

①

$$\lim_{u \rightarrow \infty} \frac{\sqrt{2u-3} - \sqrt{2u+3}}{\sqrt[3]{u+4} - \sqrt[3]{u-2}}$$

$$= \lim_{u \rightarrow \infty} \frac{\sqrt{2u-3} - \sqrt{2u+3}}{\sqrt[3]{u+4} - \sqrt[3]{u-2}} \cdot \frac{\sqrt{2u-3} + \sqrt{2u+3}}{(\sqrt[3]{u+4})^2 + \sqrt[3]{u+4}\sqrt[3]{u-2} + (\sqrt[3]{u-2})^2} \cdot \frac{(\sqrt[3]{u+4})^2 + \sqrt[3]{u+4}\sqrt[3]{u-2} + (\sqrt[3]{u-2})^2}{\sqrt{2u-3} + \sqrt{2u+3}}$$

$$= \lim_{u \rightarrow \infty} \frac{2u-3 - (2u+3)}{u+4 - (u-2)} \cdot \frac{\sqrt[3]{(u+4)^2} + \sqrt[3]{u+4}\sqrt[3]{u-2} + \sqrt[3]{(u-2)^2}}{\sqrt{2u-3} + \sqrt{2u+3}}$$

$$= \lim_{u \rightarrow \infty} \frac{-6}{6} \cdot \frac{\sqrt[3]{n^2}}{\sqrt{n}} \cdot \frac{\sqrt[3]{(1+\frac{4}{n})^2} + \sqrt[3]{1+\frac{4}{n}}\sqrt[3]{1-\frac{2}{n}} + \sqrt[3]{(1-\frac{2}{n})^2}}{\sqrt{2-\frac{3}{n}} + \sqrt{2+\frac{3}{n}}}$$

$n^{2/3} = n^{1/6}$ $\sqrt[3]{(1+0)^2} \rightarrow \sqrt[3]{1+0}$ $\sqrt[3]{1-0} \rightarrow \sqrt[3]{1-0}$ $\sqrt[3]{(1-0)^2} \rightarrow \sqrt[3]{1-0}$
 $\sqrt{2-0} \rightarrow \sqrt{2+0}$

$$= -1 \cdot \infty \cdot \frac{1+1+1}{\sqrt{2}+\sqrt{2}} = -\infty$$

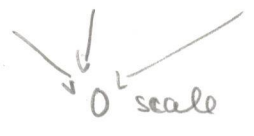
②

$$\lim_{n \rightarrow \infty} \frac{n! + 2^n - 3\cos(n^2) + 3n^4}{2n^4 + \ln n - 2(n!) - e^n}$$

$$= \lim_{n \rightarrow \infty} \frac{n!}{n!} \cdot \frac{1 + \frac{2^n}{n!} - 3 \frac{\cos(n^2)}{n!} + 3 \frac{n^4}{n!}}{2 \cdot \frac{n^4}{n!} + \frac{\ln n}{n!} - 2 - \frac{e^n}{n!}}$$

scale 0 bounded 0 • zero
 ↑ ↑ ↑
 0 2^n cos(n^2) n^4 scale 0
 ↓ ↓ ↓ ↓ ↓
 1 n! n! n! n!
 2 · n^4/n! + ln n/n! - 2 - e^n/n! scale 0

$$= \frac{1+0-0+0}{0+0-2-0} = -\frac{1}{2}$$

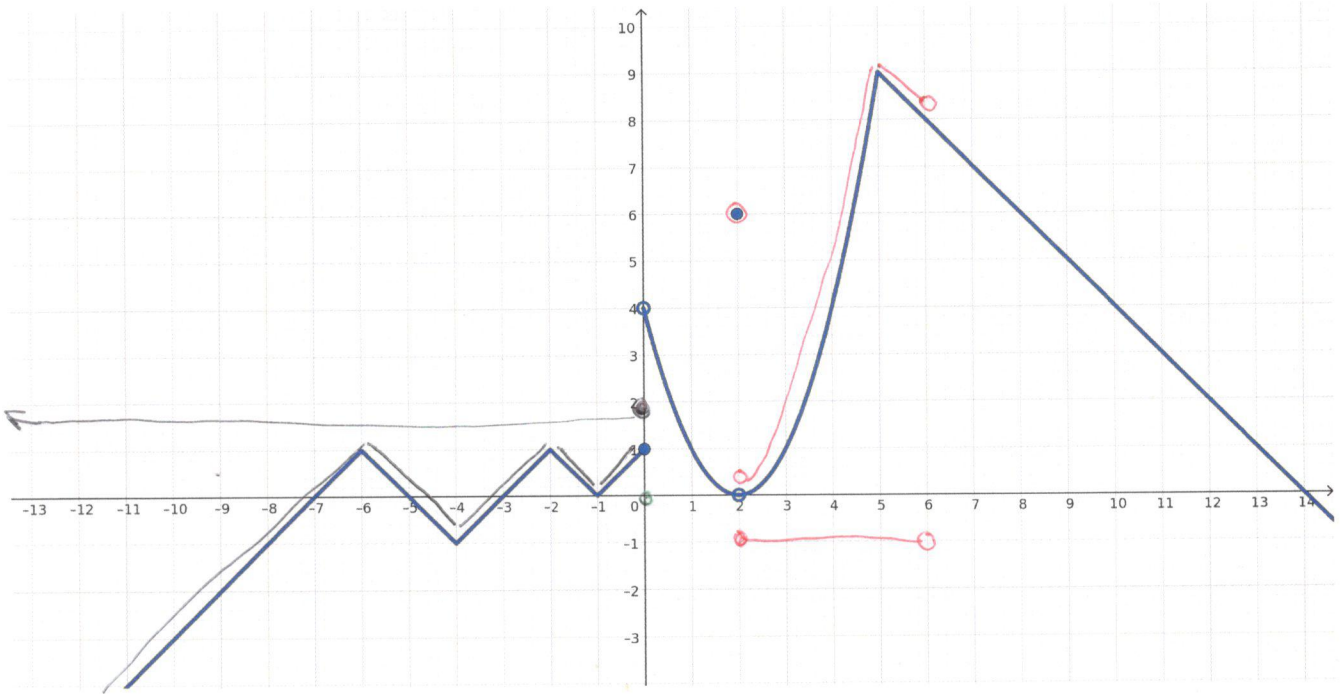


Image

$$f([2,6]) = (0, 9]$$

$$f((-\infty, 0)) = (-\infty, 1]$$

$$f(\{0\}) = \{1\}$$



Preimage

$$f^{-1}([3,7]) = (0, 1) \cup \{2\} \cup (3, 4] \cup [7, 11]$$

$$f^{-1}(\{4\}) = \{4, 10\}$$

$$f^{-1}((-\infty, 0]) = (-\infty, -7] \cup [-5, -3] \cup \{-1\} \cup [14, \infty)$$

