(1) $\arcsin(x+y) + \arctan(x+y) + xy = 0$ $[0,0]$

$F(x,y) = \arcsin(x+y) + \arctan(x+y) + xy$

- $F \in C^1(G)$ $\quad G = \{[x,y] \in \mathbb{R}^2 : -1 < x+y < 1\}$

  $[0,0] \in G$, G is open

- $F(0,0) = \arcsin 0 + \arctan 0 + 0 = 0$ ✓

- $$\frac{\partial F}{\partial y} = \frac{1}{\sqrt{1-(x+y)^2}} + \frac{1}{1+(x+y)^2} + x$$

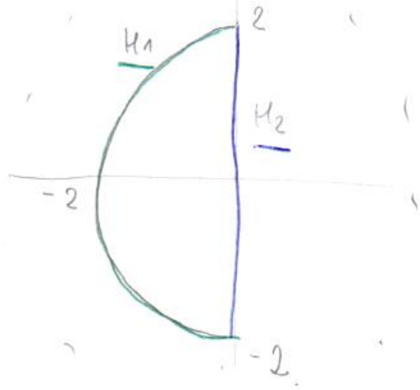
  $$\frac{\partial F}{\partial y}(0,0) = \frac{1}{\sqrt{1}} + \frac{1}{1} + 0 = 2 \neq 0$$

Impl. function thm ✓

$$\frac{\partial F}{\partial x} = \frac{1}{\sqrt{1-(x+y)^2}} + \frac{1}{1+(x+y)^2} + y \qquad \frac{\partial F}{\partial x}(0,0) = \frac{1}{\sqrt{1}} + \frac{1}{1} + 0 = 2$$

$$f'(0) = -\frac{\partial F/\partial x}{\partial F/\partial y}(0,0) = -\frac{1}{1} = \boxed{-1}$$

(2) $f(x,y) = -y^2 + x^2 + \frac{4}{3}x^3$



$M = \mathrm{bd}\,\{x^2+y^2 \le 4,\ x \le 0\}$

- M is bounded $\quad M \subset B((0,0),3)$
- M is closed $\quad$ (it is boundary)
- f is continuous

$\Big\}$ f attains extrema

(a) M_1 : $\quad x^2+y^2 = 4, \quad x < 0$

Lagrange multipliers: $\quad g = x^2+y^2-4 \qquad x \in [-2,0)$

$f, g \in C^\infty(\mathbb{R}^2)$

(1) $\nabla g : (2x, 2y) = (0,0) \to x = y = 0$, but $(0,0) \notin M_1$

$0^2+0^2 \neq 4$

(2) $\nabla f + \lambda \nabla g = (0,0)$

$$2x + 4x^2 + \lambda 2x = 0$$
$$-2y + \lambda 2y = 0 \quad \to \quad 2y(1-\lambda) = 0$$
$$x^2+y^2 = 4$$

$y = 0$:

$x^2 = 4$

$x = \pm 2$

$[-2,0]$ ✓

$[2,0] \notin M$ X

$\lambda = 1$:

$2x + 4x^2 + 2x = 0$

$4x(x+1) = 0$

$x = 0$: $\quad y^2 = 4$, $\quad y = \pm 2$

$[0, 2]$, $[0,-2]$ $\notin M_1$ X

$x = -1$: $\quad y^2 = 3$, $\quad y = \pm\sqrt{3}$

$[-1, \pm\sqrt{3}]$ ✓

(b) $M_2 \qquad x = 0 \qquad y \in (-2,2)$

$f(0,y) = -y^2 \to$ point $(0,0)$

edges

Suspect points:

$(0,0)$, $(-2,0)$, $(-1,\sqrt{3})$, $(-1,-\sqrt{3})$, $(0,2)$, $(0,-2)$

f value: 0 $\quad -20/3$ $\quad -10/3$ $\quad -10/3$ $\quad -4$ $\quad -4$

glob. max $\quad$ glob min

(3) $(1{,}04)^{2{,}02}$

$f(x,y) = x^y = e^{y \ln x}$ at point $(1,2)$ $\qquad f(1,2) = 1^2 = 1$

$\frac{\partial f}{\partial x} = y\,x^{y-1}$ at $(1,2)$: $\quad 2 \cdot 1^{2-1} = 2$

$\frac{\partial f}{\partial y} = x^y \cdot \ln x \qquad 1^2 \cdot \ln 1 = 0$

$f(1{,}04,\ 2{,}02) \approx 1 + 0\,(1{,}04 - 1) + 2\,(2{,}02 - 2)$

$= 1 + 0{,}04 = \underline{\underline{1{,}04}}$