Exam - sample

The exam consists of written and oral part. Their dates will be in SIS. Please, let us know, if you have some special needs (the sooner the better).

Written part

The written part comprises 4 questions:

1. inverse/determinant of a matrix - 7 points;
2. find extrema over a set (extrema + Lagrange multipliers) - 9 points;
3. implicit functions - 9 points;

Students have 120 minutes and can use any literature (notes, tables, textbooks...), but no technical devices (phone, calculator, watches...).

Example of the written part

1. (7 points) Compute the determinant of the following matrix

$$\begin{pmatrix} 1 & 2 & 3 & -3 \\ 2 & 1 & 1 & -3 \\ 3 & 0 & 4 & 0 \\ -5 & 3 & 2 & 1 \end{pmatrix}$$

   or

   Find the inverse matrix of $\mathbf{A} = \begin{pmatrix} -2 & -1 & 1 \\ 2 & 0 & 0 \\ -2 & 1 & 1 \end{pmatrix}$

2. (9 points) Show that

$$\arcsin(x+y) + \arctan(x+y) + xy = 0$$

   determines at some neighborhood of the point $[0,0]$ implicitly given function with variable x. Compute the first derivative of this function at the point $[0,0]$.

3. (9 points) Find global maximum and minimum of the function f on the set M.

$$f(x,y) = -y^2 + x^2 + \frac{4}{3}x^3, \qquad M = \{[x,y] \in \mathbb{R}^2; x^2 + y^2 \leq 4, x \leq 0\}.$$

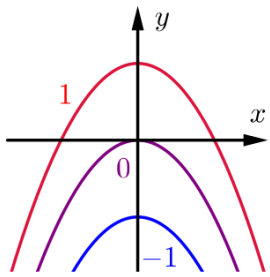
Oral part

The oral part tests knowledge of definitions, theorems, some proofs (those from the lecture) and concept questions (similar to those in the lecture).

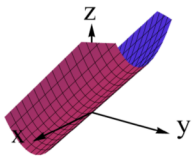
The students have (almost arbitrary) time for preparation, but can not use any literature or electronic devices.

Example of the oral part

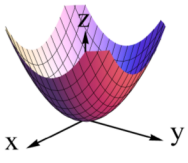
1. (2x2 points) Definition of: **inverse matrix**, **open set**.
2. (3.5 points) Statement of the theorem: **Implicit function theorem**.
3. (3.5+10 points) Statement and proof of the theorem: **Linearity of a primitive function**.
4. (6x1.5 points) Answer concept questions:
   (a) Find bounded sets
      i. $x \in [-1, 3]$, $0 < y \leq 100$
      ii. $x^2 + y^2 + z^2 \leq 5$
      iii. $|x + y| < 6$
   (b) Find the suitable graph for given contour lines:



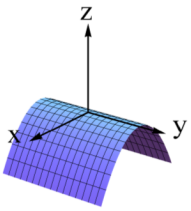
A.



B.

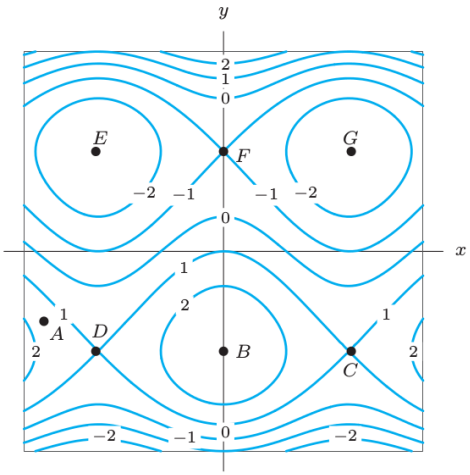


C.



http://www.cpp.edu/~conceptests/question-library/mat214.shtml

(c) i. Find the critical points.
ii. Find the points of
A. local minimum,
B. local maximum,
C. saddle?



Source 1: Calculus, 6th Edition; Hughes-Hallett, Gleason, McCallum et al.

(d) Let matrices A and B be invertible. Let $AB = C$. Find C^{-1}:

i. $A^{-1}B^{-1}$
ii. $B^{-1}A^{-1}$
iii. AB^{-1}
iv. BA^{-1}
v. C does not have to be invertible.

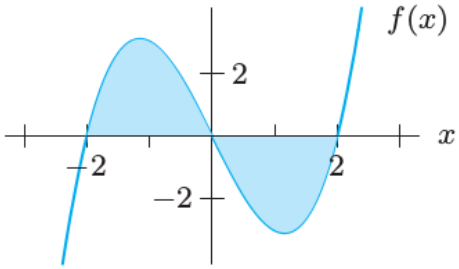
(e) Decide, if it is better to use per partes or substitution

i. $\int \arcsin x \, \mathrm{d}x$
ii. $\int \frac{x}{1+x^2} \, \mathrm{d}x$
iii. $\int (x^2 - 3) \ln x \, \mathrm{d}x$
iv. $\int \frac{1}{x \ln x} \, \mathrm{d}x$
v. $\int x^2 \cos 2x \, \mathrm{d}x$

(f) There is an odd function on the picture. If you now that $\int_{-2}^{0} f(x)\,\mathrm{d}x = 4$, find

i. $\int_{0}^{2} f(x)\,\mathrm{d}x$

ii. $\int_{-2}^{2} f(x)\,\mathrm{d}x$



Source 2: Calculus: Single and Multivariable, Hughes-Hallet