Limit of a function

Theory

Theorem 1 (Squeeze theorem)**.** Let I be an interval having the point a as a limit point. Let g, f, and h be function defined on I, except possibly at a itself. Suppose that for every x in I not equal to a, we have

$$g(x) \leq f(x) \leq h(x)$$

and also suppose that

$$\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L.$$

Then

$$\lim_{x \to a} f(x) = L.$$

Facts

1. $\beta > 0$, $a > 1$: $\lim_{x \to +\infty} \frac{x^\beta}{a^x} = 0$.
2. $\alpha > 0$, $\beta > 0$: $\lim_{x \to +\infty} \frac{\ln^\alpha x}{x^\beta} = 0$.

Exercises

1. Find all the mistakes in the joke:

Know your limits

$$\lim_{x \to 8} \frac{1}{x - 8} = \infty.$$

Therefore

$$\lim_{x \to 5} \frac{1}{x - 5} = \text{ש}.$$

Zdroj 1: https://kityates.com/public-engagement/

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2. Find limits:

(a) $\lim_{x\to\infty} \dfrac{-2x+3}{3x^2+1}$

(b) $\lim_{x\to\infty} \dfrac{-2x^2+3}{3x^2+1}$

(c) $\lim_{x\to\infty} \dfrac{-2x^3+3}{3x^2+1}$

(d) $\lim_{x\to\infty} \dfrac{1}{x^2-x-1}$

(e) $\lim_{x\to\infty} \dfrac{x}{\sqrt{3x^2+2}}$

(f) $\lim_{x\to\infty} \dfrac{x-\cos x}{x}$

(g) $\lim_{x\to 0} x^2 \sin\dfrac{1}{x}$

(h) $\lim_{x\to\infty} \dfrac{2^x+3^x}{2^{x+1}+3^{x+1}}$

(i) $\lim_{x\to\infty} \dfrac{1^x+2^x+3^x+4^x+5^x}{5,0001^x}$

(j) $\lim_{x\to\infty} \dfrac{\ln x + x^3 + \frac{1}{x} + e^x + 5^x}{\ln_{10} x + x^4 + 5^x + x^3 + 4^x}$

(k) $\lim_{x\to\infty} \dfrac{\sin x}{x}$

(l) $\lim_{x\to\infty} e^{-x}\cos x$

(m) $\lim_{x\to\infty} \dfrac{x+\sin x}{x-\sin x}$

(n) $\lim_{x\to 0+} x\cos\left(\dfrac{x+3}{\sqrt{x}-1}\right)$

(o) $\lim_{x\to\infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$

(p) $\lim_{x\to\infty} e^x\cos x$

(q) $\lim_{x\to\infty} \dfrac{x}{\sin x}$

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3. Find limits:

(a) $\lim_{x\to 1} \dfrac{x-1}{x^2+x-2}$

(b) $\lim_{x\to -3} \dfrac{x^2+x-6}{x+3}$

(c) $\lim_{x\to 2} \dfrac{x^2+3x-4}{x^2-4x+4}$

(d) $\lim_{x\to 0} \dfrac{1}{\sin x}$

(e) $\lim_{x\to -2} \dfrac{-4}{x+2}$

(f) $\lim_{x\to 4} \dfrac{3}{(4-x)^3}$

(g) $\lim_{x\to 3} \dfrac{2x}{x-3}$

(h) $\lim_{x\to 4} \dfrac{x^2}{x^2-16}$

(i) $\lim_{x\to -3} \dfrac{x^2-2x-3}{x^2+6x+9}$

(j) $\lim_{x\to -\infty} \dfrac{1}{e^x}$

(k) $\lim_{x\to 0} \dfrac{|2x|}{x}$