① $\lim\limits_{x\to 0} \dfrac{\arctan(\sin x)}{e^{\tan x}-1}$

6pt

(4 × 0,5) ①

$= \lim\limits_{x\to 0} \dfrac{\arctan(\sin x)}{\sin x} \cdot \dfrac{\sin x}{x} \cdot \dfrac{\tan x}{e^{\tan x}-1} \cdot \dfrac{x}{\tan x} \overset{AL}{=} 1\cdot 1\cdot 1\cdot 1 = \underline{\underline{1}}$

(2 × 1,5)

CF: $f(y) = \dfrac{\arctan y}{y}$ $\qquad \lim\limits_{y\to 0} \dfrac{\arctan y}{y} = 1$ $\qquad$ (I)

$g(x) = \sin x$ $\qquad \lim\limits_{x\to 0} \sin x = 0$ $\qquad \sin x \neq 0 \quad x \in P(0, \pi/4)$

CF $\quad f(y) = \dfrac{y}{e^y - 1}$ $\qquad \lim\limits_{y\to 0} \dfrac{y}{e^y-1} = 1$ $\qquad$ (I)

$g(x) = \tan x$ $\qquad \lim\limits_{x\to 0} \tan x = 0$ $\qquad \tan x \neq 0 \quad x \in P(0, \pi/4)$

another way

$\overset{LH}{\underset{0/0}{=}} \lim\limits_{x\to 0} \dfrac{\frac{1}{1+\sin^2 x}\cdot \cos x}{e^{\tan x}\cdot \frac{1}{\cos^2 x}} \overset{AL}{=} \dfrac{\frac{1}{1+0}\cdot 1}{1\cdot \frac{1}{1}} = 1$

② $\lim\limits_{x\to 0} \left(\sqrt{1+x}\right)^{1/x}$ (0,5) $= e^{\frac{1}{x}\log\left(\sqrt{1+x}\right)}$ (0,5) $= \underline{\underline{e^{1/2}}}$

6pt

(0,5)

$\lim\limits_{x\to 0} \dfrac{\log\sqrt{1+x}}{x} = \lim\limits_{x\to 0} \underbrace{\dfrac{\log\sqrt{1+x}}{\sqrt{1+x}-1}}_{\to 1} \cdot \dfrac{\sqrt{1+x}-1}{x} \overset{AL}{=} 1\cdot \dfrac{1}{2}$ ①

(1,5)

$\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{x} \cdot \dfrac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = \lim\limits_{x\to 0} \dfrac{1+x-1}{x} \cdot \dfrac{1}{\sqrt{1+x}+1} \underset{\text{cont.}}{=} \dfrac{1}{\sqrt{1+0}+1} = \dfrac{1}{2}$

CF $\quad f(y) = \dfrac{\log y}{y-1}$ $\qquad \lim\limits_{y\to 1} \dfrac{\log y}{y-1} = 1$ $\qquad$ (I) $\sqrt{1+x} \neq 1 \quad x \in P(0, 1/2)$

① $\quad g(x) = \sqrt{1+x}$ $\qquad \lim\limits_{x\to 0} \sqrt{1+x} = 1$ $\qquad 1+x \neq 1$, $x \neq 0$

another way:

$\lim\limits_{x\to 0} \dfrac{\log\sqrt{1+x}}{x} \overset{LH}{\underset{0/0}{=}} \lim\limits_{x\to 0} \dfrac{\frac{1}{\sqrt{1+x}}\cdot \frac{1}{2}\frac{1}{\sqrt{1+x}}}{1} = \dfrac{1}{1}\cdot\dfrac{1}{2}\cdot\dfrac{1}{1} = \dfrac{1}{2}$

CF $\quad f = e^y$ $\qquad \lim\limits_{y\to 1/2} f = e^{1/2}$ $\qquad$ (c) e^y cont at $1/2$

① $\quad g = \frac{1}{x}\log\sqrt{1+x}$ $\qquad \lim g = 1/2$

③ $f = e^{-x}(2x+1)$

13pt

(0,5) (a) $x \in \mathbb{R} = D_f$

(0,5) (b) f cont. on $\mathbb{R}$

(1) (c) $f(0) = e^0(1) = 1$ $[0,1]$

$0 = e^{-x}(2x+1) \rightarrow x = -\frac{1}{2}$ $[-1/2, 0]$

(0,5) (d) not even, not odd, (because of the intercepts)
not periodic

(1) (e) $\lim\limits_{x\to\infty} e^{-x}(2x+1) = \lim\limits_{x\to\infty} \frac{2x+1}{e^x} \overset{LH}{\underset{\frac{\infty}{\infty}}{=}} \lim\limits_{x\to\infty} \frac{2}{e^x} \overset{AL}{=} 0$

$\lim\limits_{x\to-\infty} e^{-x}(2x+1) \overset{AL}{=} \infty \cdot (-\infty) = -\infty$

(1) (f) $f' = -e^{-x}(2x+1) + e^{-x}\cdot 2 = e^{-x}(-2x+1)$ $D_{f'} = D_f$

(1) (h) $-2x+1 = 0$ $x = 1/2$

f': $+$ | $-$

f: $\nearrow$ 0 $1/2$ $\searrow$

f incr on $(-\infty, 1/2)$
decr. $(1/2, \infty)$

(0,5) (i) loc. max at $x = 1/2$ $f(x) = e^{-1/2}(2) \doteq 1{,}21$

(1) (j) $f'' = -e^{-x}(-2x+1) + e^{-x}(-2) = e^{-x}(2x-3)$ $D_{f''} = D_f$

(1) (k) $2x-3 = 0$
$x = 3/2$

f'': $-$ | $+$

f: $\cap$ $3/2$ $\cup$

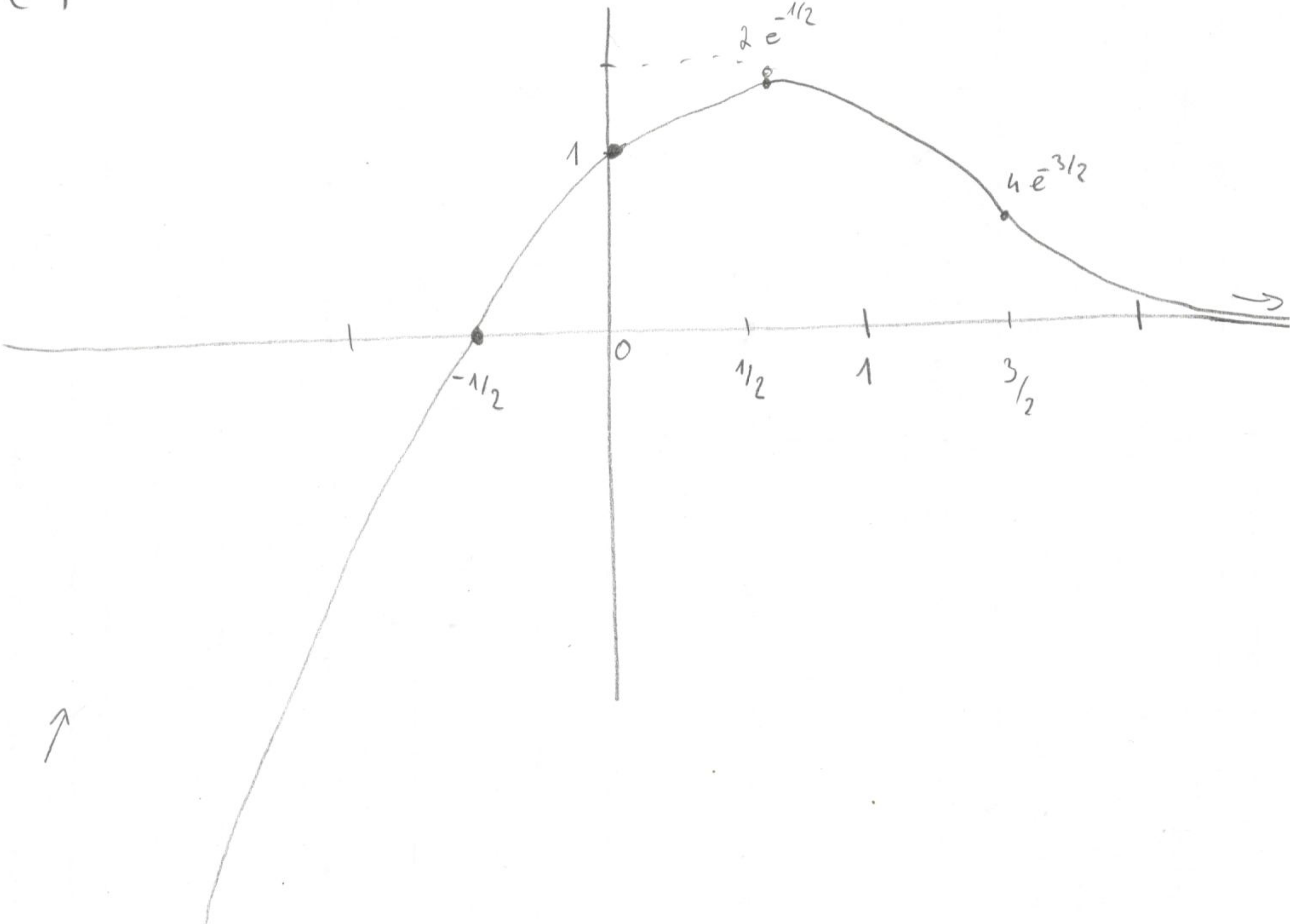
f concave on $(-\infty, 3/2)$
convex $(3/2, \infty)$
$f(3/2) = e^{-3/2}\cdot 4 \doteq 0{,}89$

(1) (l) $a_1 = \lim\limits_{x\to\infty} \frac{e^{-x}(2x+1)}{x} = \lim\limits_{x\to\infty} e^{-x} \frac{2+1/x}{1} = 0 \cdot 2 = 0$

$b_1 = \lim\limits_{x\to\infty} e^{-x}(2x+1) = 0$ $y = 0x + 0$

$a_2 = \lim\limits_{x\to-\infty} \frac{e^{-x}(2x+1)}{x} = \infty \cdot 2 = \infty$ no as.

③ (m)



(n) global max at $x = 1/2$

(0,5) min $\nexists$

(o) range $H_f = (-\infty, 2e^{-1/2}]$

(0,5)