A

1 6pt $\lim_{x \to 0} \frac{\cos x}{\sin x} \cdot \tan(2\sin x)$

$= \lim_{x\to 0} \cos x \cdot \frac{\tan(2\sin x)}{2 \cdot \sin x} \cdot 2 \overset{AL}{=} 1 \cdot 1 \cdot 2 = 2$

1 1 1,5

composed function:

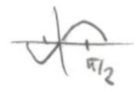
2,5 1,5 1

$f(y) = \frac{\tan y}{y}$ $\lim_{y\to 0} f = 1$

$g(x) = 2\sin x$ $\lim_{x \to 0} g = 0$

(±) $2\sin x \neq 0$

$x \in P(0, \pi/4)$



2 6pt $\lim_{x\to 0} (1+\sin(3x^2))^{\frac{1}{x^2}} \overset{1}{=} \lim e^{\frac{1}{x^2}\log(1+\sin 3x^2)} \overset{1}{=} e^3$

1 $\lim_{x\to 0} \frac{\log(1+\sin 3x^2)}{\sin 3x^2} \cdot \frac{\sin 3x^2}{3x^2} \cdot 3 \overset{AL}{=} 1 \cdot 1 \cdot 3$

3x1

- $f(y) = e^y$ $\lim_{y\to 3} f = e^3$ (c) e^y cont on $\mathbb{R}$

  $g(x) = \frac{1}{x^2}\log(1+\sin 3x^2)$ $\lim_{x\to 0} g = 3$

- $f(y) = \frac{\log(1+y)}{y}$ $\lim_{y\to 0} f = 1$

  $g(x) = \sin 3x^2$ $\lim_{x\to 0} g = 0$

  (I) $\sin 3x^2 \neq 0$

  $3x^2 < \pi/2$

  $x \in P(0, \pi/6)$

- $f(y) = \frac{\sin y}{y}$ $\lim_{y\to 0} f = 1$

  $g(x) = 3x^2$ $\lim_{x\to 0} g = 0$

  (I)

  $3x^2 \neq 0$ $x \in P(0, 1)$

③ $f(x) = x^2(-2 + \ln x^4)$

(a) $x \neq 0 \quad \rightarrow \quad x \in (-\infty, 0) \cup (0, \infty) = D_f$
0,5

(b) f is continuous at D_f
0,5
(compos. + product of cont. funct.)

(c) $x \neq 0$
1
$$f(x) = 0 \iff -2 + \ln x^4 = 0$$
$$\ln x^4 = 2$$
$$x^4 = e^2$$
$$x = \pm e^{1/2}$$

(d) $f(-x) = (-x)^2(-2 + \ln(-x)^4) = x^2(-2 + \ln x^4) = f(x)$
0,5
f is even

(e)
1
$$\lim_{x \to 0} x^2(-2 + \ln x^4) = \lim_{x \to 0} \frac{-2 + \ln x^4}{\frac{1}{x^2}} \underset{\frac{\infty}{\infty}}{\overset{LH}{=}} \lim_{x \to 0} \frac{\frac{1}{x^4} \cdot 4x^3}{-2 \frac{1}{x^3}}$$
$$= \lim_{x \to 0} -2x^2 = 0$$
$$\lim_{x \to \infty} x^2(-2 + \ln x^4) = \infty(-2 + \infty) = \infty$$

(f) $f' = 2x(-2 + \ln x^4) + x^2\left(0 + \frac{1}{x^4} \cdot 4x^3\right)$
1
$$= -4x + 2x\ln x^4 + 4x = \underline{\underline{2x \ln x^4}} \qquad x \in D_{f'} = D_f$$

(h) $\ln x^4 = 0 \iff x^4 = 1 \qquad x = \pm 1$
1
$2x = 0 \iff x = 0$

		-1		0		1	
f'	$-$		$+$		$-$		$+$
f	↘		↗		↘		↗

f is incr. on $(-1, 0), (1, \infty)$
decr. $(-\infty, -1), (0, 1)$

(i) local min at $x = \pm 1$ $\quad f(1) = -2$
0,5

(j) $f'' = (2x \ln x^4)' = 2\ln x^4 + 2x \cdot \frac{1}{x^4} \cdot 4x^3 = 2\ln x^4 + 8$

1

$D_{f''} = D_f$

(k) $2\ln x^4 + 8 = 0 \iff \ln x^4 = -4$

1

$x^4 = e^{-4}$

$x = \pm e^{-1}$

		$-1/e$		0		$1/e$	
f''	$+$		$-$		$-$		$+$
f	$\cup$		$\cap$		$\cap$		$\cup$

f is convex at $(-\infty, -1/e)$, $(1/e, \infty)$

concave $(-1/e, 0)$, $(0, 1/e)$

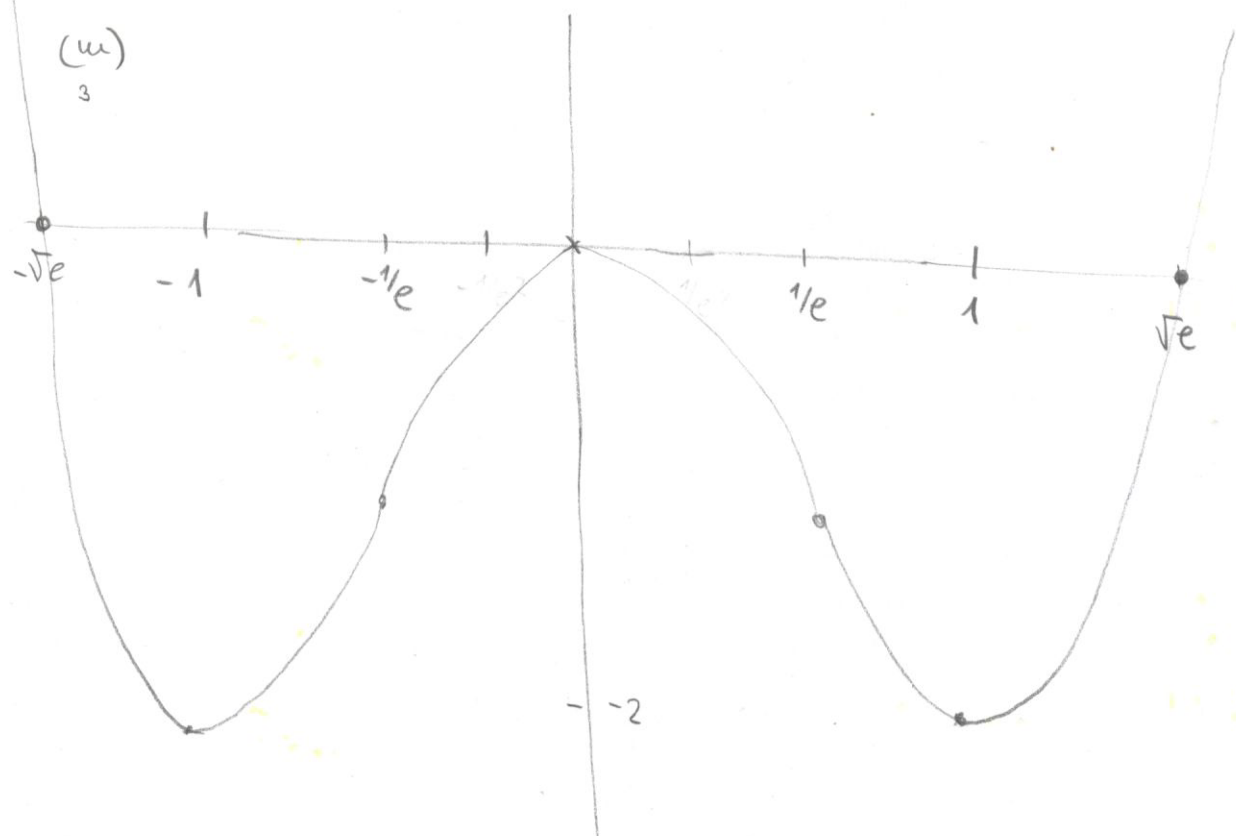
(l) $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} x(-2 + \ln x^4) = \infty(-2+\infty) = \infty$

1

$\rightarrow$ no asymptotes

(m)

3



(n) global extr. $\nexists$

0,5

glob min at $x = \pm 1$, value -2.

(o) range: $H_f = [-2, \infty)$

0,5