1

$$\lim_{n\to\infty} \frac{\sqrt{2n-3}-\sqrt{2n+3}}{\sqrt[3]{n+4}-\sqrt[3]{n-2}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{2n-3}-\sqrt{2n+3}}{\sqrt[3]{n+4}-\sqrt[3]{n-2}} \cdot \frac{\sqrt{2n-3}+\sqrt{2n+3}}{(\sqrt[3]{n+4})^2+\sqrt[3]{n+4}\,\sqrt[3]{n-2}+(\sqrt[3]{n-2})^2} \cdot \frac{(\sqrt[3]{n+4})^2+\sqrt[3]{n+4}\,\sqrt[3]{n-2}+(\sqrt[3]{n-2})^2}{\sqrt{2n-3}+\sqrt{2n+3}}$$

$$= \lim_{n\to\infty} \frac{2n-3-(2n+3)}{n+4-(n-2)} \cdot \frac{\sqrt[3]{(n+4)^2}+\sqrt[3]{n+4}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt{2n-3}+\sqrt{2n+3}} =$$

$$= \lim_{n\to\infty} \frac{-6}{6} \cdot \frac{\sqrt[3]{n^2}}{\sqrt{n}} \cdot \frac{\sqrt[3]{(1+4/n)^2}+\sqrt[3]{1+4/n}\,\sqrt[3]{1-2/n}+\sqrt[3]{(1-2/n)^2}}{\sqrt{2-3/n}+\sqrt{2+3/n}}$$

$n^{2/3-1/2} = n^{1/6}$

$\sqrt[3]{(1+0)^2}$ $\quad \sqrt[3]{1+0}$ $\quad \sqrt[3]{1-0}$ $\quad \sqrt[3]{(1-0)^2}$

$\sqrt{2-0}$ $\quad \sqrt{2+0}$

$$= -1 \cdot \infty \cdot \frac{1+1+1}{\sqrt{2}+\sqrt{2}} = -\infty$$

(2)

$$\lim_{x\to 0+} \left(1-\sqrt{\arcsin x}\right)^{\frac{1}{\sqrt[4]{1-\cos x}}}$$

$$= \lim_{x\to 0+} e^{\frac{1}{\sqrt[4]{1-\cos x}} \ln\left(1-\sqrt{\arcsin x}\right)} = e^{-\sqrt[4]{2}}$$

- inner function

$$\lim_{x\to 0+} \frac{\ln(1-\sqrt{\arcsin x})}{\sqrt[4]{1-\cos x}} = \lim_{x\to 0+} \frac{\ln(1-\sqrt{\arcsin x})}{-\sqrt{\arcsin x}} \cdot \frac{-\sqrt{\arcsin x}}{\sqrt[4]{x^2}} \cdot \frac{\sqrt[4]{x^2}}{\sqrt[4]{1-\cos x}}$$

$$\overset{AL}{=} 1 \cdot (-1) \cdot \sqrt[4]{2} = -\sqrt[4]{2}$$

◦ because • $\lim_{x\to 0+} \frac{\ln(1-\sqrt{\arcsin x})}{-\sqrt{\arcsin x}} = 1$.

$$\lim_{t\to 0} \frac{\ln(1+t)}{t} = 1 \qquad \lim_{x\to 0+} -\sqrt{\arcsin x} = -\sqrt{0} = 0$$

• $\lim_{x\to 0+} \frac{-\sqrt{\arcsin x}}{\sqrt[4]{x^2}} = \lim_{x\to 0+} -\sqrt{\underbrace{\frac{\arcsin x}{x}}_{\text{known limit}}} = -\sqrt{1}$

• $\lim_{x\to 0+} \frac{\sqrt[4]{x^2}}{\sqrt[4]{1-\cos x}} = \lim_{x\to 0+} \sqrt[4]{\frac{x^2}{1-\cos x}} = \sqrt[4]{\frac{1}{\frac{1}{2}}} = \sqrt[4]{2}$

because $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

- outer function

$$\lim_{u\to -\sqrt[4]{2}} e^u = e^{-\sqrt[4]{2}}$$

(3) $f(x) = \arctan \frac{x+3}{x+4}$

(1) domain: $x \neq -4$ $D_f = (-\infty, -4) \cup (-4, \infty)$

(2) $f(0) = \arctan \frac{3}{4} \doteq 0{,}64$ $[0, \arctan \frac{3}{4}]$

$0 = \arctan \frac{x+3}{x+4} \Rightarrow 0 = \frac{x+3}{x+4} \Rightarrow x = -3$ $[-3, 0]$

(3) f is not even, is not odd, is not periodic

has not symmetric domain

(4) $\lim\limits_{x \to \infty} \arctan \underbrace{\frac{x+3}{x+4}}_{\to 1} = \arctan 1 = \pi/4$

$\lim\limits_{x \to -\infty} \arctan \underbrace{\frac{x+3}{x+4}}_{\to 1} = \pi/4$

$\lim\limits_{x \to -4+} \arctan \underbrace{\frac{x+3}{x+4}}_{\frac{-1}{0+} \to -\infty} = -\pi/2$ $\lim\limits_{x \to -4-} \arctan \underbrace{\frac{x+3}{x+4}}_{\frac{-1}{0-} \to +\infty} = \pi/2$

($\arctan$ "$-\infty$" $= -\pi/2$)

(5) f is continuous at its domain

(6) $f' = \frac{1}{1+\left(\frac{x+3}{x+4}\right)^2} \cdot \frac{1(x+4) - 1\cdot(x+3)}{(x+4)^2} = \frac{1}{\frac{x^2+8x+16+x^2+6x+9}{(x+4)^2}} \cdot \frac{1}{(x+4)^2} =$

$= \frac{1}{\underbrace{2x^2+14x+25}_{\text{no roots}}}$ $D_{f'} = D_f$

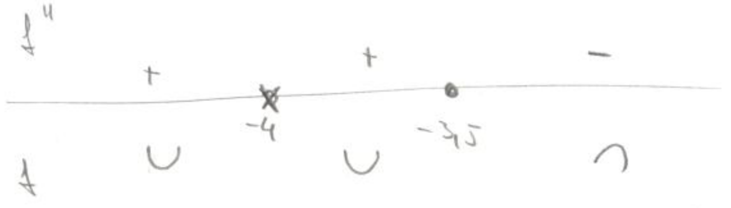
(7) no special points for derivative

(8) Monotonicity: $f' > 0$ on $D_{f'}$ $\Rightarrow$ f is increasing at $(-\infty, -4)$ and at $(-4, \infty)$

(9) No extrema suspects

(10) $f'' = -\dfrac{4x+14}{(2x^2+14x+25)^2}$ $\qquad D_{f''} = D_{f'} = D_f$

(11) $4x+14=0 \to 4x=-14 \to x = \frac{-14}{4} = -\frac{7}{2} = -3{,}5$



f is convex at $(-\infty,-4)$, $(-4,-3{,}5)$
concave $(-3{,}5;\ \infty)$

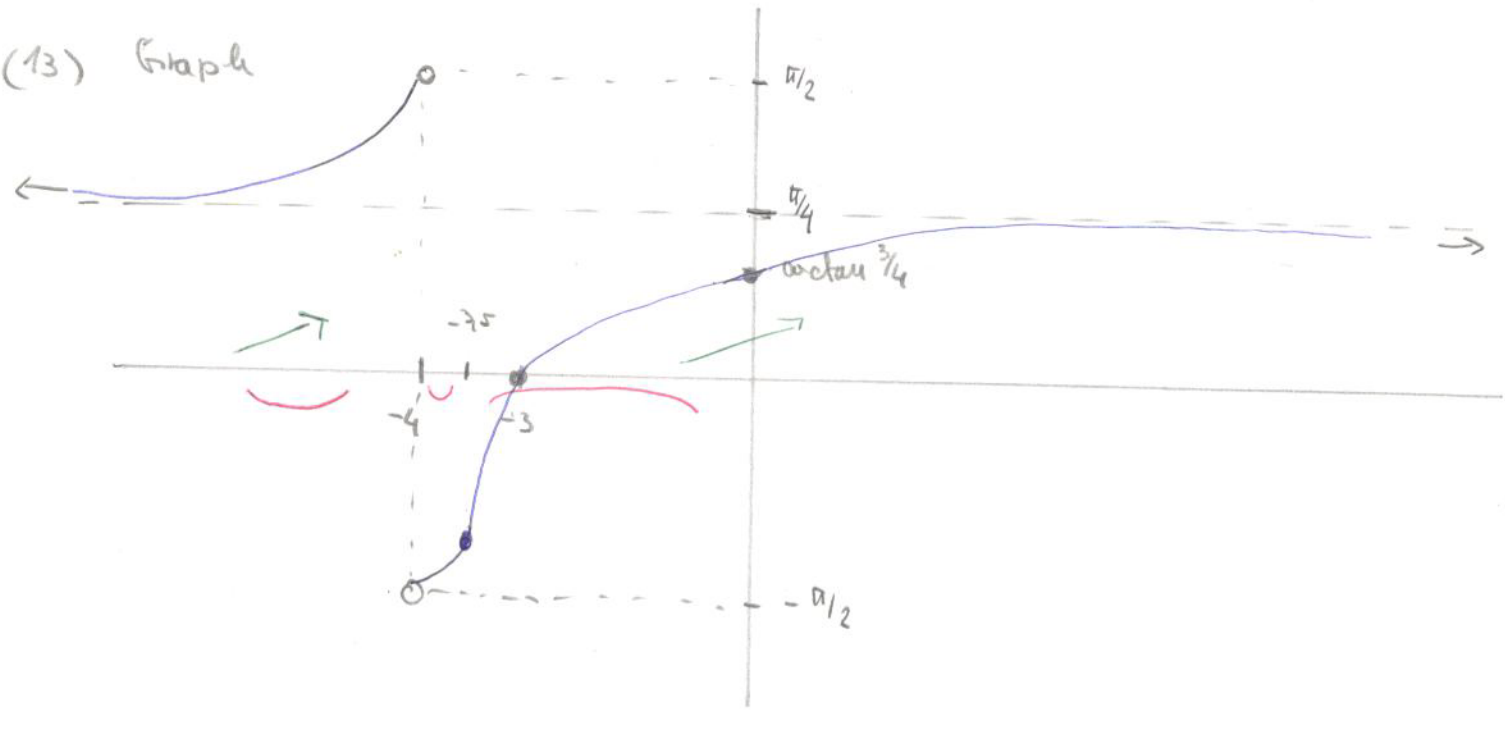
(12) Asymptotes:

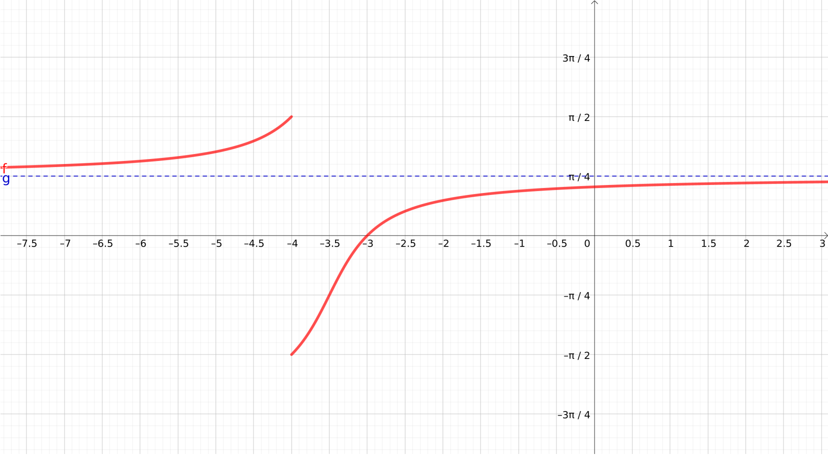
$a = \lim\limits_{x\to\pm\infty} \dfrac{\arctan\frac{x+3}{x+4}}{x} = \dfrac{"+\pi/4"}{\infty} = 0$

$b = \lim\limits_{x\to\pm\infty} \arctan\frac{x+3}{x+4} - 0\cdot x = + \frac{\pi}{4}$

asymptotes: $y = \pi/4$ (both at ∞ and $-\infty$)

(13) Graph





3π / 4
π / 2
π / 4
−π / 4
−π / 2
−3π / 4
−7.5
−7
−6.5
−6
−5.5
−5
−4.5
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
2.5
3
f
g