② $\arcsin(x+y) + \arctan(x+y) + xy = 0$ $[0,0]$

$F(x,y) = \arcsin(x+y) + \arctan(x+y) + xy$

- $F \in C^1(G)$ $G = \{[x,y] \in \mathbb{R}^2 : -1 < x+y < 1\}$

  $[0,0] \in G$, $G$ is open

- $F(0,0) = \arcsin 0 + \arctan 0 + 0 = 0$ ✓

- $$\frac{\partial F}{\partial y} = \frac{1}{\sqrt{1-(x+y)^2}} + \frac{1}{1+(x+y)^2} + x$$

  $$\frac{\partial F}{\partial y}(0,0) = \frac{1}{\sqrt{1}} + \frac{1}{1} + 0 = 2 \neq 0$$

Impl. function thm ✓

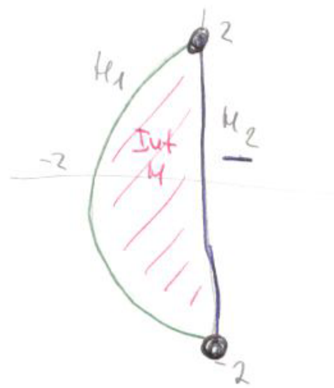
$$\frac{\partial F}{\partial x} = \frac{1}{\sqrt{1-(x+y)^2}} + \frac{1}{1+(x+y)^2} + y \qquad \frac{\partial F}{\partial x}(0,0) = \frac{1}{\sqrt{1}} + \frac{1}{1} + 0 = 2$$

$$f'(0) = -\frac{\partial F/\partial x}{\partial F/\partial y}(0,0) = -\frac{1}{1} = \boxed{-1}$$

③ $f(x,y) = -y^2 + x^2 + \frac{4}{3}x^3$ $\qquad M = \{x^2+y^2 \le 4,\ x \le 0\}$

- M is bounded (half circle)
- M closed (with its boundary)
- f is continuous (polynomial)

$\}$ f attains extrema




① Int M $\qquad x^2+y^2<4,\ x<y$

$\frac{\partial f}{\partial x} = 2x + 4x^2 \qquad 2x(1+2x)=0 \qquad x=0 \ \vee\ x=-\frac{1}{2}$

$\frac{\partial f}{\partial y} = -2y \ \to\ y=0 \qquad [0,0] \notin \text{Int } M \qquad [-\frac{1}{2},0]$

② $M_1$ $\qquad x^2+y^2=4 \quad x<0 \quad : \quad x\in[-2,0), \quad f,g \in C^\infty(\mathbb{R}^2)$

Lagrange (a) $\nabla g = (2x,2y) = (0,0) \to [0,0] \notin M_1$

$0^2+0^2 \neq 4$

(b) $\nabla f + \lambda \nabla g = (0,0)$

$2x+4x^2+\lambda 2x = 0$

$-2y + \lambda 2y = 0 \quad \to \quad 2y(1-\lambda)=0$

$x^2+y^2=4$

$y=0$ $\qquad\qquad \lambda = 1$

$x^2=4 \qquad\qquad 2x+4x^2+2x=0$

$x=\pm 2 \qquad\qquad 4x(x+1)=0$

$[-2,0]$ ✓ $\quad [2,0] \notin M$

$x=0 \qquad\qquad x=-1$

$y^2=4 \qquad\qquad y^2=3$

$y=\pm 2 \qquad\qquad y=\pm\sqrt{3}$

$[0,\pm 2] \notin M_1 \qquad [-1,\pm\sqrt{3}]$ ✓

$M_2$ : $\quad x=0 \quad y\in(-2,2)$

$f(0,y) = -y^2 \ \to\ [0,0]$

(3)

Suspect points

| | $[-\frac{1}{2},0]$ | $[0,0]$ | $[-2,0]$ | $[-1,\sqrt{3}]$ | $[-1,-\sqrt{3}]$ | $[0,2]$ | $[0,-2]$ |
|---|---|---|---|---|---|---|---|
| f: | $+1/12$ | $0$ | $-20/3$ | $-10/3$ | $-10/3$ | $-4$ | $-4$ |
| | ↑ glob. max | | ↑ glob. min | | | | |

(1)

$$\rightarrow \begin{vmatrix} 1 & 2 & 3 & -3 \\ 2 & 1 & 1 & -3 \\ 3 & 0 & 4 & 0 \\ -5 & 3 & 2 & 1 \end{vmatrix} = (-1)^{1+3}\cdot 3\cdot \begin{vmatrix} 2 & 3 & -3 \\ 1 & 1 & -3 \\ 3 & 2 & 1 \end{vmatrix} + (-1)^{3+3}\cdot 4\cdot \begin{vmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -5 & 3 & 1 \end{vmatrix}$$

$$= 3\cdot\left(2+(-27)+(-6)-[-9-12+3]\right)$$

$$+4\left(1+30-18-[15-9+4]\right) =$$

$$= -39 + 12 = \underline{\underline{-27}}$$

Or

$$\left(\begin{array}{ccc|ccc} -2 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ -2 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 0 & 1 & 0 \\ -2 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right) \sim \begin{array}{c} :2 \\ \\ \\ \end{array}\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right)$$

$$\sim \begin{array}{c} \\ \cdot(-1) \\ :2 \end{array}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & +1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1/2 & 1 & 1/2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & -1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1 & 1/2 \end{array}\right) \rightarrow A^{-1} = \begin{pmatrix} 0 & 1/2 & 0 \\ -1/2 & 0 & 1/2 \\ 1/2 & 1 & 1/2 \end{pmatrix}$$