

## Exam - A

You have 120 minutes and can use any literature (notes, tables, textbooks...), but no technical devices (phone, calculator, watches...). Please, be honest.

Good luck.

1. (6 points) Find the limit of a function:

$$\lim_{x \rightarrow 0} \frac{\cos x}{\sin x} \cdot \tan(2 \sin x)$$

2. (6 points) Find the limit of a function:

$$\lim_{x \rightarrow 0} (1 + \sin(3x^2))^{\frac{1}{x^2}}$$

3. (13 points) Sketch the graph of the function:

$$f(x) = x^2 (-2 + \ln(x^4))$$

You may appreciate following numbers:  $\sqrt{e} \doteq 1.65$ ,  $1/e \doteq 0.37$ .

- (a) Find the **domain**.
- (b) Is the function **continuous**?
- (c) Find the **intercepts with the axes**.
- (d) Is the function **even, odd or periodic** or not?
- (e) Find the **limits** at the endpoints of the domain.
- (f) Find **the first derivative of  $f$** . Find the domain of  $f'$ .
- (g) ~~Find the first derivative of  $f$  at the special points.~~
- (h) Decide about the **monotonicity of  $f$** .
- (i) Find the **local extrema**.
- (j) Compute **the second derivative of  $f$** . Find the domain of  $f''$ .
- (k) Decide about **convexity/concavity**. Find points of inflection.
- (l) Find the **asymptotes**.
- (m) **Sketch the graph**.
- (n) Check the **extrema**.
- (o) Find the **range**.