Mathematics I - List of concept questions

21/22

Sets

Exercise (True or false)

A - set of all animals living in Australia.

A $a \in A$ **B** $b \in A$ **C** $c \in A$ **D** $d \in A$ **E** $e \in A$

a

c

b

d

e

Sets

Exercise (True or false)

A - set of all animals living in Australia.

A $a \notin A$ **B** $b \notin A$ **C** $c \notin A$ **D** $d \notin A$ **E** $e \notin A$

Sets - questions

Exercise

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{1, 2, 3, 4, 5\}$. Find

1. $A \cup B$
2. $A \cap B$
3. A^c
4. $(B^c)^c$
5. $A \setminus B$
6. $B \setminus A$

Exercise (True or false)

Let A be a set.

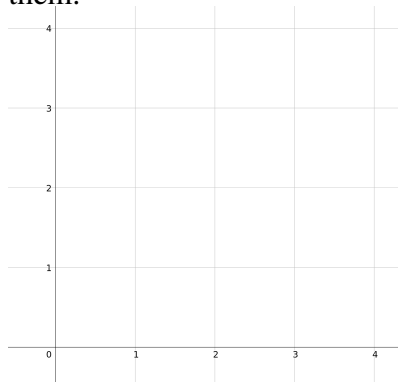A. $\emptyset \in A$
B. $\emptyset \subset A$
C. $0 = \emptyset$
D. $\{x\} \in \{x, y, z\}$
E. $x \in \{x, y, z\}$

Sets

Exercise

Let $A = \{1, 2, 3\}$, $B = \{2, 4\}$. Find $A \times B$, $B \times B$ and sketch them.

Exercise

Let $A_1 = \{0, 1\}, A_2 = \{0, 2\}, A_3 = \{0, 3\}$. Find

1. $\displaystyle\bigcup_{i=1}^{3} A_i$

2. $\displaystyle\bigcap_{i \in \{1,2,3\}} A_i$

Exercise

Which sets are bounded from below? Bounded from above?
Bounded?

A $\mathbb{N}$

B $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\}$

C $\mathbb{R} \setminus \mathbb{Q} \cap (-3, 2]$

D $\{x \in \mathbb{R} : x < \pi\}$

E $(-\infty, -1) \cup \{0\} \cup [1, \infty)$

Exercise

Find infimum, minimum, maximum and supremum:

1. $\{1, 2, 3, 4\}$
2. $[-2, 3]$
3. $(-2, 3)$
4. $(-2, 3]$
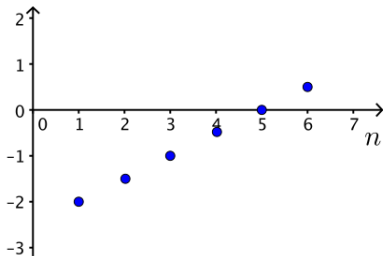5. $[-2, -1) \cup (0, 25]$

6. $(-7, -0) \cup (1, 2)$
7. $[0, \infty)$
8. $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$
9. $\mathbb{N}$

Exercise

Find the formula for a_n.

A. $a_n = (-\frac{1}{2})^n - \frac{3}{2}$

B. $a_n = \frac{1}{2}n + 5$

C. $a_n = \frac{1}{2}n - 2$

D. $a_n = -\frac{1}{2}n + \frac{5}{2}$

E. $a_n = \frac{1}{2}n - \frac{5}{2}$

Figure:
https://www.cpp.edu/conceptests/question-library/mat116.shtml

Exercise

Find the first 4 terms of a sequences

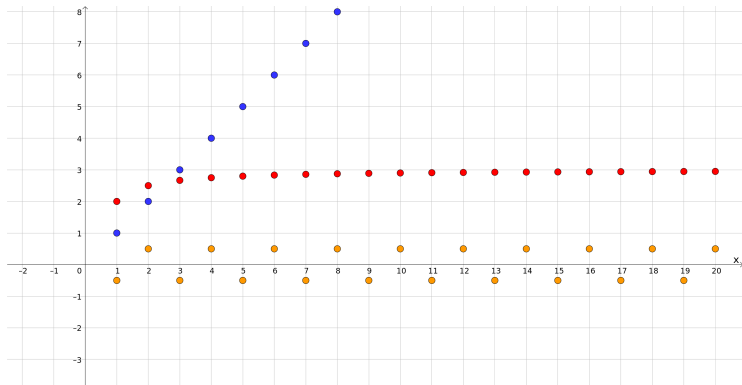A $a_n = \dfrac{(-1)^n}{n}$

B $a_n = \dfrac{n+1}{n}$

Exercise

Find the formula for the following sequence

A $1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \cdots$

B $-1, \dfrac{1}{2}, \dfrac{-1}{3}, \dfrac{1}{4}, \dfrac{-1}{5} \cdots$

Exercise

Which of these sequences are bounded?

A blue   B red   C yellow

Exercise

Find non-decreasing sequences.

A $a_n = \ln n$

B $a_n = e^{-n}$

C $a_n = -4$

D $a_n = \dfrac{(-1)^n}{3^n}$

E $a_n = (-2)^n$

Exercise

Check, if the sequence is monotone:

1. $a_n = \dfrac{n}{4 + n^2}$

2. $a_n = \dfrac{n}{n + 1}$

Exercise

Let $a_n = 1, 2, 3, 4, 5, \ldots$, $b_n = (-1)^n$. Find

A $a_n + b_n$ B a_n / b_n C $3a_n$

Exercise

Find a sequence, which is

1. bounded and covergent
2. bounded and divergent
3. unbounded and covergent
4. unbounded and divergent

Exercise

Let $a_n = 3, 7, 4, 1/2, \pi, -1$. Find $b_n = a_{2n}$:

A $6, 14, 8 \ldots$

B $5, 9, 6 \ldots$

C $7, 1/2, -1 \ldots$

D $4, 1/2, \pi \ldots$

By:https://www.cpp.edu/conceptests/
question-library/mat116.shtm

Exercise (True or false)

Let $\lim a_n = A \in \mathbb{R}$ and $\lim b_n = B \in \mathbb{R}$.
If $a_n < b_n$, then $A < B$.

Exercise

Find the sandwich for the sequence $a_n = \dfrac{\cos n}{n}$.

Exercise

Give an example of $a_n \to \infty$ and find its lower bound.

Exercise

1. $2 + \infty$
2. $-\infty + 3$
3. $\pi\infty$
4. $-4(-\infty)$
5. -7∞
6. $\frac{\infty}{-3}$
7. $\frac{5}{\infty}$

Exercise

Find a sequence $\{x_n\}$, $x_n \to \sup M$ for a set $M = [2, 5)$.

Exercise

Find the convergent subsequence:

A $a_n = (-1)^n$

B $a_n = \{0, 2, 0, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 2, \ldots\}$

Exercise

Find the domain and range for the following mappings:
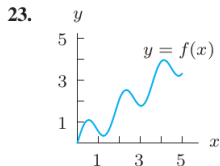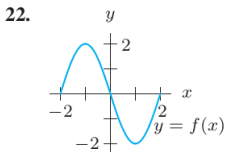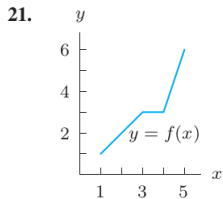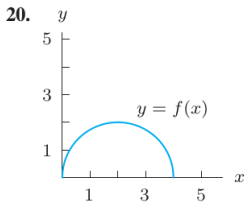
Figure: Calculus: Single and Multivariable, 6th Edition, Hughes-Hallett, col.

Exercise

Which of the following functions has its domain the same as its range?

A x^2 B $\sqrt{x}$ C x^3 D $|x|$ E $2x - 3$

(Inspired by: Active Calculus & Mathematical Modeling,
Carroll College Mathematics Department)

Exercise

Find the image:

A $[-6, -2]$ B $[-1, 1)$ C $[0, 2)$ D $[2, \infty)$
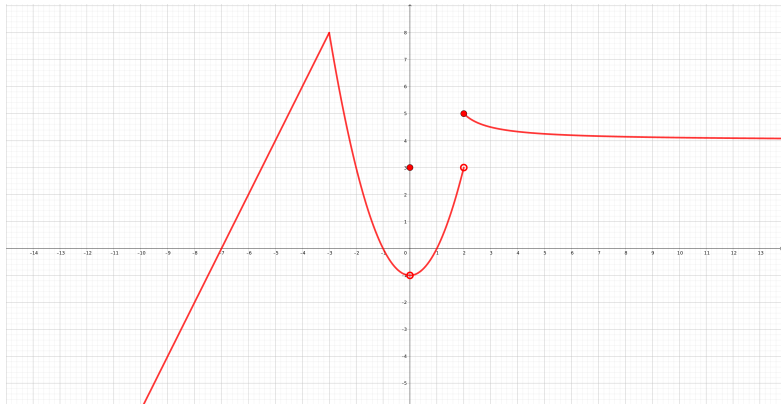
Exercise

Find the preimage:

A $\{-1\}$ B $[2,3]$ C $[0,1]$ D $[0,1)$
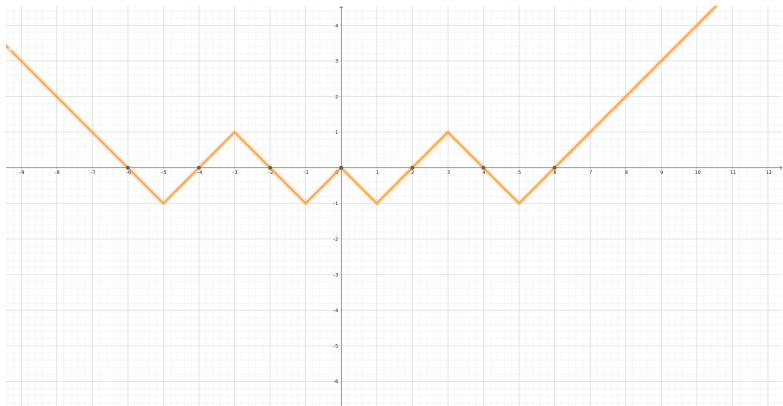
Exercise
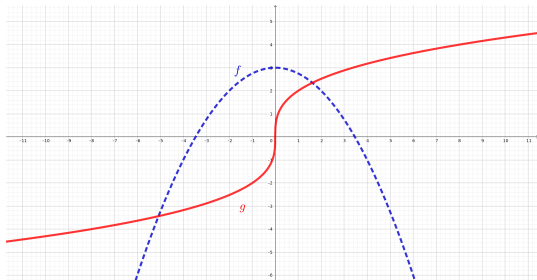
Find $g(f(4))$.

   A -2   B -1   C 0   D 1   E 2

Find x, if $f(g(x)) = 2$.

Exercise

In the tables we can find values of functions f and g.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

Find $g(f(1))$.

  A -2   B -1   C 0   D 1   E 2

Find $f(f(0))$.

  A -2   B -1   C 0   D 1   E 2

Exercise

In the tables we can find values of functions f and g. If $f(g(x)) = -2$, find x.

x	-2	-1	0	1	2
$f(x)$	1	0	-2	2	-1
$g(x)$	-1	1	2	0	-2

A -2   B -1   C 0   D 1   E 2

Exercise

  A e^x B x^3 C $\sin x$ D $\tan x$ E $\frac{1}{x}$

Which functions are onto?
Which functions are one-to-one?
Which functions are bijections?

Exercise
Find inverse mappings at $\mathbb{R}$:

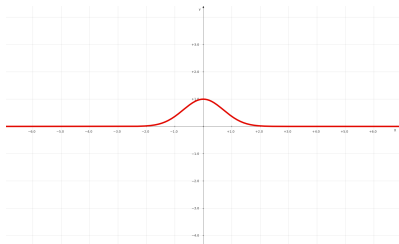 A $2x + 1$ C $\sqrt[3]{x}$

  B e^x D x^2

Exercise

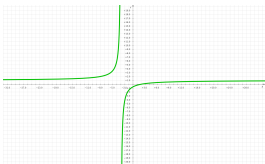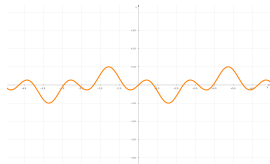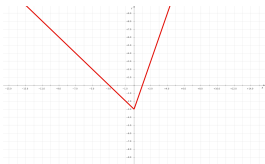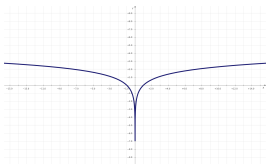Decide, which functions are monotone on its domain:
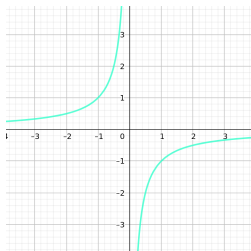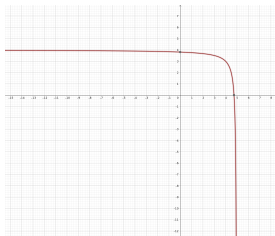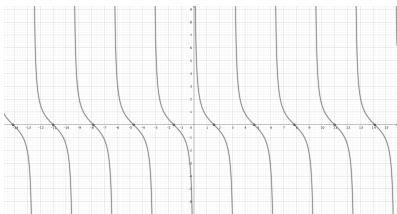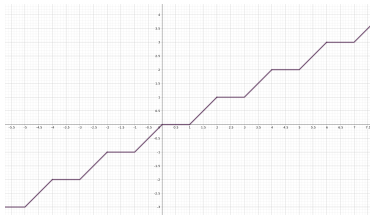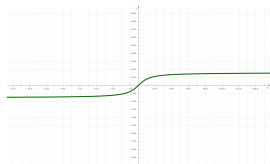
Exercise

Decide, which functions are bounded, bounded from above,
below:
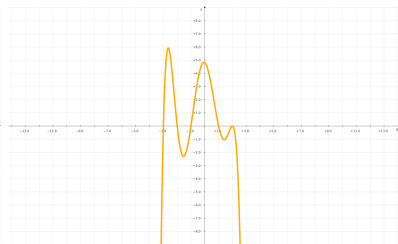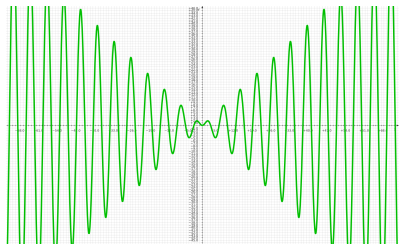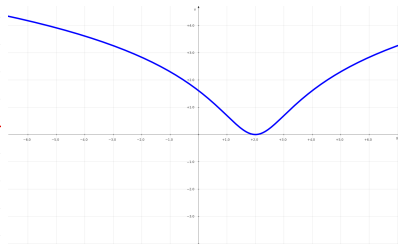
Exercise

Decide, which functions are even or odd:

Decide, which functions are even or odd:
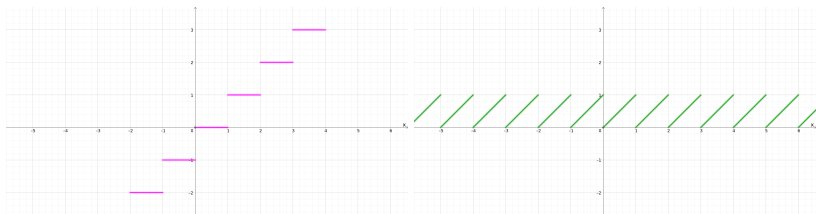
A $x^3 + 1$ C $|x - 2|$ E $|1 + \cos x|$
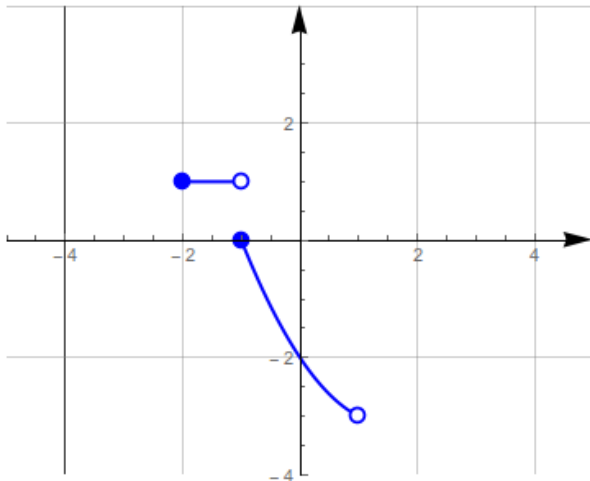B $x(x^2 + 1)$ D $e^{x^2} \sin x$

Exercise

Decide, which functions are periodic

Exercise

Sketch in the function so that it is periodic with the smallest possible period

Exercise

Find $\lim_{x \to 0} f(x)$

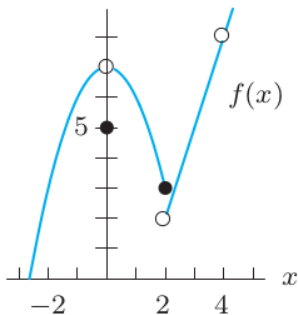A -3   B 0   C 5   D 7   E ∞

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find $\lim_{x \to 2} f(x)$

A ∞

B 3

C 2

D 0

E does not exist
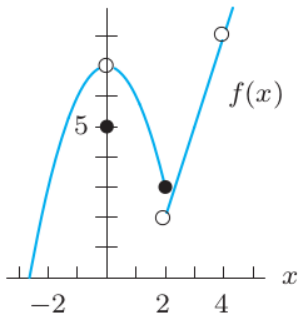

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find $\lim_{x \to 4} f(x)$

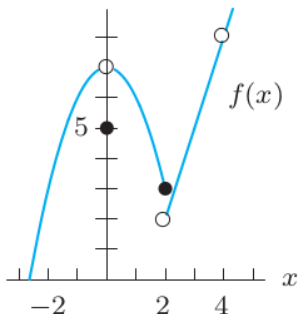A 4   C 0   E does   exists

B 8   D ∞ not

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find $\lim_{x \to 2^-} f(x)$.

Find $\lim_{x \to 2^+} f(x)$.

A 0   C 2   E does not exist

B 1   D 3

$f(x)$

5

x

-2 2 4
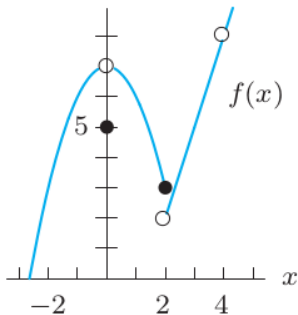
Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find $\lim_{x \to 1+} f(x) + 2g(x)$

A 13

B 9

C 8

D 6

E 3
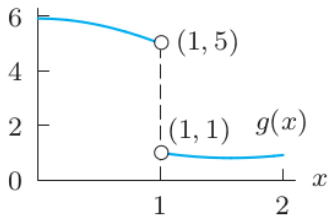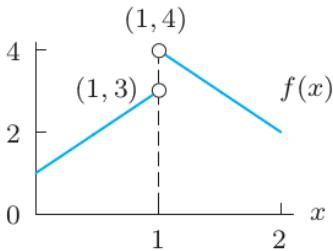

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

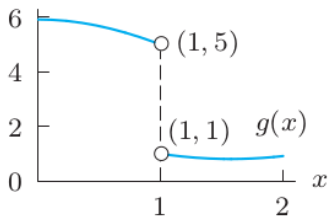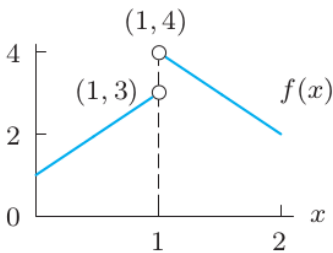Find $\lim_{x \to 1-} f(x)g(x)$

A 20

C 4

E does not exist

B 15

D 3

Figure: Calculus: Single and Multivariable, Hughes-Hallet

Exercise
Which functions are continuous at $\mathbb{R}$?

A $x^3 + \sin(4 - x)$ C $\frac{2+x}{e^x}$ E $\ln(2 + x^2)$

B $\frac{e^x}{2+x}$ D $\cos(e^{\sqrt[3]{x}})$

Example

$$\lim_{x \to 0} (\sin x)(\operatorname{sign} x)$$

Exercise

$$\lim_{x \to \infty} \ln\left(\frac{x-1}{x+2}\right)$$

A 0   B 1   C $\ln 1$ D $-\frac{1}{2}$ E ∞

Exercise

$$\lim_{x \to -\infty} \cos \frac{1}{x}$$

A 0   C π E does not exist

B 1   D $-\infty$

Exercise

$$\lim_{x \to 0} \arctan \frac{1}{x^2}$$

A 0   B 1   C $\frac{\pi}{2}$ D $-\frac{\pi}{4}$ E ∞

Exercise

Is there a solution between 0 and 2?

- $x^5 - 2x - 1 = 0$
- $x^3 - 4x^2 + 4x + 1 = 0$
- $5x^3 - 15x^2 + 10x + 1 = 0$

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https:
//www.geogebra.org/calculator/pqbtmk54
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Exercise

Find the image of the interval $(-1, 2]$ under the functions
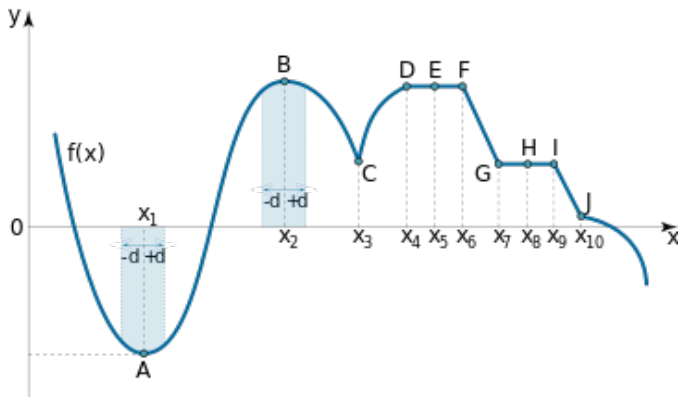
- x^2
- $\operatorname{sign} x$

Exercise

Find local extrema:

Exercise

$f = \cos x \sin x$. Find f'.

A $\cos^2 x$ C $\cos^2 x - \sin^2 x$

B $\sin^2 x$ D $-\sin x \cos x$

Exercise

$f = e^7$. Find f'.

A $7e^6$ B e^7 C 0

Exercise

$f = \dfrac{e^x}{x^2}$ Find f'.

A $\dfrac{e^x}{2x}$ C $\dfrac{e^x x^2 - 2xe^x}{x^4}$

B $\dfrac{e^x(x-2)}{x^3}$ D $\dfrac{e^x 2x + x^2 e^x}{x^4}$

Exercise

$f = \sin x + e^{\sin x}$ Find f'.

  A $\cos x + e^{\cos x}$

  B $\cos x + e^{\sin x}$

  C $\cos x + \sin x\, e^{\cos x}$

  D $\cos x + \cos x\, e^{\sin x}$

Exercise (True or false?)

1. If $f'(x) = g'(x)$, then $f(x) = g(x)$. (For every x.)
2. If $f'(a) \neq g'(a)$, then $f(a) \neq g(a)$.
   (We are talking about particular point a.)

$$\lim_{x \to \infty} \frac{\ln x}{x} =$$

A ∞ B 0 C 1 D $\nexists$

Exercise

Let us assume that a function $y = f(x)$ is continuous at $\mathbb{R}$.
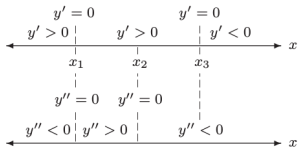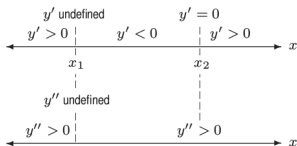Sketch f.

Exercise

Let us assume that a function $y = f(x)$ is continuous at $\mathbb{R}$.
Sketch f.

Figure: Calculus, Hughes-Hallet, Gleason, McCallum