# Exam - sample

The exam consists of written and oral part. Their dates will be in SIS. Please, enroll to both parts of the exam. And let us know, if you have some special needs (the sooner the better).

## Written part

The written part comprises 3 questions:

1. limit of a sequence (especially roots or scale) - 6 points;
2. limit of a function (especially compound functions) - 6 points;
3. graph sketching - 13 points.

Students have 120 minutes and can use any literature (notes, tables, textbooks…), but no technical devices (phone, calculator, watches…).

### Example of the written part

1. (6 points) Find the limit of a sequence:

$$\lim_{n\to\infty} \frac{\sqrt{2n-3}-\sqrt{2n+3}}{\sqrt[3]{n+4}-\sqrt[3]{n-2}}.$$

2. (6 points) Find the limit of a function:

$$\lim_{x\to 0+} \left(1-\sqrt{\arcsin x}\right)^{\frac{1}{\sqrt[4]{1-\cos x}}}.$$

3. (13 points) Sketch the graph of a function:

$$f(x) = \arctan \frac{x+3}{x+4}.$$

## Oral part

The oral part tests knowledge of definitions, theorems, some proofs (those from the lecture) and concept questions (similar to those in the lecture).

The students have (almost arbitrary) time for preparation, but can not use any literature or electronic devices.

## Example of the oral part

1. (2x2 points) Definition of: **limit of a sequence**, **asymptote**.
2. (3 points) Statement of the theorem: **L'Hospital rule**.
3. (3+3 points) Statement and proof of the theorem: **Arithmetics of derivatives**.
4. (6x2 points) Answer concept questions:

   (a) Let $\{a_n\}$ is a sequnce such that $\lim_{n\to\infty} \frac{a_n}{n} = 0$. Which of the following statements needs to be true?

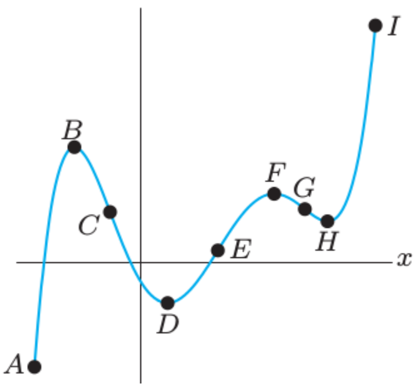
   i. $\lim_{n\to\infty} \frac{a_n}{e^n} = 0$
   ii. $\lim_{n\to\infty} \frac{a_n}{\sqrt{n}} = 0$
   iii. $\lim_{n\to\infty} \frac{a_n}{n!} = 0$
   iv. $\lim_{n\to\infty} \frac{a_n}{\ln n} = 0$
   v. $\lim_{n\to\infty} a_n = 0$

   (b) True or false?

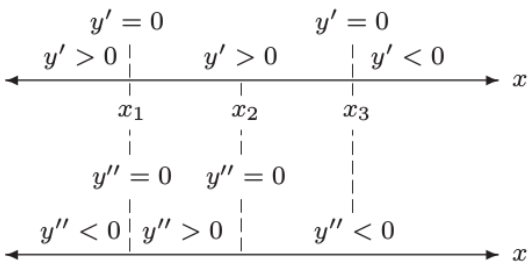
   i. If $f'(x) = g'(x)$, then $f = g$.
   ii. If $f'(a) \neq g'(a)$, then $f(a) \neq g(a)$.

   (c) Find intervals, where the function is:

   i. increasing and convex
   ii. increasing and concave
   iii. decreasing and convex
   iv. decreasing and concave



(d) Sketch a graph of a function $y = f(x)$. Function is continuous at $\mathbb{R}$.

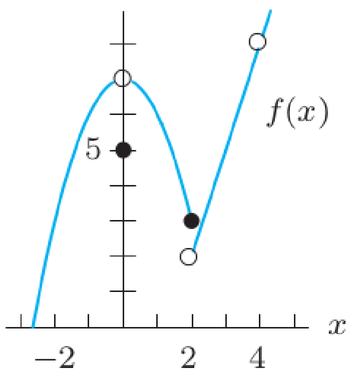



(e) Find $k \in \mathbb{R}$, such that the following function is continuous at $\mathbb{R}$.

$$f(x) = \begin{cases} kx, & x < 1, \\ x + 3, & 1 \leq x \end{cases}$$

(f) Find $\lim_{x\to 0} f(x)$

i. -3 ii. 0 iii. 5 iv. 7 v. $\infty$




Source 1: Calculus: Single and Multivariable, Hughes-Hallet