

(4) Sine, Cosine

$$(1) \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = 1$$

$$\lim_{x \rightarrow 0} 5x = 0 \quad \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$$

$$(2) \lim_{x \rightarrow 0} \frac{\sin x^2}{x^2} = 1$$

$$\lim_{x \rightarrow 0} x^2 = 0 \quad \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$$

$$(3) \lim_{x \rightarrow 0} \frac{\sin 3x^2}{x^2} = \lim_{x \rightarrow 0} \frac{\sin 3x^2}{3x^2} \cdot 3 = 1 \cdot 3 = \underline{3}$$

$$\lim_{x \rightarrow 0} 3x^2 = 0 \quad \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$$

$$(4) \lim_{x \rightarrow 0} \frac{\lg x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x} = \lim_{x \rightarrow 0} \cos x \cdot \frac{\sin x}{x} = 1 \cdot 1 = \underline{1}$$

$$(5) \lim_{x \rightarrow 0^+} \frac{\lg \sqrt{x}}{\sqrt{x}} = 1$$

$$\lim_{x \rightarrow 0^+} \sqrt{x} = 0^+ \quad \lim_{y \rightarrow 0} \frac{\lg y}{y} = 1$$

$$(6) \lim_{x \rightarrow 0^+} \frac{\lg \sqrt{x}}{\sqrt{2x}} = \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{2}} \cdot \frac{\lg \sqrt{x}}{\sqrt{x}} = \frac{1}{\sqrt{2}} \cdot 1 = \underline{\frac{1}{\sqrt{2}}}$$

$$(7) \lim_{x \rightarrow 0} \ln \left(\frac{x}{\sin x} \right) = \underline{0}$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = \lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x}} = 1 \quad \lim_{y \rightarrow 1} \ln y = 0$$

$$(8) \lim_{x \rightarrow 0} \frac{1 - \cos x^2}{x^4} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} x^2 = 0 \quad \lim_{y \rightarrow 0} \frac{1 - \cos y}{y^2} = \frac{1}{2}$$

$$(9) \lim_{x \rightarrow 0} \frac{x^4}{1 - \cos x^2} = \lim_{x \rightarrow 0} \frac{1}{\frac{1 - \cos x^2}{x^2}} = \frac{1}{\frac{1}{2}} = 2$$

$$(10) \lim_{x \rightarrow 0} \frac{x^4}{1 - \cos(4x^2)} = \lim_{x \rightarrow 0} \frac{1}{16} \cdot \frac{16x^4}{1 - \cos 4x^2} = \frac{1}{16} \cdot 2 = \frac{1}{8}$$

$$\lim_{x \rightarrow 0} 4x^2 = 0 \quad \lim_{y \rightarrow 0} \frac{y^2}{1 - \cos y} = 2$$

$$(11) \lim_{x \rightarrow 0} x \cot 3x = \lim_{x \rightarrow 0} \frac{3x \cdot \cos 3x}{3 \sin 3x} = \frac{1}{3} \cdot 1 \cdot 1 = \frac{1}{3}$$

$$\lim_{x \rightarrow 0} 3x = 0 \quad \lim_{y \rightarrow 0} \frac{y}{\sin y} = 1$$

(5) Exponenciale a logaritmus

$$(1) \lim_{x \rightarrow 0} \frac{\ln(1+3x)}{3x} = 1$$

$$\lim_{x \rightarrow 0} 3x = 0 \quad \lim_{y \rightarrow 0} \frac{\ln(1+y)}{y} = 1$$

$$(2) \lim_{x \rightarrow 0} \frac{\ln(1+3x)}{x} = \lim_{x \rightarrow 0} 3 \frac{\ln(1+3x)}{3x} = 3 \cdot 1 = 3$$

$$(3) \lim_{x \rightarrow \infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = 1$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0 \quad \lim_{x \rightarrow 0} \frac{\ln(1+y)}{y} = 1$$

$$(4) \lim_{x \rightarrow \infty} x \ln(1+\frac{1}{x}) = \lim_{x \rightarrow \infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = 1$$

$$(5) \quad \lim_{x \rightarrow 1} \frac{\ln(1+(x-1))}{x-1} = 1$$

$$\lim_{x \rightarrow 1} x-1 = 0 \quad \lim_{y \rightarrow 0} \frac{\ln(1+y)}{y} = 1$$

$$(6) \quad \lim_{x \rightarrow 1} \frac{\ln x}{x-1} = \lim_{x \rightarrow 1} \frac{\ln(1+(x-1))}{x-1} = 1$$

$$(7) \quad \lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{x^2} = 1$$

$$\lim_{x \rightarrow 0} x^2 = 0 \quad \lim_{y \rightarrow 0} \frac{\ln(1+y)}{y} = 1$$

$$(8) \quad \lim_{x \rightarrow 0} \frac{-x^2}{\ln(1-x^2)} = \lim_{x \rightarrow 0} \frac{1}{\frac{\ln(1-x^2)}{-x^2}} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0} -x^2 = 0 \quad \lim_{y \rightarrow 0} \frac{\ln(1+y)}{y} = 1$$

$$(9) \quad \lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{\ln(1-x^2)} = \lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{x^2} \cdot \frac{-x^2}{\ln(1-x^2)} \cdot (-1) = -1$$

$$(10) \quad \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = \lim_{x \rightarrow 0} 2 \cdot \frac{e^{2x} - 1}{2x} = 2 \cdot 1 = 2$$

$$\lim_{x \rightarrow 0} 2x = 0 \quad \lim_{y \rightarrow 0} \frac{e^y - 1}{y} = 1$$

$$(11) \quad \lim_{x \rightarrow \infty} \frac{e^{1/x} - 1}{1/x} = 1$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0 \quad \lim_{y \rightarrow 0} \frac{e^y - 1}{y} = 1$$

$$(12) \quad \lim_{x \rightarrow \infty} x(e^{1/x} - 1) = \lim_{x \rightarrow \infty} \frac{e^{1/x} - 1}{1/x} = 1$$

$$(13) \quad \lim_{x \rightarrow 0} \frac{e^{bx} - 1}{bx} = 1$$

$$\lim_{x \rightarrow 0} bx = 0 \quad \lim_{y \rightarrow 0} \frac{e^y - 1}{y} = 1$$

$$(14) \lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x} = \lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{\sin x} \cdot \frac{\sin x}{x} = 1 \cdot 1 = \underline{1}$$

(6) Bonusové příklady

$$(1) \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = \lim_{x \rightarrow 0} (-1) \cdot \frac{1 - \cos x}{x^2} \cdot x = (-1) \cdot \frac{1}{2} \cdot 0 = \underline{0}$$

$$(2) \lim_{x \rightarrow 0^+} \frac{\sin \sqrt{x}}{x} = \lim_{x \rightarrow 0^+} \frac{\sin \sqrt{x}}{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} = \infty$$

\downarrow \downarrow
 1 "0+"

$$(3) \lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{\sin x} = \lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{x} \cdot \frac{x}{\sin x} =$$

$$= \lim_{x \rightarrow 0} \left(5 \cdot \frac{\sin 5x}{5x} - 3 \cdot \frac{\sin 3x}{3x} \right) \cdot \frac{x}{\sin x} = (5 \cdot 1 - 3 \cdot 1) \cdot 1 = \underline{2}$$

\downarrow \downarrow \downarrow
 1 1 1

$$(4) \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{\tan(\tan x)} = \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{\sin x} \cdot \frac{\sin x}{x} \cdot \frac{x}{\tan x} \cdot \frac{\tan x}{\tan(\tan x)} = 1$$

\downarrow \downarrow \downarrow \downarrow
 1 1 1 1

$$\lim_{x \rightarrow 0} \sin x = 0 \quad \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \Rightarrow \lim_{x \rightarrow 0} \frac{\sin(\sin x)}{\sin x} = 1$$

$$\lim_{x \rightarrow 0} \tan x = 0 \quad \lim_{y \rightarrow 0} \frac{y}{\tan y} = 1 \Rightarrow \lim_{x \rightarrow 0} \frac{\tan x}{\tan(\tan x)} = 1$$

$$(5) \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \lim_{x \rightarrow 0} \frac{e^{x \ln a} - 1}{x} = \lim_{x \rightarrow 0} \frac{e^{x \ln a} - 1}{x \ln a} \cdot \ln a = 1 \cdot \ln a$$

$$\lim_{x \rightarrow 0} x \ln a = 0$$

\downarrow
 ln a je číslo

$$\lim_{y \rightarrow 0} \frac{e^y - 1}{y} = 1$$

(7) AVO

Ne - lim. fy musí být stejné

Ne - třeba různé

Ne - - u -