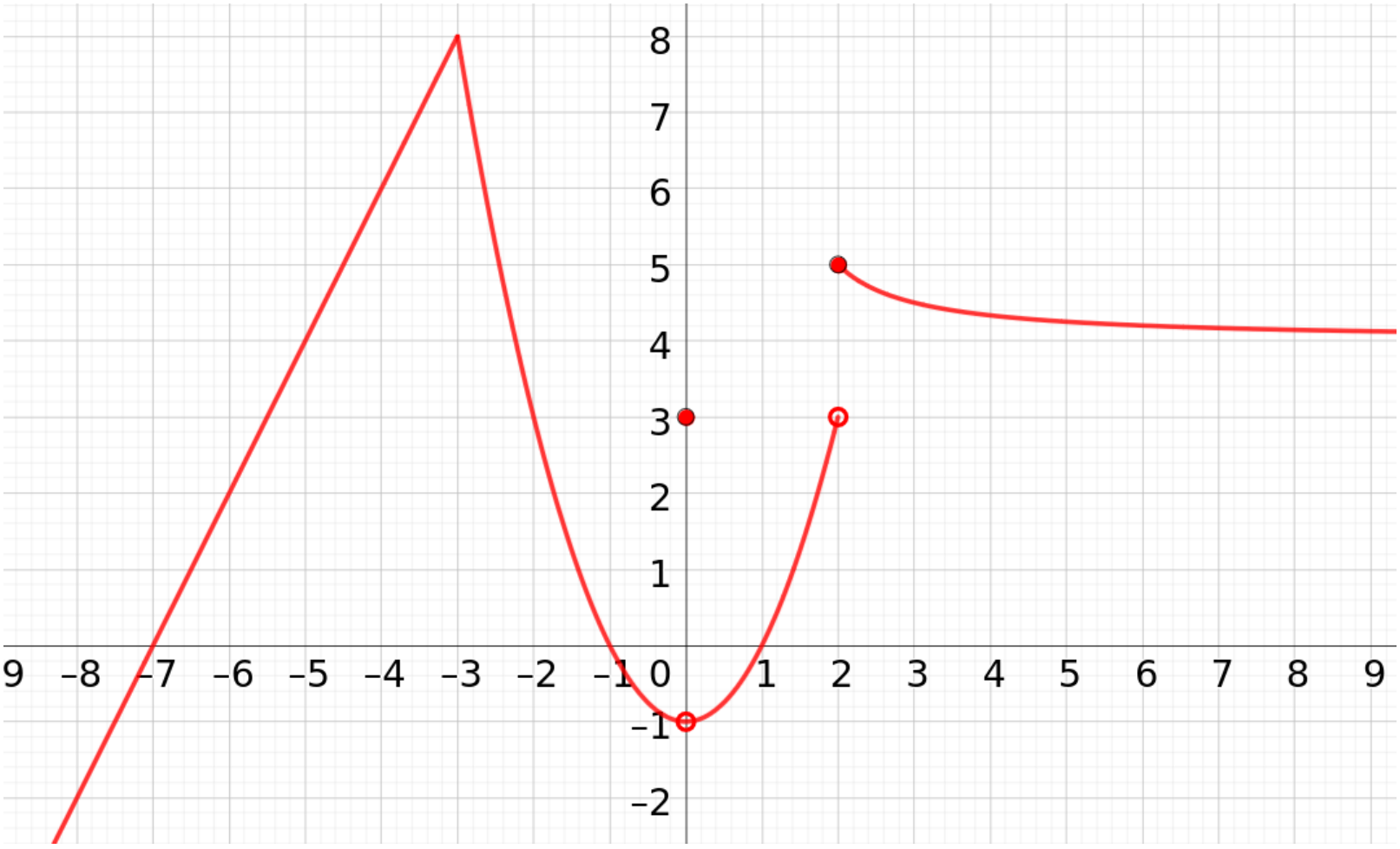
A $[-6, -2]$ B $[-1, 1)$ C $[0, 2)$ D $[2, \infty)$



Exercise

Find the image:

A $[-6,-2]$ B $[-1,1)$ C $[0,2)$ D $[2,\infty)$

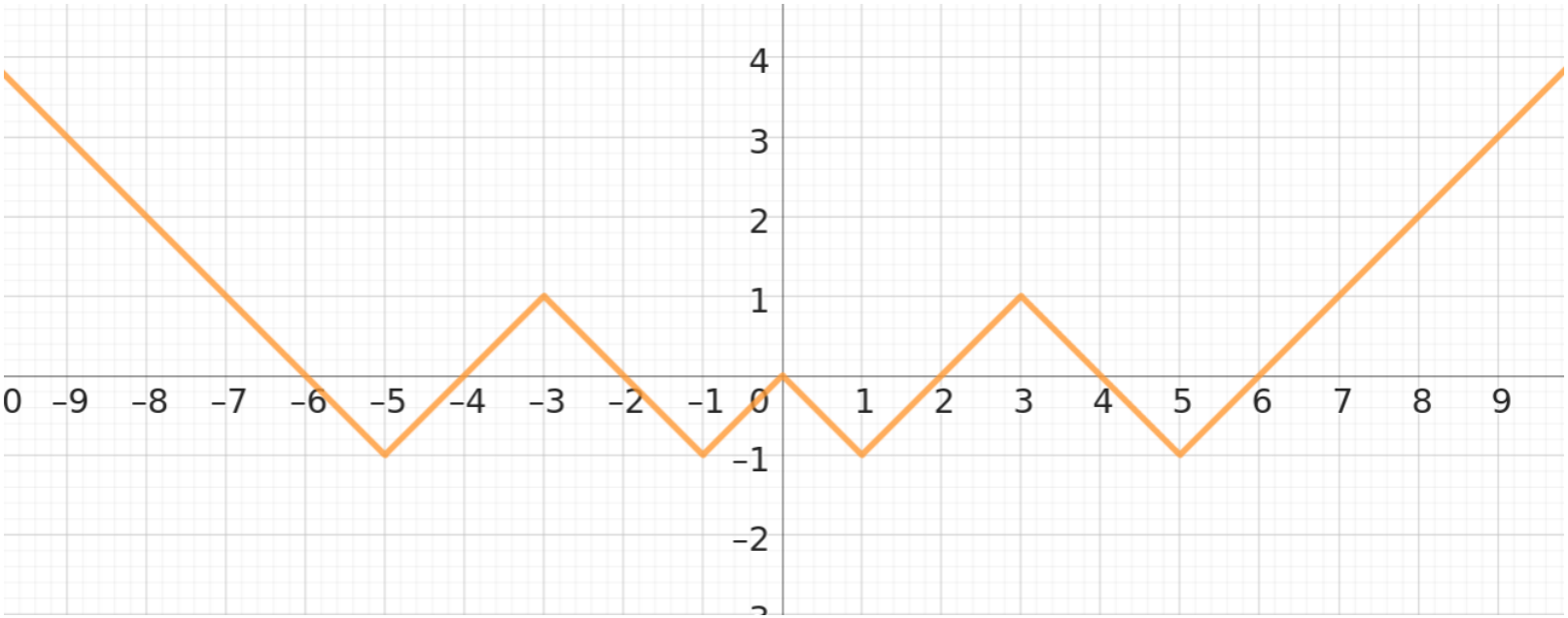


A $[2,8]$, B $(-1,0]\cup\{3\}$, C $(-1,3]$, D $(4,5]$.

Exercise

Find the preimage:

A $\{-1\}$ B $[2,3]$ C $[0,1]$ D $[0,1)$



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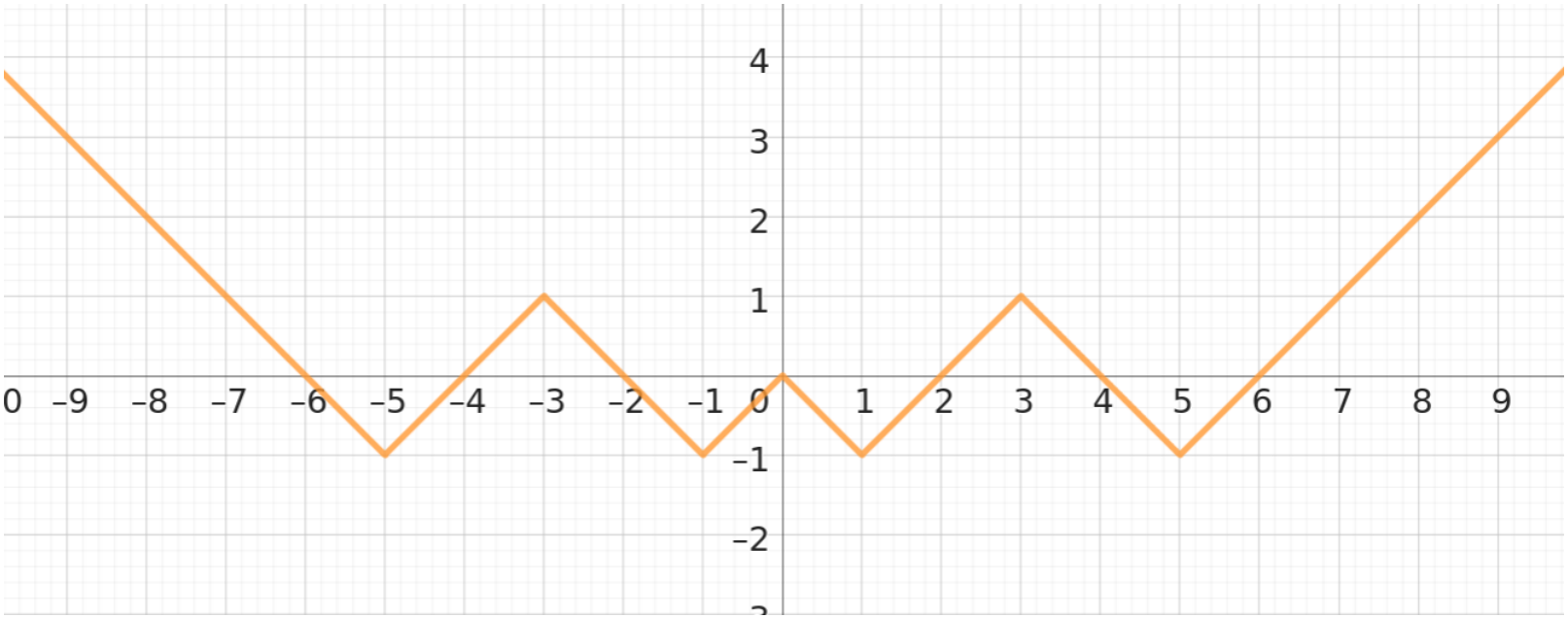
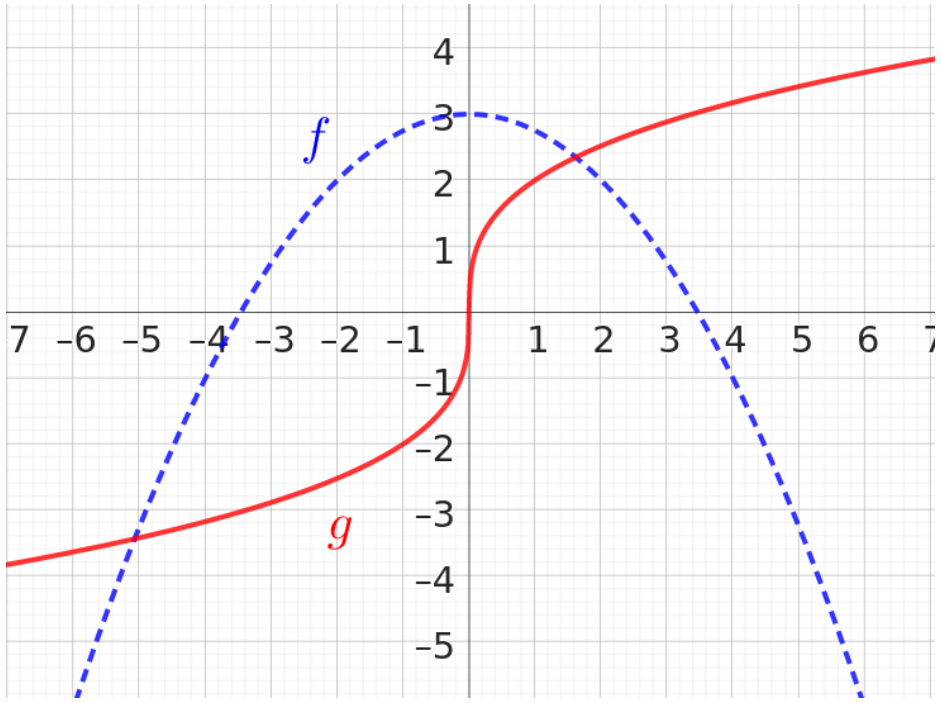


A $\{-5,-1,1,5\}$, B $[-9,-8]\cup[8,9]$,
C $[-7,-6]\cup[-4,-2]\cup\{0\}\cup[2,4]\cup[6,7]$,
D $(-7,-6]\cup[-4,-3)\cup(-3,-2]\cup\{0\}\cup[2,3)\cup(3,4]\cup[6,7)$

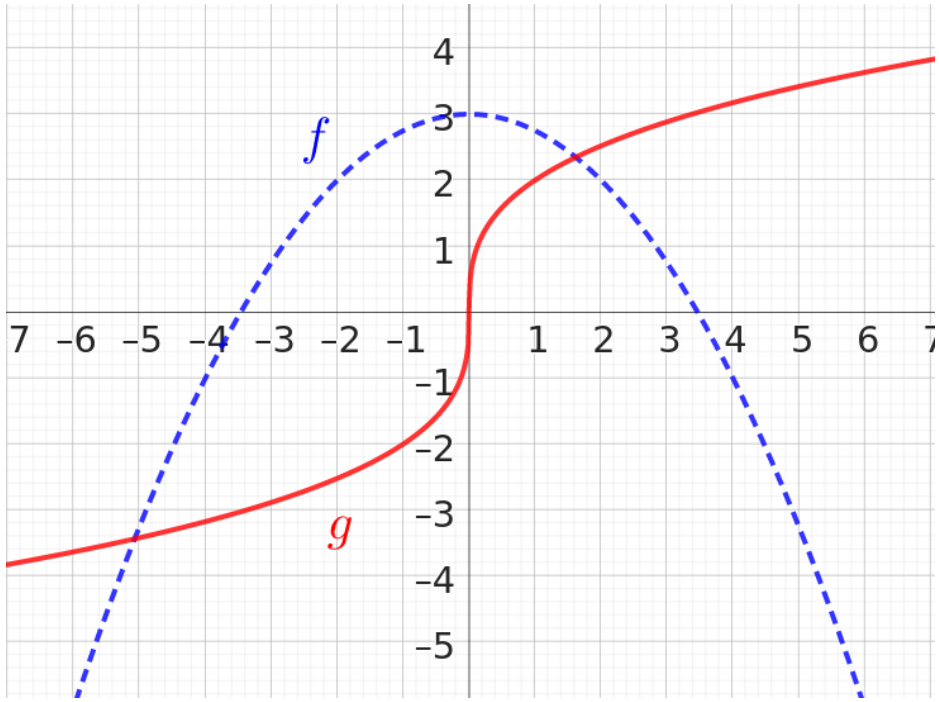
Exercise



Find $g(f(4))$.

A -2 B -1 C 0 D 1 E 2

Exercise



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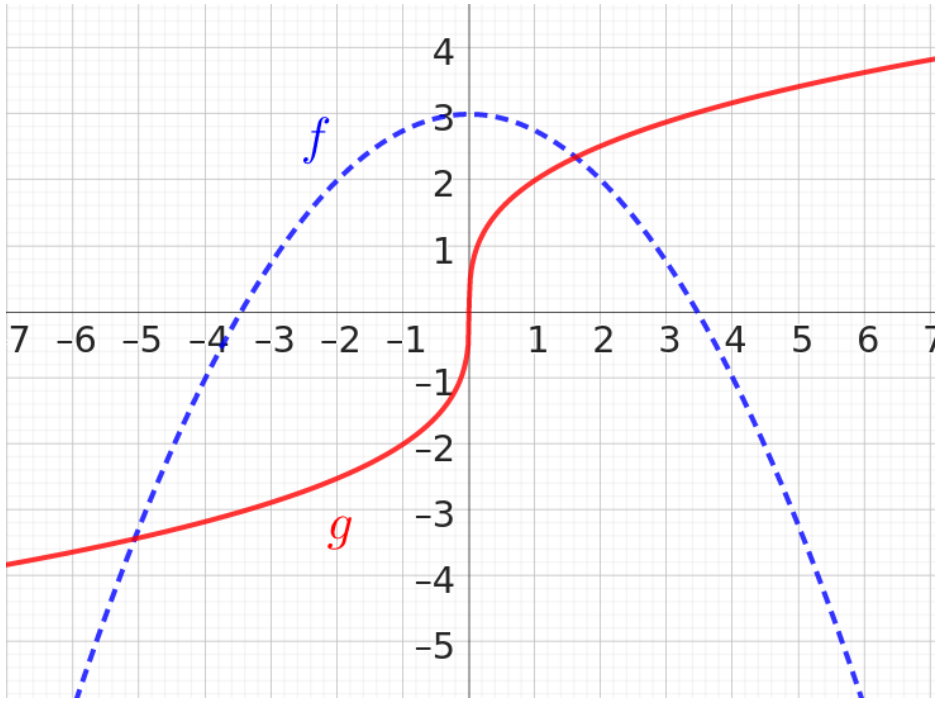
A

Exercise



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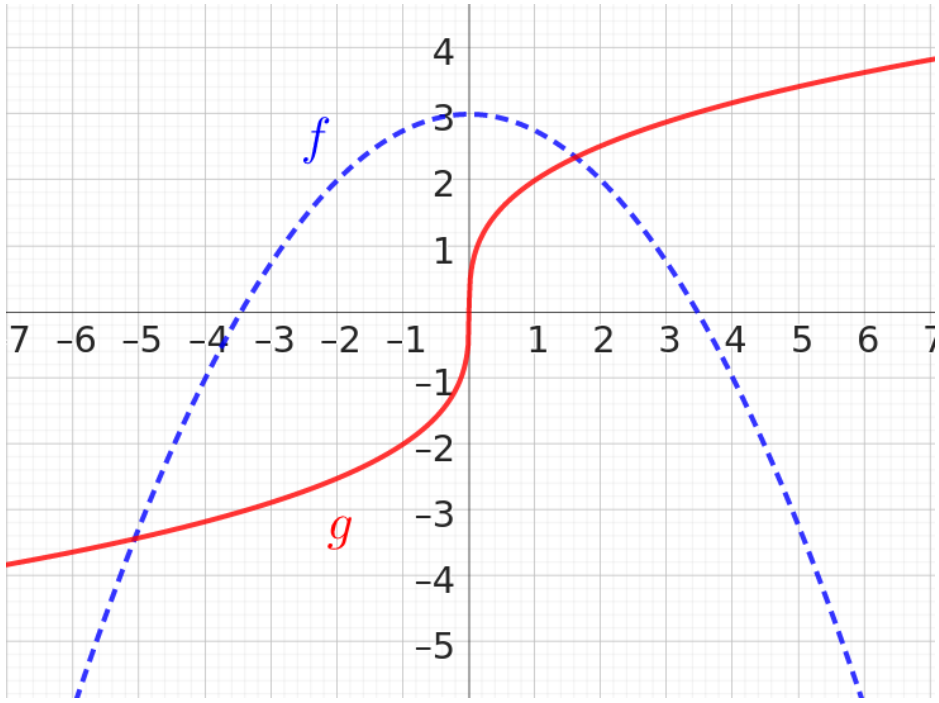
A

Find x, if $f(g(x)) = 2$.

Exercise



Find $g(f(4))$.

A -2 B -1 C 0 D 1 E 2

A

Find x, if $f(g(x)) = 2$.

B, D

Exercise

In the table we can find values of functions f and g.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

Find $g(f(1))$.

A -2 B -1 C 0 D 1 E 2

Exercise

In the table we can find values of functions f and g.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

Find $g(f(1))$.

A -2 B -1 C 0 D 1 E 2

A

Exercise

In the table we can find values of functions f and g.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

Find $g(f(1))$.

A -2 B -1 C 0 D 1 E 2

A

Find $f(f(0))$.

A -2 B -1 C 0 D 1 E 2

Exercise

In the table we can find values of functions f and g.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

Find $g(f(1))$.

A -2 B -1 C 0 D 1 E 2

A

Find $f(f(0))$.

A -2 B -1 C 0 D 1 E 2

D

Exercise

In the table we can find values of functions f and g. If $f(g(x)) = -2$, find x.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

A -2 B -1 C 0 D 1 E 2

Exercise

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$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

A -2 B -1 C 0 D 1 E 2

D

Exercise

A e^x B x^3 C $\sin x$ D $\tan x$ E $\frac{1}{x}$

Which functions are onto $\mathbb{R}$?
Which functions are one-to-one?
Which functions are bijections?

Exercise

A e^x B x^3 C $\sin x$ D $\tan x$ E $\frac{1}{x}$

Which functions are onto $\mathbb{R}$?
Which functions are one-to-one?
Which functions are bijections?
B, D
A, B, E
B

Exercise

Find inverse mappings at $\mathbb{R}$:

A e^x

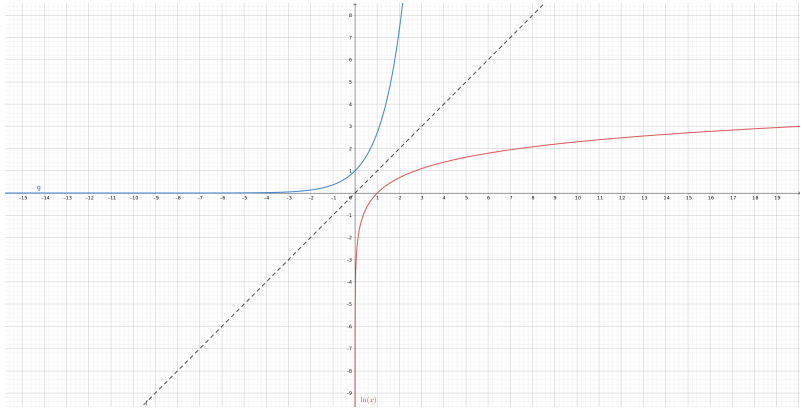
B $2x + 1$

C $\sqrt[3]{x}$

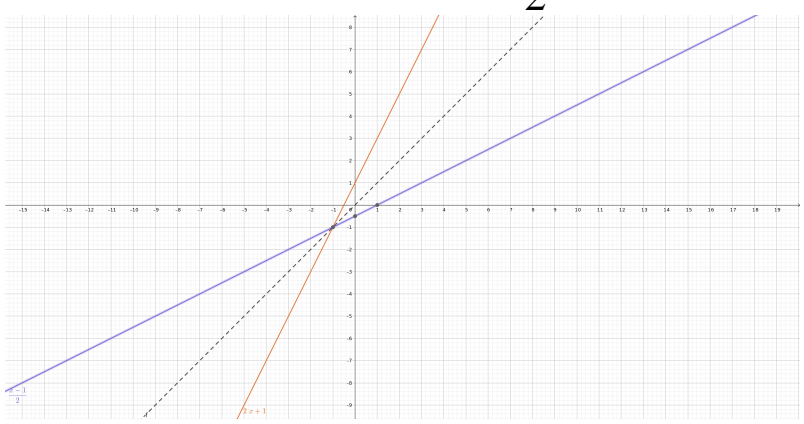
D x^2

Exercise

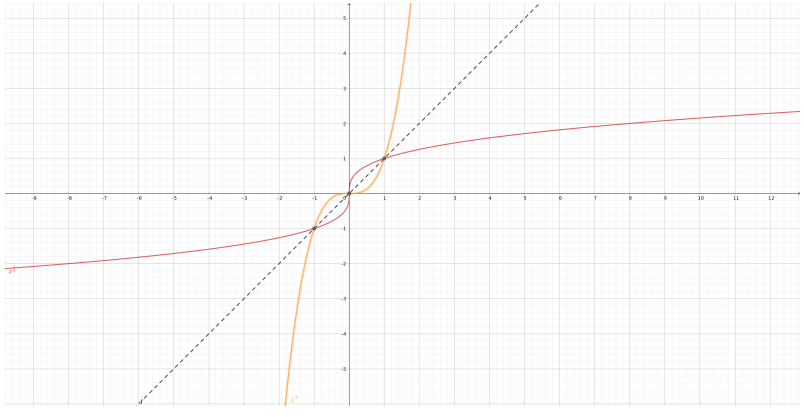
e^x vs $\log x$



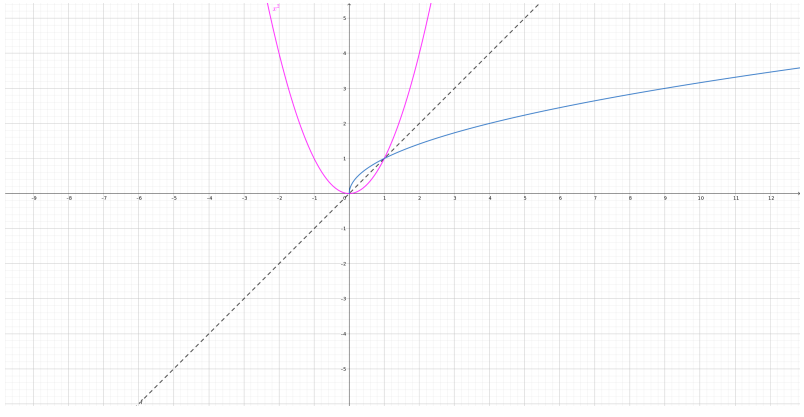
$2x + 1$ vs $\frac{x-1}{2}$



x^3 vs $\sqrt[3]{x}$

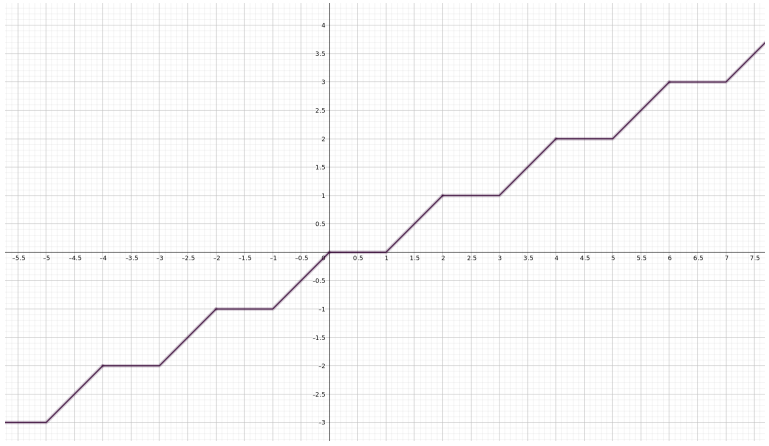


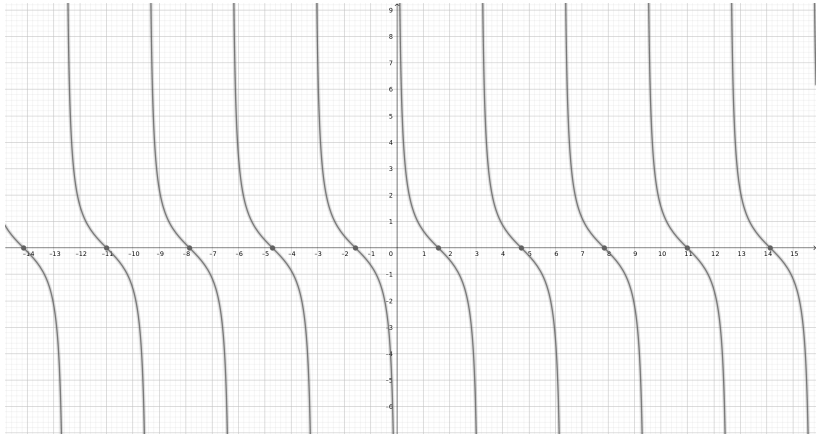
x^2 vs $\sqrt{x}$

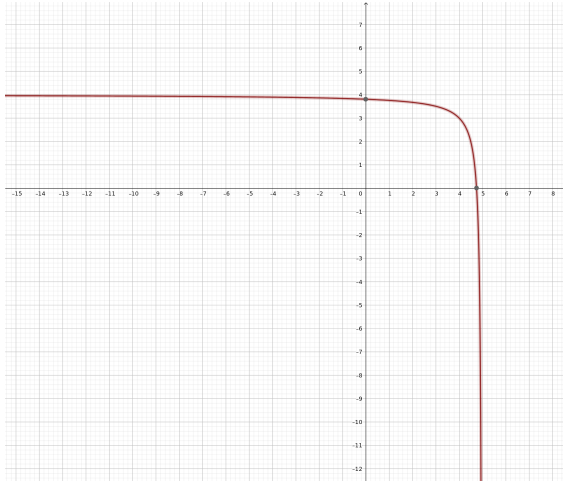


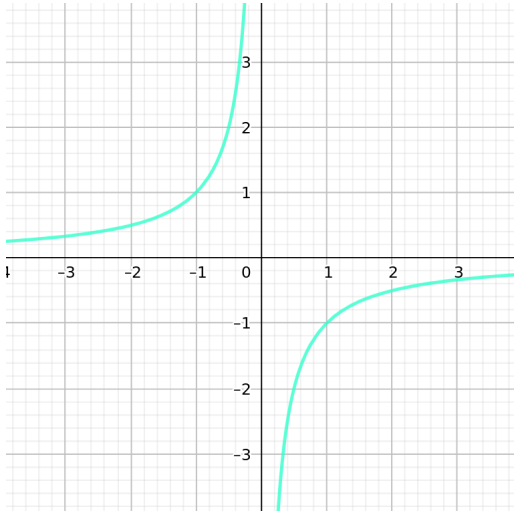
Exercise

Decide, which functions are monotone on its domain:



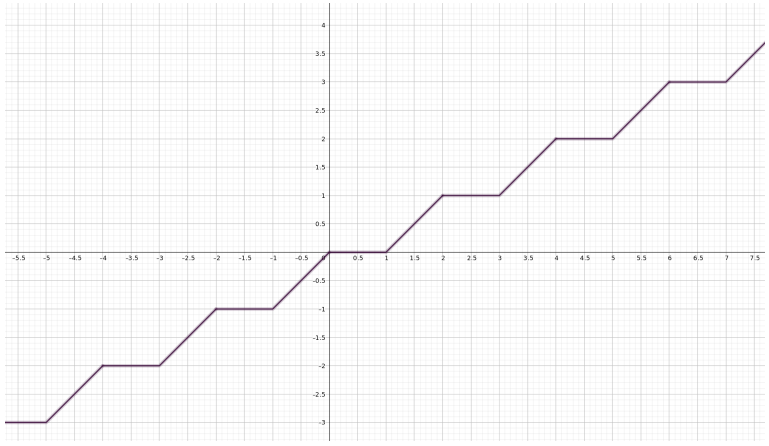


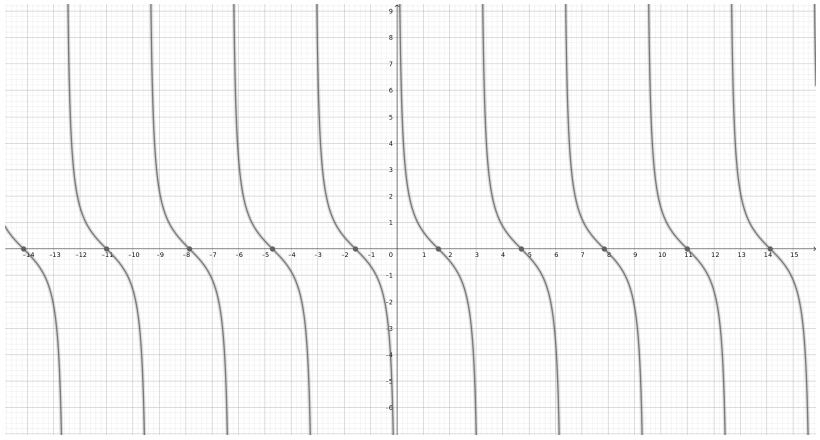


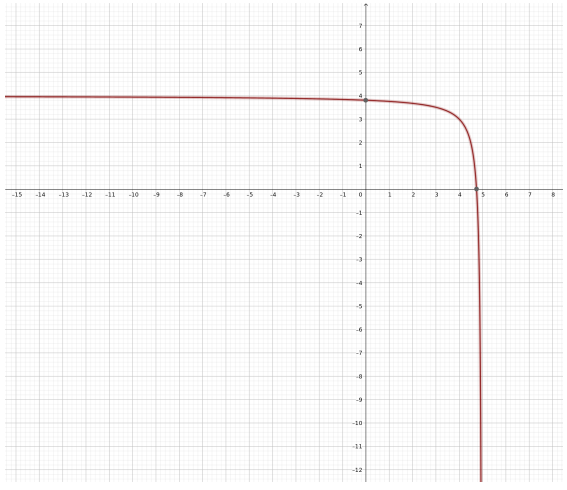


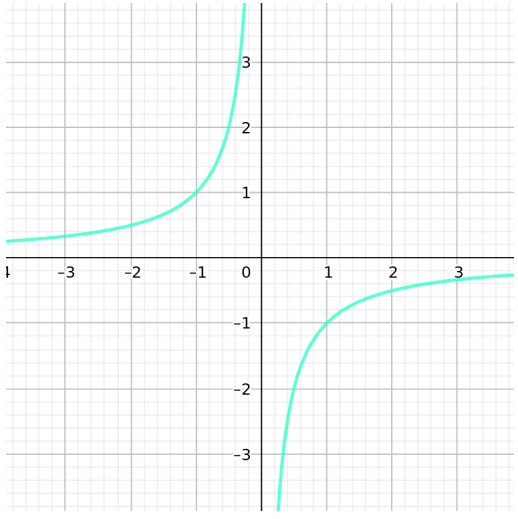
Exercise

Decide, which functions are monotone on its domain:





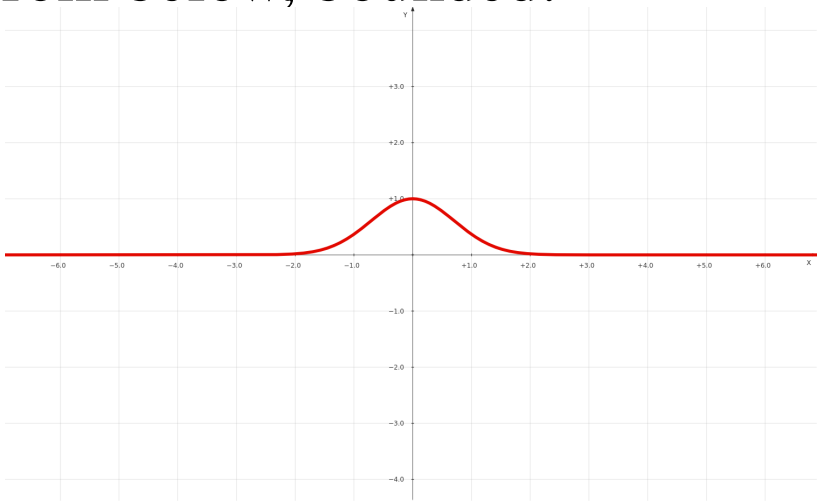


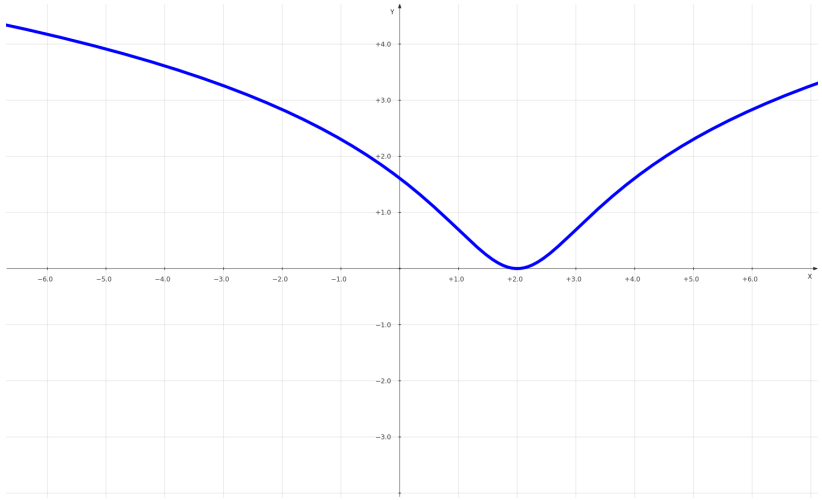


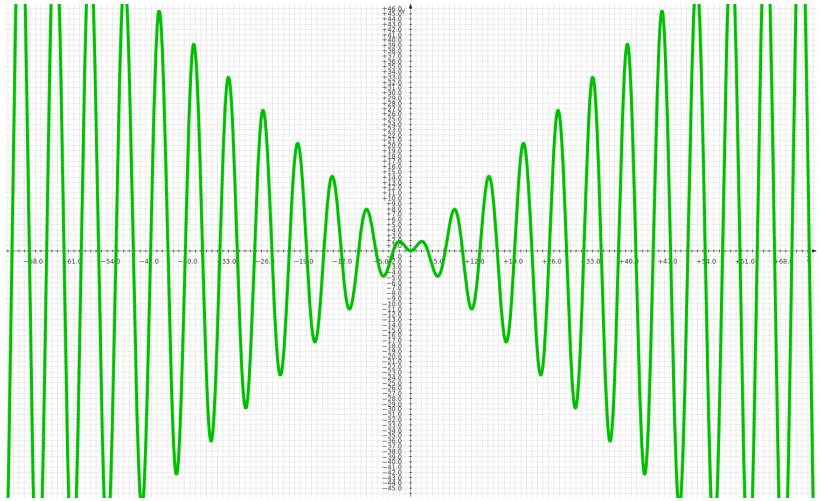
non-decreasing, nothing,
decreasing, nothing

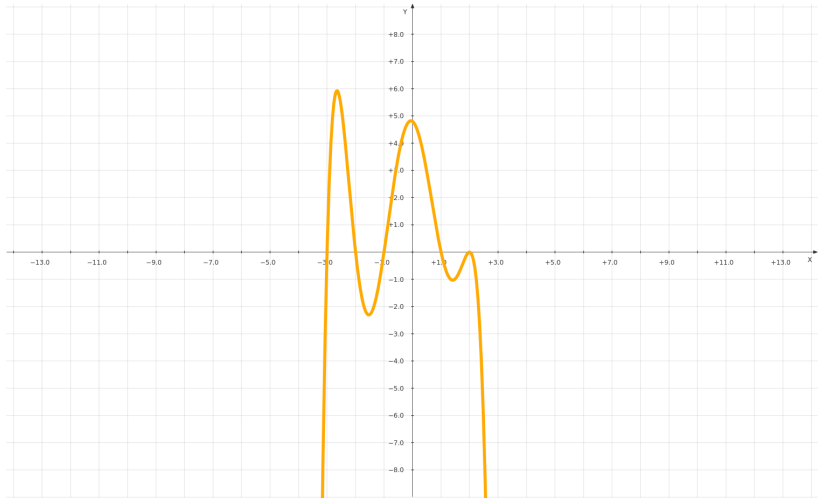
Exercise

Decide, which functions are bounded from above, bounded from below, bounded:



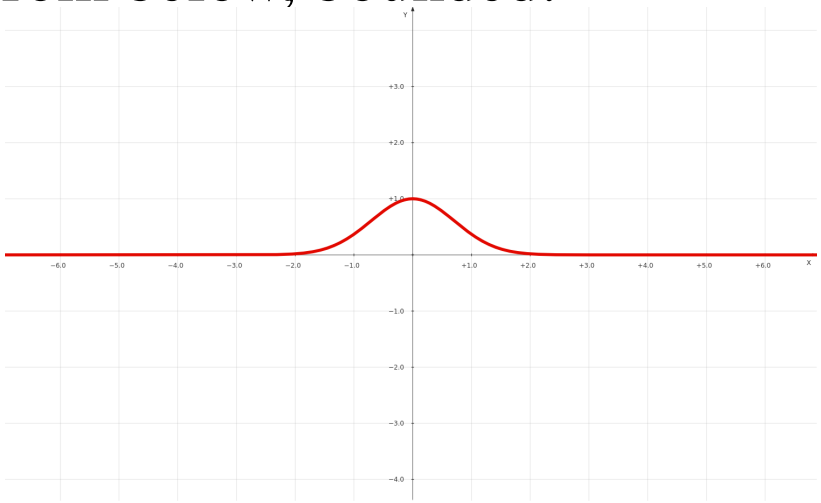


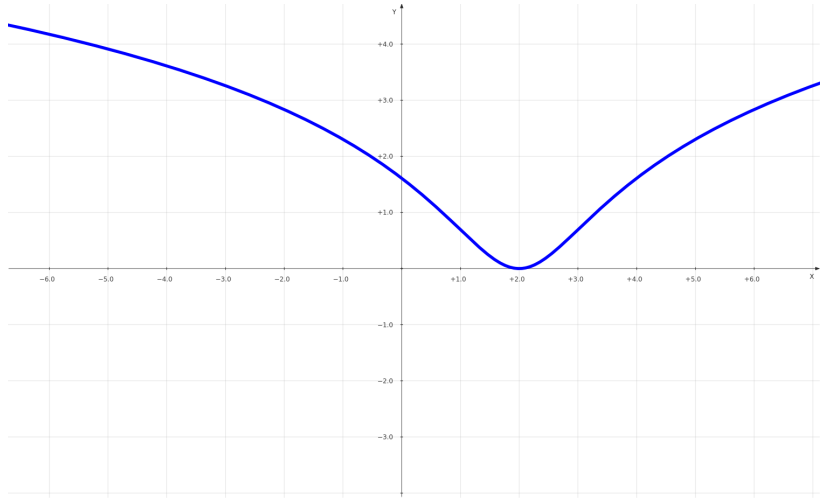


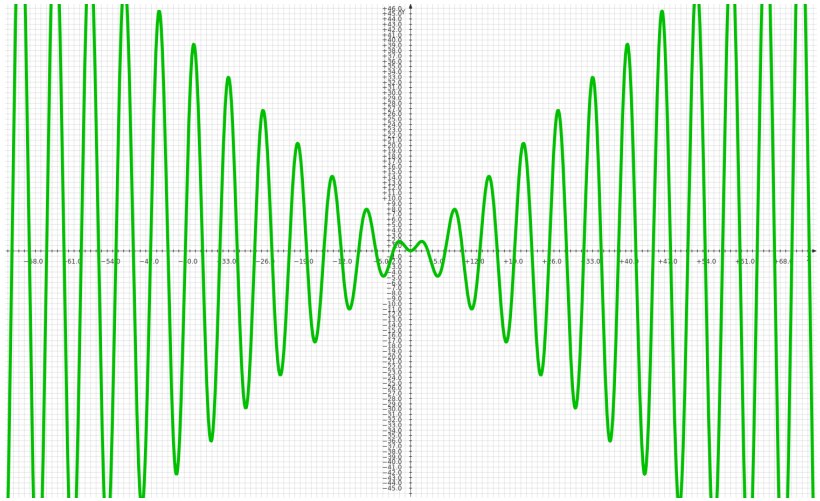


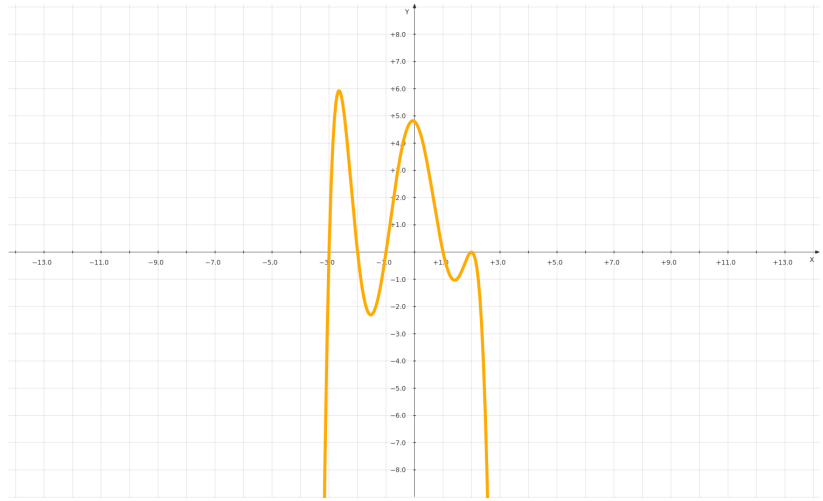
Exercise

Decide, which functions are bounded from above, bounded from below, bounded:





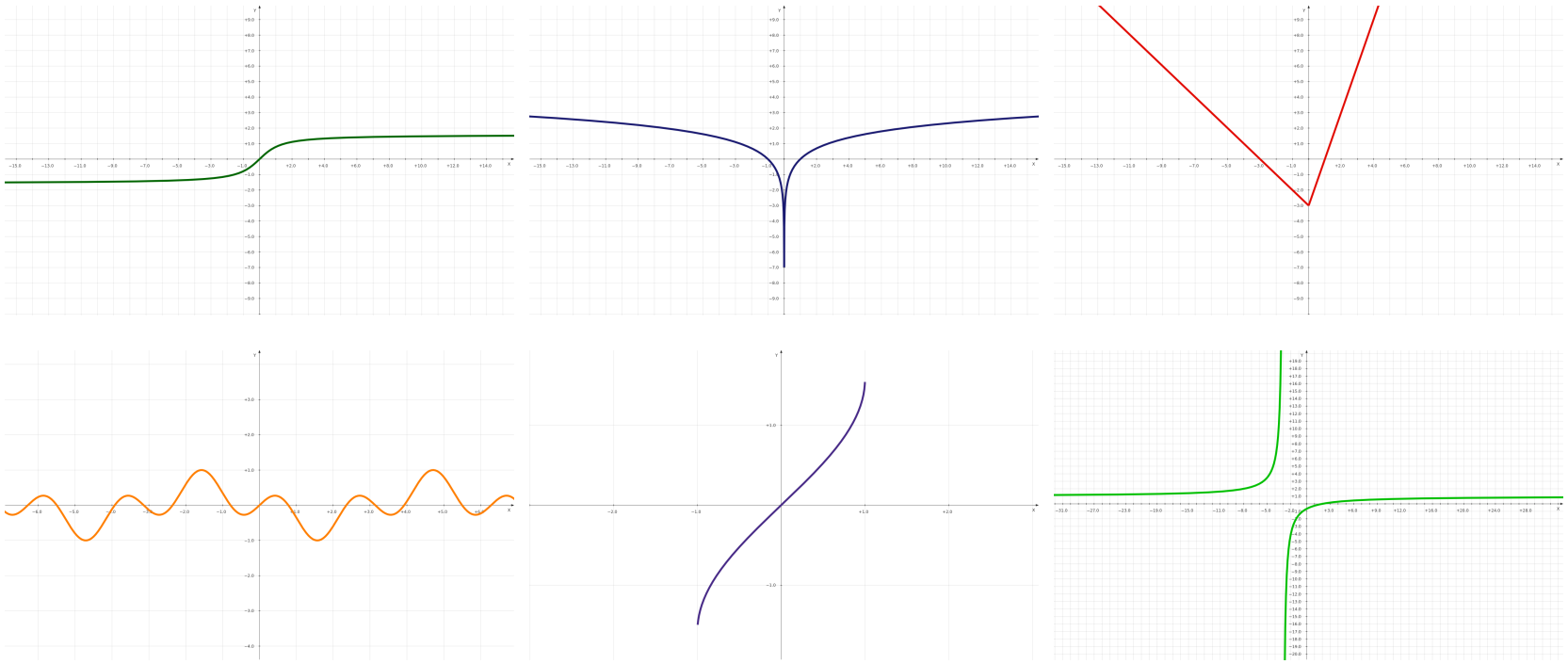




red: bounded, blue: bounded from below,
green: unbounded, yellow: bounded from above

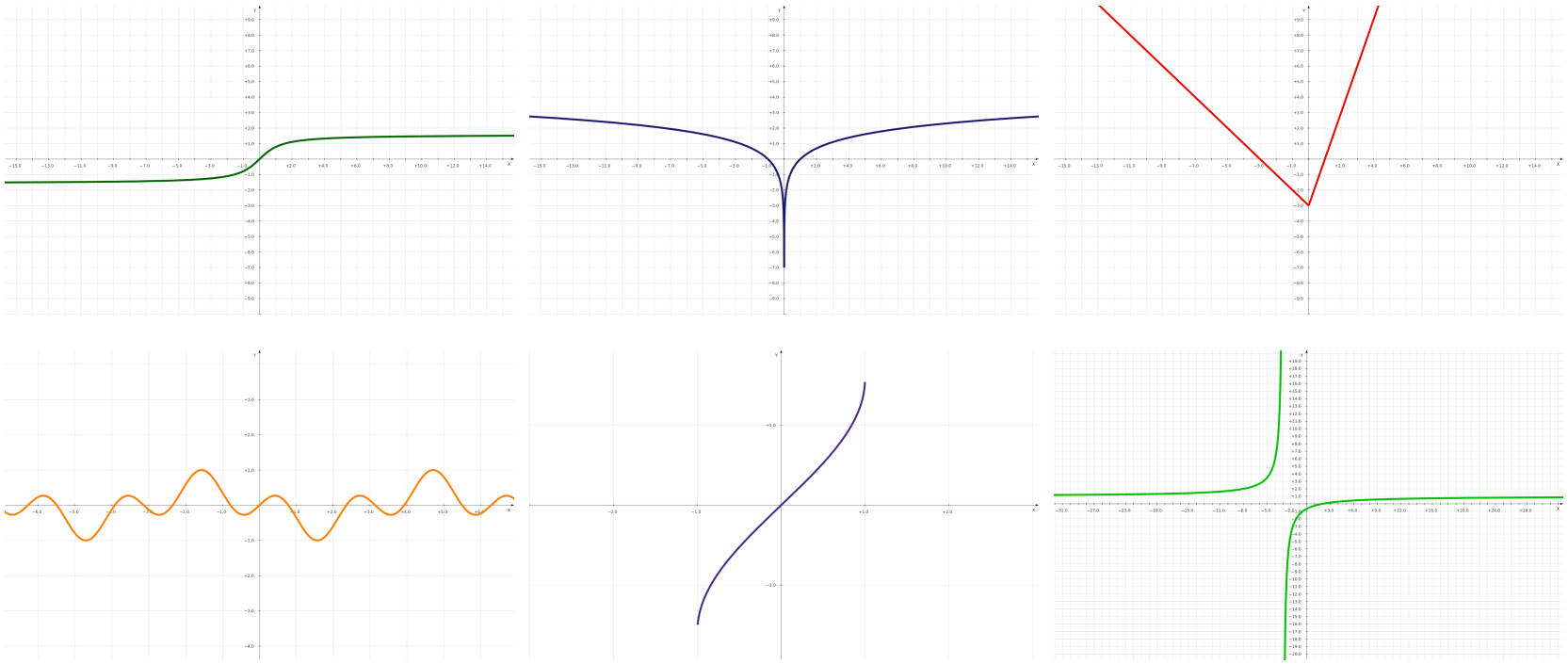
Exercise

Decide, which functions are even or odd:



Exercise

Decide, which functions are even or odd:



A odd, B even, D odd, E odd

Exercise

Decide, which functions are even or odd:

A $x^3 + 1$

B $x(x^2 + 1)$

C $|x - 2|$

D $e^{x^2} \sin x$

E $|1 + \cos x|$

Exercise

Decide, which functions are even or odd:

A $x^3 + 1$

B $x(x^2 + 1)$

C $|x - 2|$

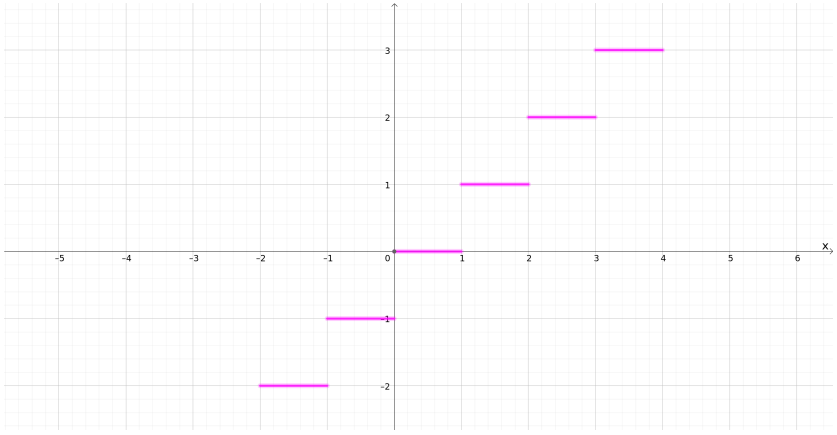
D $e^{x^2} \sin x$

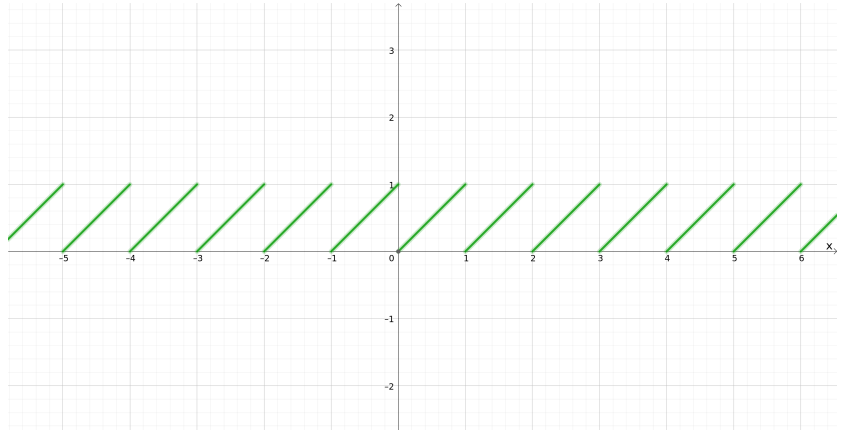
E $|1 + \cos x|$

B odd, D odd, E even

Exercise

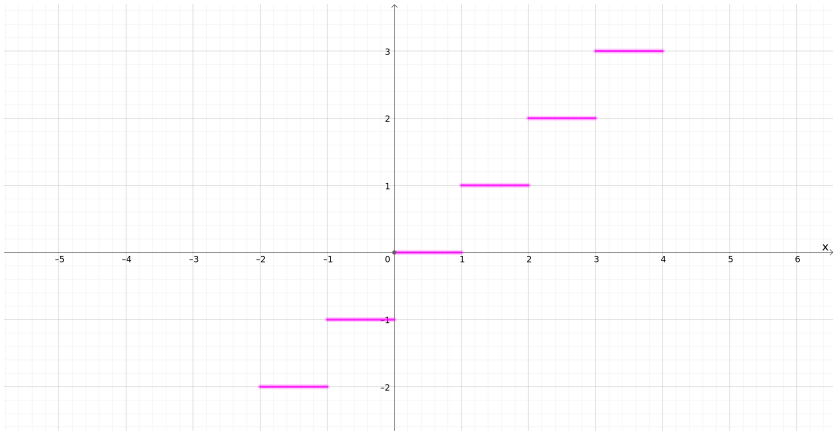
Decide, which functions are periodic

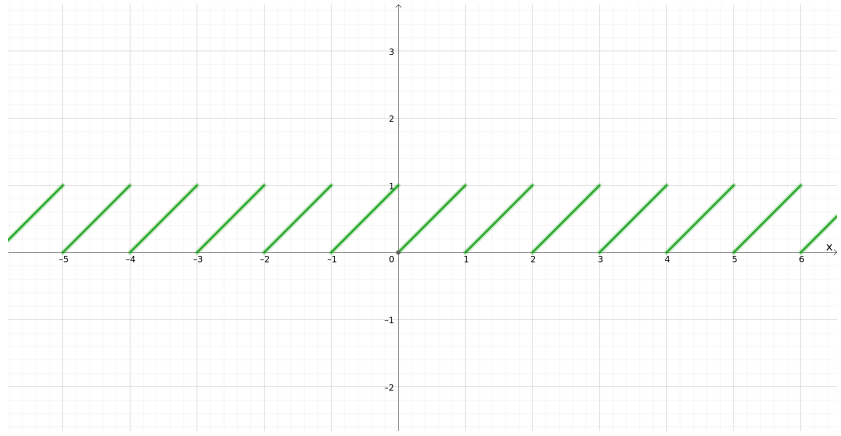




Exercise

Decide, which functions are periodic

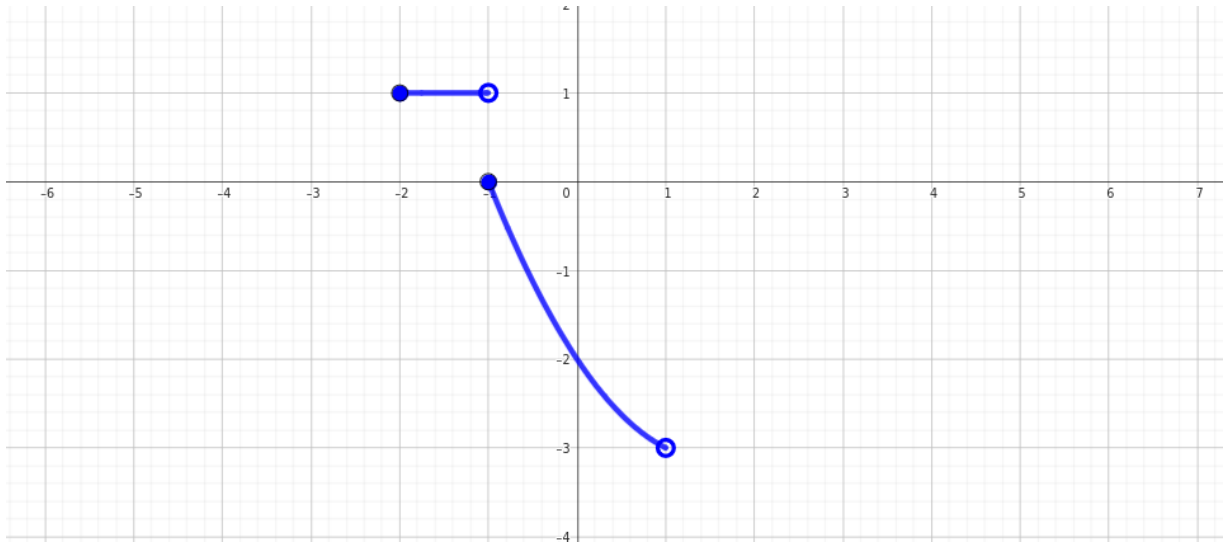




No, yes

Exercise

Sketch in the function so that it is periodic with the smallest possible period



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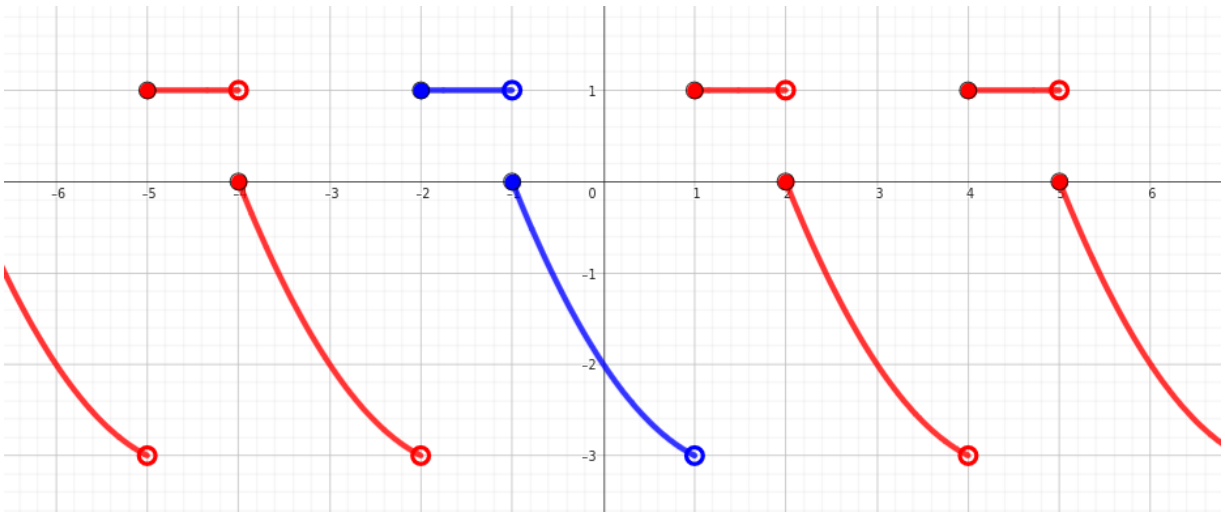

Exercise

Find $\lim_{x \to 0} f(x)$

A -3 B 0 C 5 D 7 E ∞

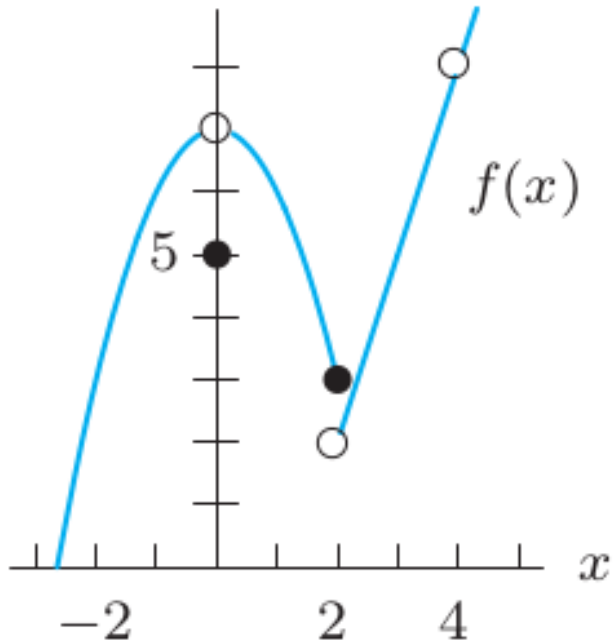

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

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A -3 B 0 C 5 D 7 E ∞

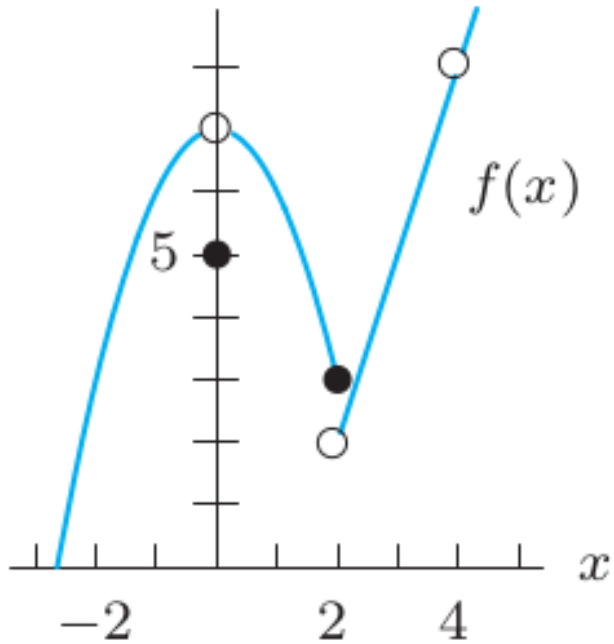

Figure: Calculus: Single and Multivariable, Hughes-Hallet

D

Exercise

Find $\lim_{x \to 2} f(x)$

A ∞

B 3

C 2

D 0

E does not exist

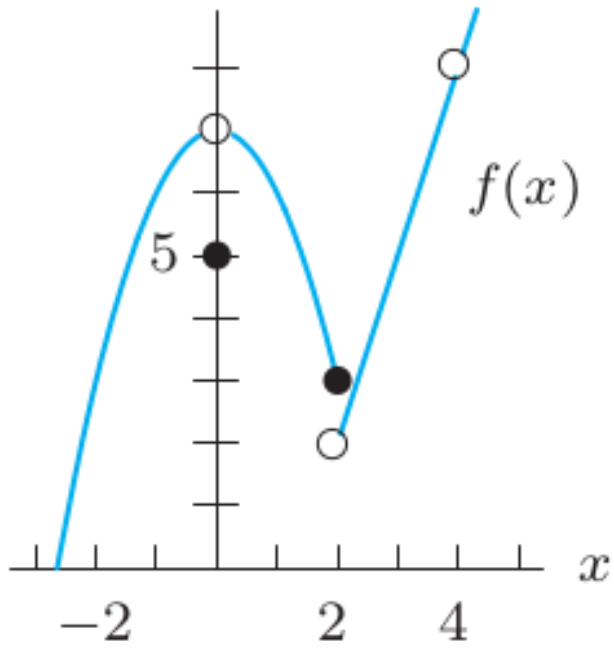

Figure: Calculus: Single and Multivariable, Hughes-Hallet

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A ∞

B 3

C 2

D 0

E does not exist

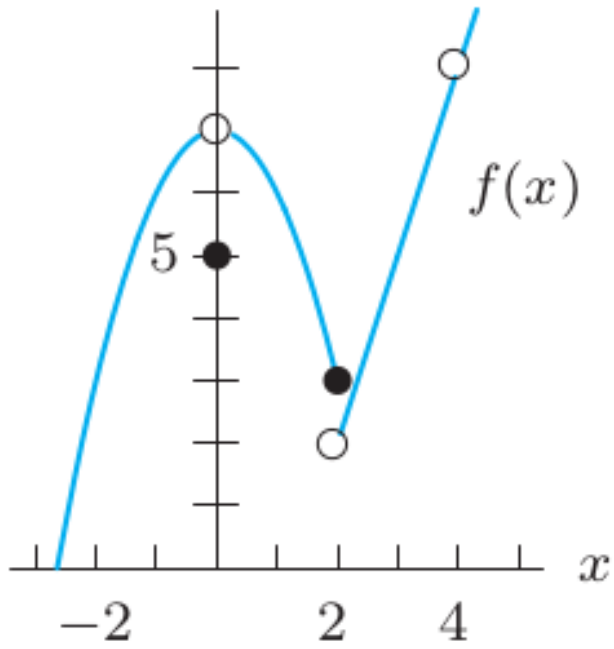

Figure: Calculus: Single and Multivariable, Hughes-Hallet

E

Exercise

Find $\lim_{x \to 4} f(x)$

A 4
B 8
C 0
D ∞
E does not exist

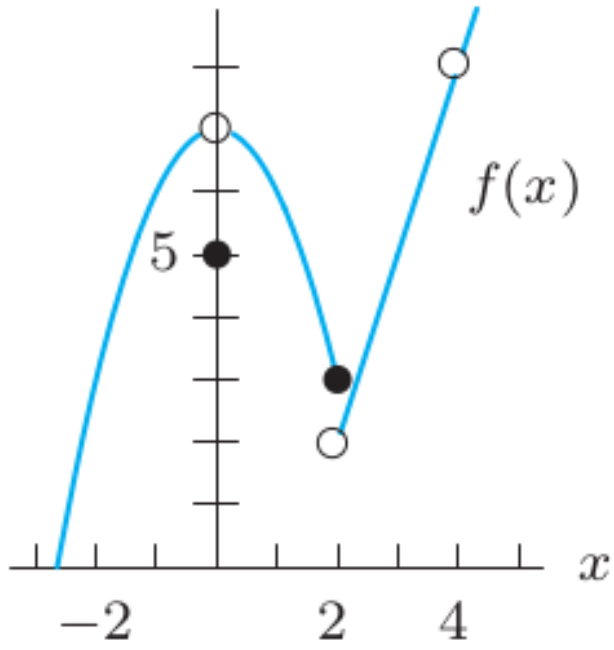

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find $\lim_{x \to 4} f(x)$

A 4	C 0	E does	exists
B 8	D ∞	not	

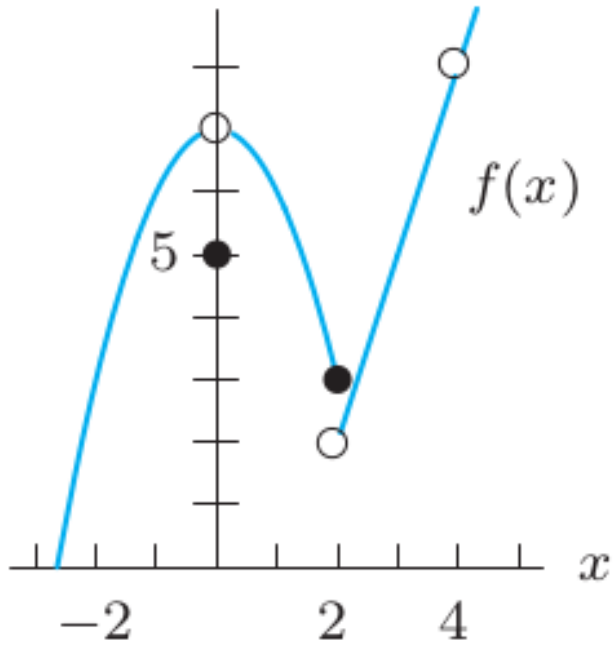

Figure: Calculus: Single and Multivariable, Hughes-Hallet

B

Exercise

Find

A $B^+(1, 1/2)$

B $P^-(-2, 1/4)$

C $B^-(+\infty, 1/50)$

D $P^+(-\infty, 1/42)$

Exercise

Find

A $B^+(1, 1/2)$

B $P^-(-2, 1/4)$

C $B^-(+\infty, 1/50)$

D $P^+(-\infty, 1/42)$

A $[1, 1.5)$

B $(-2.25, -2)$

C $(50, \infty)$

D $(-\infty, -42)$

Exercise

Find $\lim_{x \to 2-} f(x)$.

Find $\lim_{x \to 2+} f(x)$.

A 0 B 1 C 2 D 3 E $\nexists$

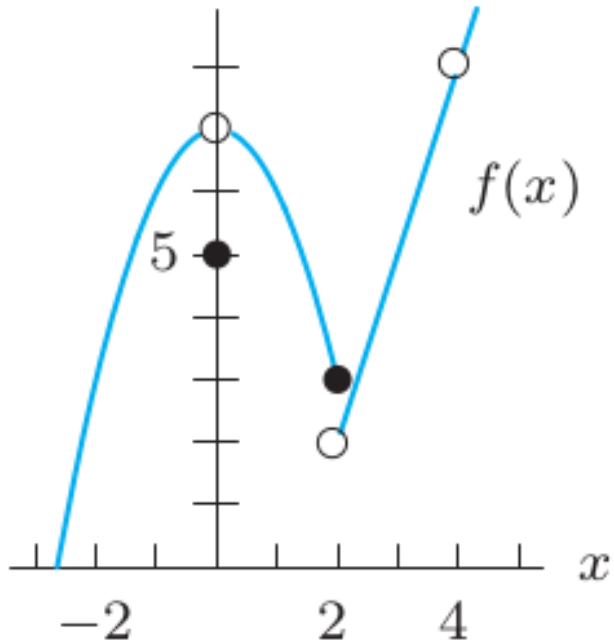

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find $\lim_{x \to 2-} f(x)$.

Find $\lim_{x \to 2+} f(x)$.

A 0 B 1 C 2 D 3 E $\nexists$

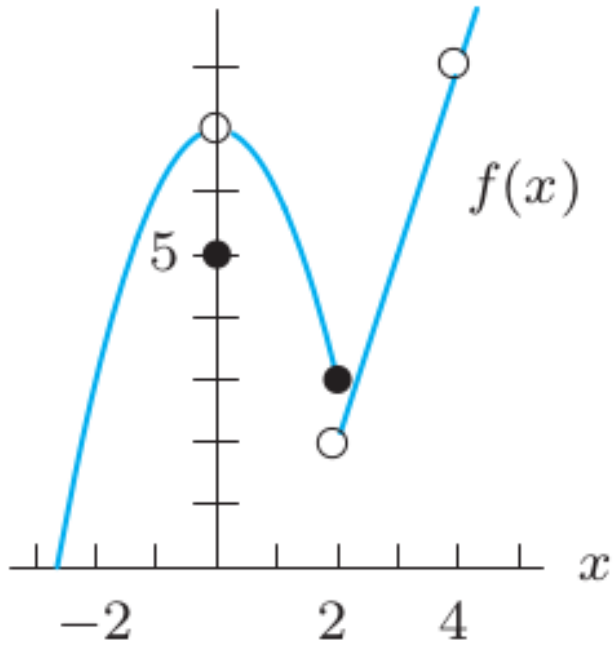

Figure: Calculus: Single and Multivariable, Hughes-Hallet

D, C

Exercise

Find $\lim_{x \to 1+} f(x) + 2g(x)$

A 13
B 9
C 8
D 6
E 3

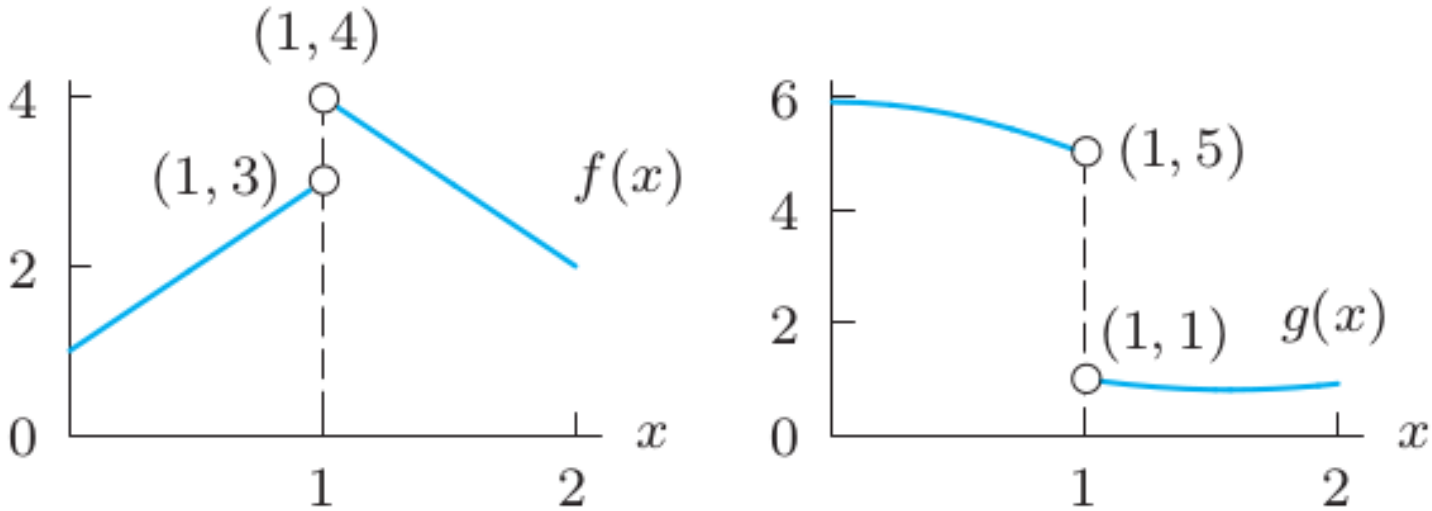

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find $\lim_{x \to 1+} f(x) + 2g(x)$

A 13
B 9
C 8
D 6
E 3

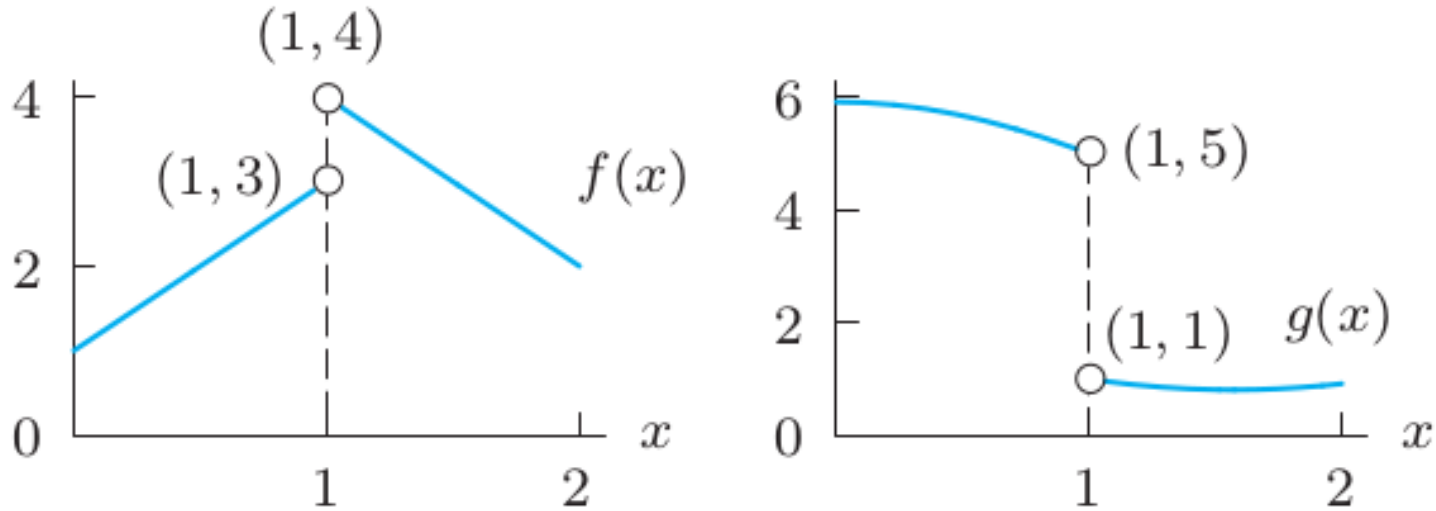

Figure: Calculus: Single and Multivariable, Hughes-Hallet

D

Exercise

Find $\lim_{x \to 1-} f(x)g(x)$

A 20
B 15
C 4
D 3
E does not exist

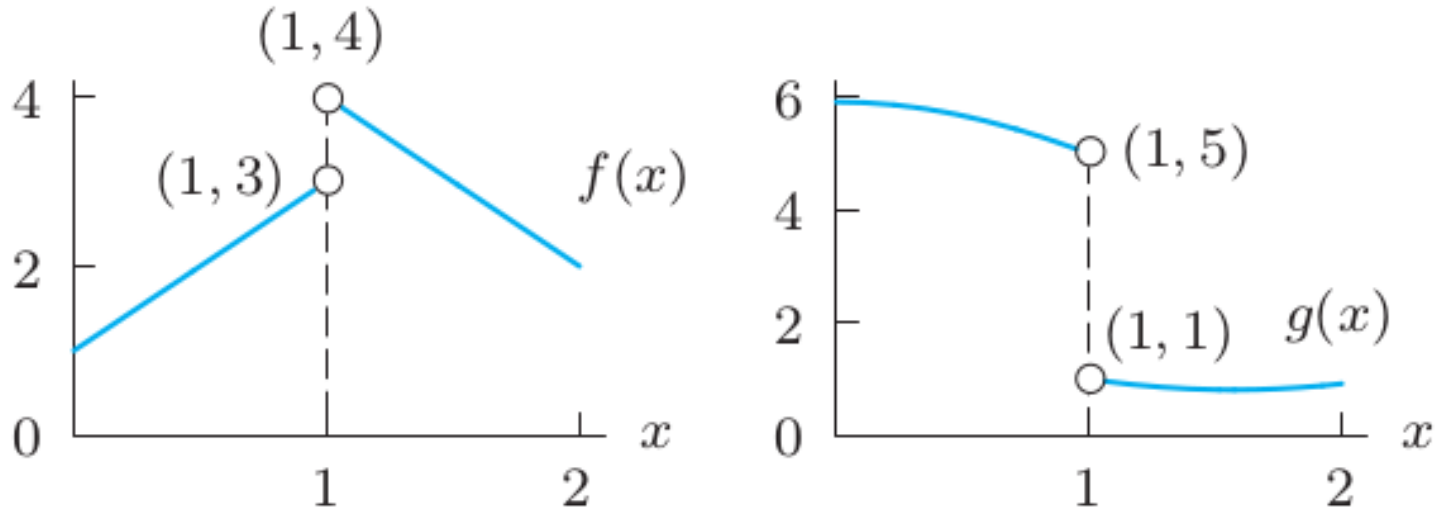

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find $\lim_{x \to 1-} f(x)g(x)$

A 20
B 15
C 4
D 3
E does not exist

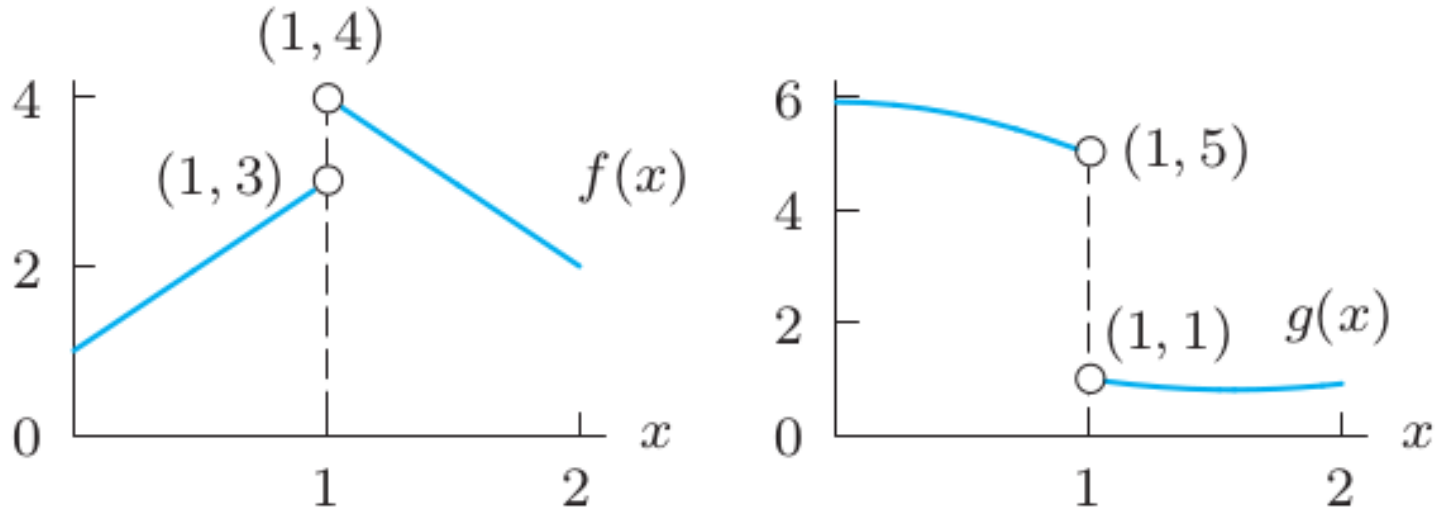

Figure: Calculus: Single and Multivariable, Hughes-Hallet

B

Exercise

Which functions are continuous at $\mathbb{R}$?

A $x^3 + \sin(4 - x)$

B $\frac{e^x}{2+x}$

C $\frac{2+x}{e^x}$

D $\cos(e^{\sqrt[3]{x}})$

E $\ln(2 + x^2)$

Exercise

Which functions are continuous at $\mathbb{R}$?

A $x^3 + \sin(4 - x)$

B $\frac{e^x}{2+x}$

C $\frac{2+x}{e^x}$

D $\cos(e^{\sqrt[3]{x}})$

E $\ln(2 + x^2)$

A, C, D, E

Exercise

$$\lim_{x\to\infty} \ln\left(\frac{x-1}{x+2}\right)$$

A 0 B 1 C $\ln 1$ D $-\frac{1}{2}$ E ∞

Exercise

$$\lim_{x\to\infty} \ln\left(\frac{x-1}{x+2}\right)$$

A 0 B 1 C $\ln 1$ D $-\frac{1}{2}$ E ∞

Exercise

$$\lim_{x\to-\infty} \cos\frac{1}{x}$$

A 0
B 1
C π
D $-\infty$
E does not exist

Exercise

$$\lim_{x\to\infty} \ln\left(\frac{x-1}{x+2}\right)$$

A 0 B 1 C $\ln 1$ D $-\frac{1}{2}$ E ∞

Exercise

$$\lim_{x\to-\infty} \cos\frac{1}{x}$$

A 0 C π E does not exist

B 1 D $-\infty$

Exercise

$$\lim_{x\to 0} \arctan\frac{1}{x^2}$$

A 0 B 1 C $\frac{\pi}{2}$ D $-\frac{\pi}{4}$ E ∞

Exercise

Is there $x \in [0, 2]$ such that

- $x^5 - 2x - 1 = 0$
- $x^3 - 4x^2 + 4x + 1 = 0$
- $5x^3 - 15x^2 + 10x + 1 = 0$

https://www.geogebra.org/calculator/pqbtmk54

Exercise

Is there $x \in [0, 2]$ such that

- $x^5 - 2x - 1 = 0$
- $x^3 - 4x^2 + 4x + 1 = 0$
- $5x^3 - 15x^2 + 10x + 1 = 0$

https://www.geogebra.org/calculator/pqbtmk54

Yes, Hard to say, Hard to say

Exercise

Find the derivative of a function $f(x) = x^2$ at the point $a = 2$.

Exercise

$f = \cos x \sin x$. Find f'.

A $\cos^2 x$

B $\sin^2 x$

C $\cos^2 x - \sin^2 x$

D $-\sin x \cos x$

Exercise

$f = \cos x \sin x$. Find f'.

A $\cos^2 x$

B $\sin^2 x$

C $\cos^2 x - \sin^2 x$

D $-\sin x \cos x$

C

Exercise

$f = \cos x \sin x$. Find f'.

A $\cos^2 x$

B $\sin^2 x$

C $\cos^2 x - \sin^2 x$

D $-\sin x \cos x$

C

Exercise

$f = e^7$. Find f'.

A $7e^6$

B e^7

C 0

Exercise

$f = \cos x \sin x$. Find f'.

A $\cos^2 x$

B $\sin^2 x$

C $\cos^2 x - \sin^2 x$

D $-\sin x \cos x$

C

Exercise

$f = e^7$. Find f'.

A $7e^6$

B e^7

C 0

C

Exercise

$f = \dfrac{e^x}{x^2}$ Find f'.

A $\dfrac{e^x}{2x}$

B $\dfrac{e^x(x-2)}{x^3}$

C $\dfrac{e^x x^2 - 2xe^x}{x^4}$

D $\dfrac{e^x 2x + x^2 e^x}{x^4}$

Exercise

$f = \dfrac{e^x}{x^2}$ Find f'.

A $\dfrac{e^x}{2x}$

B $\dfrac{e^x(x-2)}{x^3}$

C $\dfrac{e^x x^2 - 2xe^x}{x^4}$

D $\dfrac{e^x 2x + x^2 e^x}{x^4}$

B, C

Exercise

$f = \sin x + e^{\sin x}$ Find f'.

A $\cos x + e^{\cos x}$

B $\cos x + e^{\sin x}$

C $\cos x + \sin x\, e^{\cos x}$

D $\cos x + \cos x\, e^{\sin x}$

Exercise

$f = \sin x + e^{\sin x}$ Find f'.

A $\cos x + e^{\cos x}$

B $\cos x + e^{\sin x}$

C $\cos x + \sin x\, e^{\cos x}$

D $\cos x + \cos x\, e^{\sin x}$

D

Exercise (True or false?)

1. If $f'(x) = g'(x)$, then $f(x) = g(x)$. (For every x.)
2. If $f'(a) \neq g'(a)$, then $f(a) \neq g(a)$.
   (We are talking about particular point a.)

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2. If $f'(a) \neq g'(a)$, then $f(a) \neq g(a)$.
   (We are talking about particular point a.)
   False. For example $f(x) = x^2$, $g(x) = x^2 + 4$.
   False. For example $f(x) = x^2$, $g(x) = x$.

Exercise

$$\lim_{x\to\infty} \frac{\ln x}{x} =$$

A ∞ B 0 C 1 D 不存在

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Decide, when it is a good idea to use l'Hospital's rule:

A $\lim_{x\to\pi} \frac{\cos x}{x}$

B $\lim_{x\to\infty} e^{-x}x^2$

C $\lim_{x\to 0+} \frac{e^{-\frac{1}{x}}}{x}$

D $\lim_{x\to 0} \frac{\arctan x}{x}$

E $\lim_{x\to 0} \frac{\sin x - x}{\cos(2x) - 1}$

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B, D, E

Exercise

Find

$$\lim_{x \to 4} \frac{f(x)}{g(x)}.$$

A 4

B 1

C 0

D -1

E -2

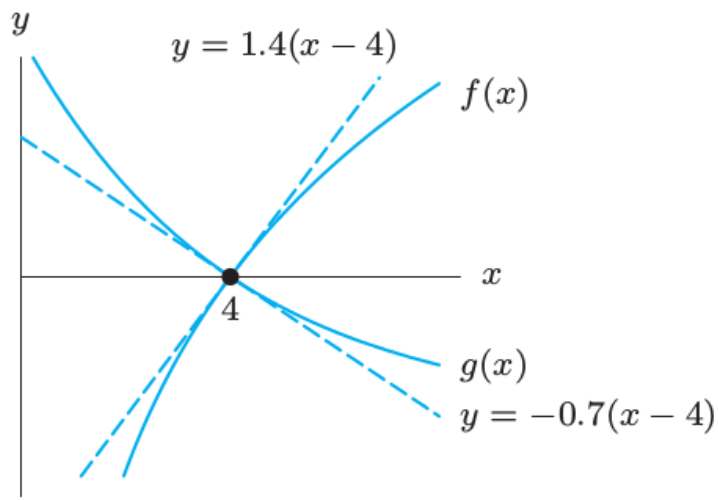

Figure: Calculus: Single and Multivariable, 6th Ed., Hughes-Hallett, col.

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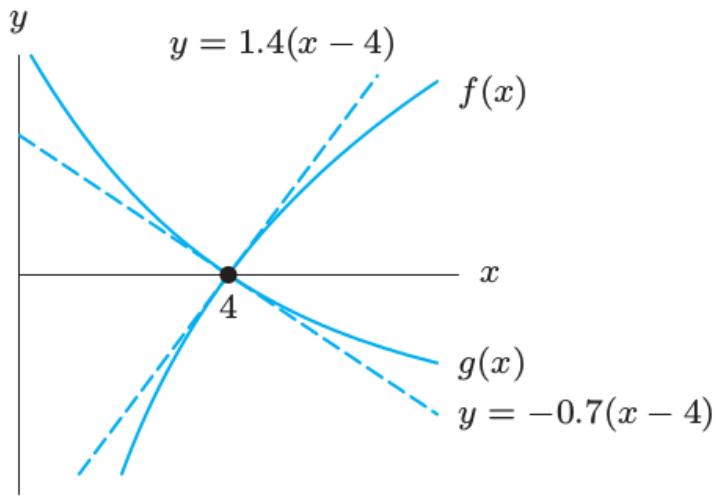

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E

Exercise

Find the asymptote of the function $f(x) = e^x$

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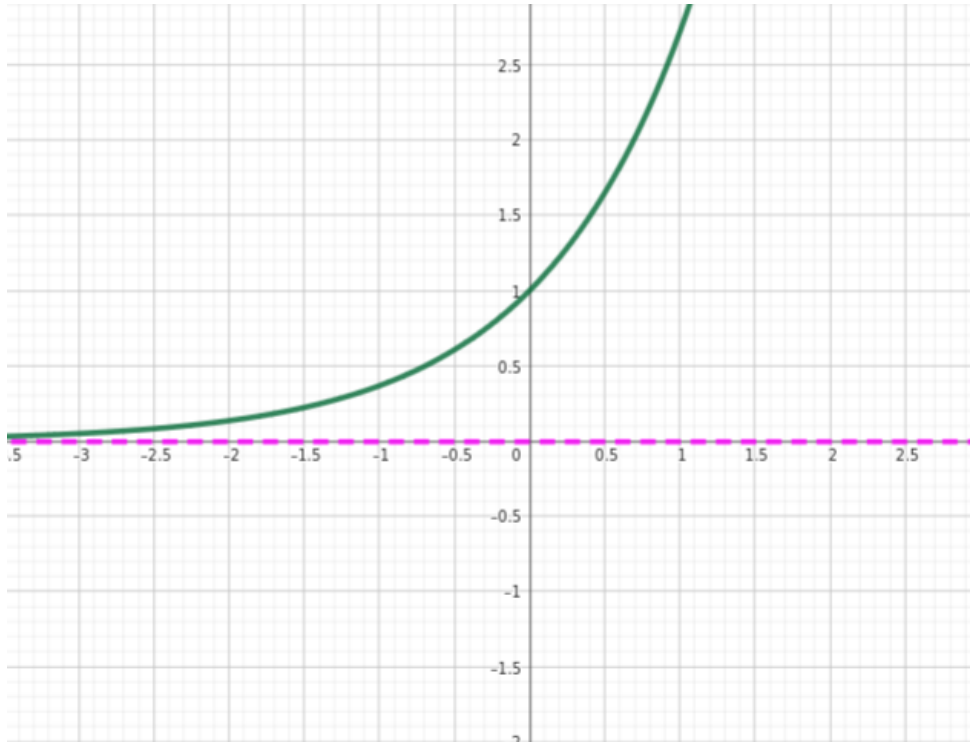
$y = 0$, 无



Exercise

Find the asymptote of the function $f(x) = x + \arctan(x^2 - 1)$

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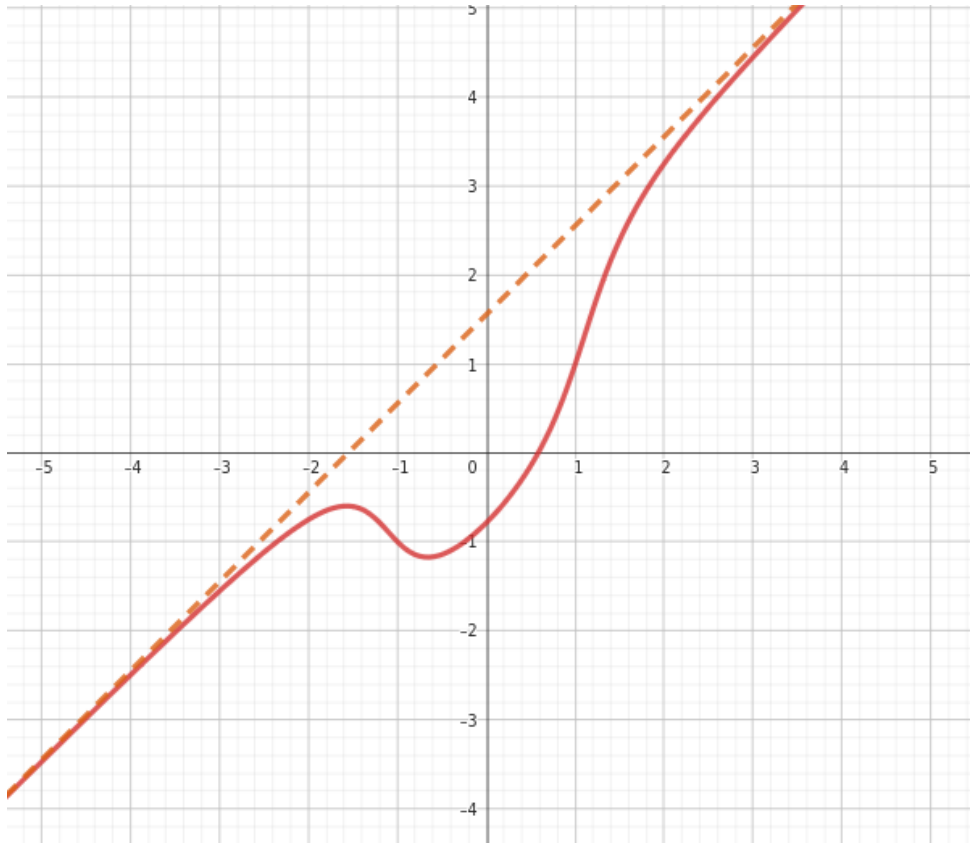
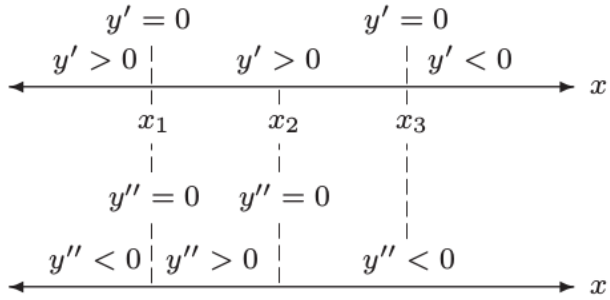
$y = x + \frac{\pi}{2}$



Exercise

Let us assume that a function $y = f(x)$ is continuous at $\mathbb{R}$. Sketch f.



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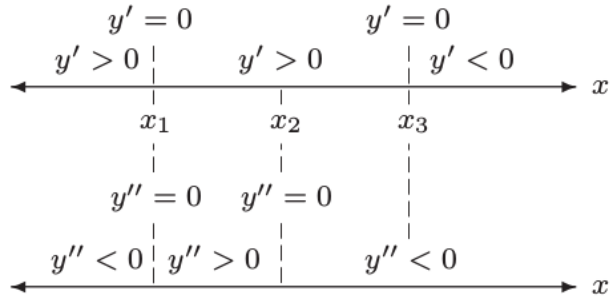

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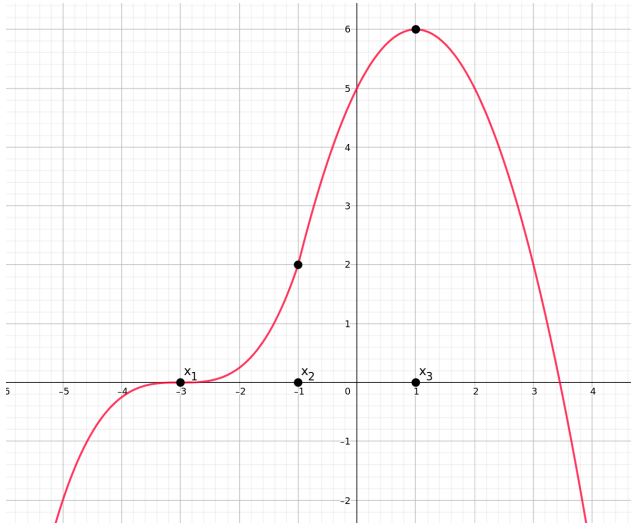

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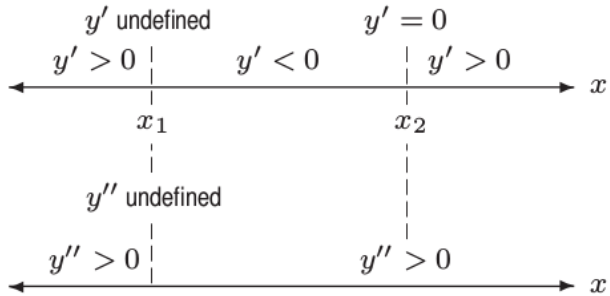

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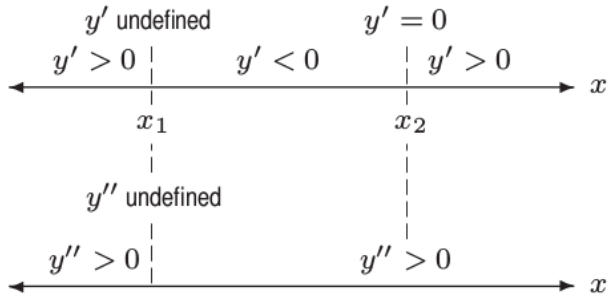

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