D

① $\lim\limits_{x\to 0} \dfrac{\cos(\tan x) - 1}{\log(1+x^2)}$

3×0,5

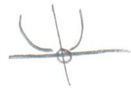
0,5

$= \lim\limits_{x\to 0} \dfrac{x^2}{\log(1+x^2)} \cdot \dfrac{\tan^2 x}{x^2} \cdot \dfrac{\cos(\tan x) - 1}{\tan^2 x} \overset{AL}{=} 1 \cdot 1 \cdot 1 \cdot \left(-\frac{1}{2}\right) = \underline{\underline{-\frac{1}{2}}}$

comp. f.

$\cdot\; f(y) = \dfrac{y}{\log(1+y)}$ $\quad \lim\limits_{y\to 0} \dfrac{y}{\log(1+y)} = 1$ $\quad$ (I) $\quad x^2 \neq 0$ $\quad P(0,1)$

2×2



$g(x) = x^2$ $\quad \lim\limits_{x\to 0} x^2 = 0$

$\cdot\; f(y) = \dfrac{\cos y - 1}{y^2}$ $\quad \lim\limits_{y\to 0} \dfrac{\cos y - 1}{y^2} = -\frac{1}{2}$ $\quad$ (I) $\quad \tan x \neq 0$

$P(0, \pi/4)$

$g(x) = \tan x$ $\quad \lim\limits_{x\to 0} \tan x = 0$

② $\lim\limits_{x\to 0} (\cos x)^{\frac{\cos^2 x}{\sin^2 x}} \overset{0,5}{=} \lim\limits_{x\to 0} e^{\frac{\cos^2 x}{\sin^2 x}\log(\cos x)} \overset{0,5}{=} e^{-1/2}$

$\lim\limits_{x\to 0} \cos^2 x \cdot \dfrac{\log(\cos x)}{\cos x - 1} \cdot \dfrac{\cos x - 1}{x^2} \cdot \dfrac{x^2}{\sin^2 x} \overset{AL}{=} 1^2 \cdot 1 \cdot -\frac{1}{2} \cdot 1 = -\frac{1}{2}$

1,5 $\quad$ 0,5

comp. funct.

$\cdot\; f(y) = \dfrac{\log y}{y-1}$ $\quad \lim\limits_{y\to 1} \dfrac{\log y}{y-1} = 1$ $\quad$ (I) $\quad \cos x \neq 1$ $\quad P(0, \pi/4)$

$g(x) = \cos x$ $\quad \lim\limits_{x\to 0} \cos x = 1$

2×1,5

$\cdot\; f(y) = e^y$ $\quad \lim\limits_{y\to -1/2} e^y = e^{-1/2}$ $\quad$ (C) $\quad e^y$ cont. at $-\frac{1}{2}$

$g(x) = \dfrac{\cos^2 x}{\sin^2 x}\log(\cos x)$ $\quad \lim\limits_{x\to 0} g(x) = -\frac{1}{2}$

③ $f = \dfrac{x-3}{(x-2)^2}$

(a) $x \neq 2$ $\quad D_f = (-\infty, 2) \cup (2, \infty)$

(b) f cont. on D_f

(c) $f(0) = \dfrac{-3}{4}$ $\quad [0, -3/4]$

$0 = f(x) \Rightarrow x = 3$ $\quad [3, 0]$

(d) $\underbrace{\text{not even, not odd,}}_{\text{because of } D_f}$ not period.

(e) $\lim\limits_{x \to \pm\infty} \dfrac{x-3}{x^2-4x+4} = \lim \dfrac{1-3/x}{x-4+4/x} = \dfrac{1-0}{\infty} = 0$

$\lim\limits_{x \to 2} \dfrac{x-3}{(x-2)^2} = \text{"}\dfrac{-3}{0+}\text{"} = -\infty$

(f) $f' = \dfrac{(x-2)^2 - (x-3)\,2(x-2)}{(x-2)^4} = \dfrac{x-2-2x+6}{(x-2)^3} = \dfrac{-x+4}{(x-2)^3}$ $\quad D_{f'} = D_f$

(h) $\searrow$ $-$ 2 $+$ $\nearrow$ 4 $-$ $\searrow$ $\qquad$ f incr on $(2, 4)$

decr. $(-\infty, 2), (4, \infty)$

(i) loc. max at $x = 4$ $\quad f(4) = \dfrac{1}{4}$

(j) $f'' = \dfrac{-(x-2)^3 - (4-x)\cdot 3(x-2)^2}{(x-2)^6} = \dfrac{-x+2-12+3x}{(x-2)^4} = \dfrac{2x-10}{(x-2)^4}$ $\quad D_{f''} = D_f$

(k) $-$ $\cap$ 2 $-$ $\cap$ 5 $+$ $\cup$ $\qquad$ f is concave at $(2, 5), (-\infty, 2)$

convex $(5, \infty)$

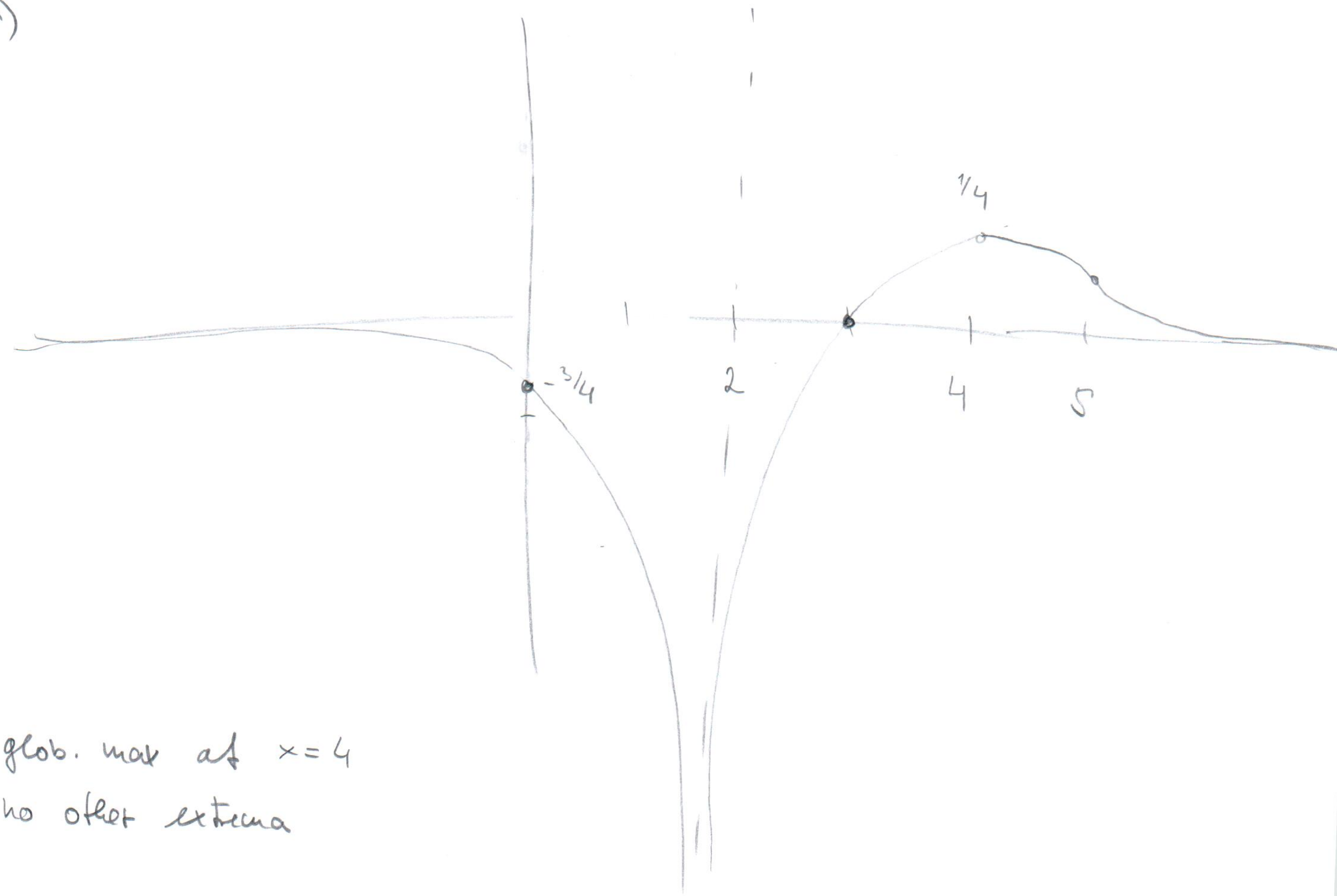
(l) asympt: $\lim\limits_{x \to \pm\infty} \dfrac{(x-3)}{(x-2)^2 \cdot x} = \lim\limits_{x \to \pm\infty} \dfrac{1-3/x}{(x-2)^2} = \text{"}\dfrac{1}{\infty}\text{"} = 0$

$\lim\limits_{x \to \pm\infty} f(x) = 0$

$y = 0x + 0$

hola asymp.

(m)



(n) glob. max at $x = 4$

no other extrema

(o) $H_f = (-\infty, 1/4]$