③ $$\int \frac{9x^3 - 56x^2 + 17x - 10}{(x-3)(x+2)(3x^2-2x+1)}\,dx$$

- $\frac{P}{Q}$ $\deg P = 3 < 4 = \deg Q$ $3x^2-2x+1 \neq 0$

- $$= \int \frac{\overset{A}{-2}}{x-3} + \frac{\overset{B}{4}}{x+2} + \frac{\overset{Cx+D}{3x-1}}{3x^2-2x+1}\,dx$$

$$A(x+2)(3x^2-2x+1) + B(x-3)(3x^2-2x+1) + (Cx+D)(x+2)(x-3)$$
$$= 9x^3 - 56x^2 + 17x - 10$$

$x=-2$: $B(-5)(17) = -72 - 224 - 34 - 10$

$\boxed{B = 4}$

$x=3$: $A \cdot 5 \cdot 22 = 243 - 504 + 51 - 10$

$\boxed{A = -2}$

$x=0$: $2\cdot(-2) + 4\cdot(-3) + D\cdot(-6) = -10$

$-6D = 6$ $\boxed{D = -1}$

$x=1$: $(-2)\cdot 3 \cdot 2 + 4\cdot(-2)\cdot 2 + (C + (-1))(-6) = 9 - 56 + 17 - 10$

$-12 \quad -16 \quad -6C + 6 \quad = -40$

$C = 3$

$$\int = -2\ln|x-3| + 4\ln|x+2| + \frac{1}{2}\ln|3x^2-2x+1|$$ $x \neq -2$, $x \neq 3$

$$\int \frac{3x-1}{3x^2-2x+1}\,dx = \frac{1}{2}\int \frac{6x-2}{3x^2-2x+1}\,dx = \frac{1}{2}\int \frac{1}{y}\,dy = \frac{1}{2}\ln|y| + c = \frac{1}{2}\ln|3x^2-2x+1| + c$$

$y = 3x^2-2x+1$

$dy = 6x-2\,dx$

(4) $\int_0^{\ln \pi} \sin(e^x)\cos(2e^x)e^x \, dx = \int_1^{\pi} \sin y \cos(2y)\, dy =$

$y = e^x$

$dy = e^x dx$

x	0	$\ln \pi$
y	1	π

$\int \sin y (\cos^2 y - \underbrace{\sin^2 y}_{1-\cos^2 y})\, dy = \int \sin y (-1 + 2\cos^2 y)\, dy =$

$= \underbrace{\int -\sin y \, dy}_{\cos y} + 2\int \cos^2 y \cdot \sin y \, dy = \cos y - \frac{2}{3}\cos^3 y$

$t = \cos y$

$dt = -\sin y \, dy$

$\to \quad -2\int t^2 \, dt \overset{c}{=} -2\frac{t^3}{3} =$

$= -\frac{2}{3}\cos^3 y$

$\int_1^{\pi} \sin y \cos(2y)\, dy = \left[\cos y - \frac{2}{3}\cos^3 y\right]_1^{\pi} = -1 + \frac{2}{3} - \left(\cos 1 - \frac{2}{3}(\cos 1)^3\right)$

$= -\frac{1}{3} - \cos 1 + \frac{2}{3}\cos^3 1$