Thm. $\forall n \in \mathbb{N}$: n^2 is odd $\Rightarrow$ n is odd

Pf **Direct** let $n = p_1 p_2 \cdots p_k$ primes

$n^2 = p_1^2 p_2^2 p_3^2 \cdots p_k^2$

none of them $p_1, p_2, \dots, p_k \neq 2$ $\Rightarrow n = p_1 p_2 \cdots p_k$ is odd $\blacksquare$

Indirect $\neg B \Rightarrow \neg A$

n is even $\Rightarrow$ n^2 is even

$n = 2k$ hence $n^2 = (2k)^2 = 2 \cdot (2k^2)$ even $\blacksquare$

contradiction A & $\neg B$

n^2 is odd & n is even

$n = 2k$ $\Rightarrow$ $n^2 = 2 \cdot (2k)^2$ is even,

which is contradiction

$\blacksquare$

$$\sum_{j=1}^{k} (2j-1) = k^2 \qquad k \in \mathbb{N}$$

1. Step $\quad k = 1$

$$\sum_{j=1}^{1} (2j-1) = 2 \cdot 1 - 1 = 1^2 \quad \checkmark$$

2. Step $\quad$ Assume for some $k \in \mathbb{N}$:

fix k: $\quad \sum_{j=1}^{k} (2j-1) = k^2$

3. Step $\quad$ Induction Step

$$? \quad \sum_{j=1}^{k+1} (2j-1) = (k+1)^2 \quad ?$$

$$\sum_{j=1}^{k+1} (2j-1) = \sum_{j=1}^{k} (2j-1) + (2(k+1)-1) = \sum_{j=1}^{k} (2j-1) + 2k+1$$

$$= k^2 + 2k + 1 = (k+1)^2$$

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