List of definitions and theorems

Functions of multiple variables

Definitions

1. Euclidean metric (distance)
2. open ball with radius r centred at x
3. interior point of M, interior of M, open set
4. boundary point of M, boundary of M, closure of M, closed set
5. convergence in $\mathbb{R}^n$, limit of the sequence in $\mathbb{R}^n$, convergent sequence in $\mathbb{R}^n$
6. bounded set, bounded sequence in $\mathbb{R}^n$
7. compact sets
8. function of two variables, function of multiple variables
9. limit of function of multiple variables, continuous function at x
10. Partial derivative.
11. function of the class C^1
12. tangent hyperplane
13. gradient
14. maximum on M, (strict) local maximum with respect to M. Minimum...
15. f attains a local maximum
16. stationary (or critical) point
17. partial derivative of the second order
18. convex set
19. (strictly) concave function, convex...
20. (strictly) quasiconcave function, quasiconvex...

Theorems

1. properties of the Euclidean metric
2. properties of open sets. **Proof**.
3. convergence is coordinatewise
4. characterisation of closed sets
5. properties of closed sets. **Proof**.
6. Properties of $\bar{M}$, $\operatorname{Int} M$, characterisation of open sets (Theorem 6)
7. Characterisation of bounded sets (Theorem 7). **Proof**.
8. characterisation of compact subsets of $\mathbb{R}^n$.
9. limit of a composed function (Theorem 9)
10. Continuity of sum/product/dividing of functions
11. Continuity of composed functions.
12. Characterizaton of levelsets of continuous functions. **Proof**.
13. tangent hyperplane
14. C^1 function and continuity **Proof**.

15. Chain rule
16. Implicit function.
17. attaining extrema
18. boundedness of a continuous function
19. necessary condition of the existence of local extremum
20. interchanging of partial derivatives
21. Lagrange multiplier theorem
22. Lagrange multipliers theorem
23. Concavity and continutiy (Theorem 22).
24. characterisation of strictly concave functions of the class C^1
25. characterisation of concave functions of the class C^1
26. Concavity and extrema (Corollary 25) **Proof**.
27. level sets of concave functions **Proof**.
28. characterization of quasiconcave functions using level sets **Proof**.
29. a uniqueness of an extremum **Proof**.
30. (sufficient condition for concave and convex functions in $\mathbb{R}^2$

Matrices

Definitions

1. Matrix of type $m \times n$, square matrix of order n.
2. ith row of a matrix, jth column
3. equal matrices
4. sum of matrices, product of real number and matrix
5. product of matrices
6. transpose of a matrix
7. symmetric matrix
8. invertible matrix, inverse of a matrix
9. determinant of a matrix
10. upper/lower triangular matrix
11. linear combination of vectors, trivial linear combination, non-trivial lin. comb.
12. linearly in/dependent vectors,
13. rank of a matrix
14. row echelon form of a matrix
15. elementary row operations on a matrix
16. matrix transformation
17. system of equations, coefficient matrix, vector of the right-hand side, vector of unknowns
18. augmented matrix of the system
19. Definiteness of matrices
20. Hessian matrix

Theorems

1. properties of the matrix multiplication
2. properties of the transpose of a matrix
3. operations with invertible matrices **Proof**.
4. cofactor expansion
5. Determinant of sum of matrices with one different row (Lemma 6). **Proof**.
6. determinant and transformations
7. determinant of a triangular matrix
8. determinant and invertibility **Proof**.
9. determinant of a product
10. determinant of a transpose
11. properties of matrix transformations
12. reprezentation of a transformation
13. transformation and identity matrix (Lemma 14)
14. invertible matrix and rank (Theorem 15)
15. solutions of a transformed system
16. Rouché-Fontené
17. solvability of an $n \times n$ system
18. Cramer's rule
19. definiteness of diagonal matrices
20. necessary conditions for definiteness
21. Sylvester's criterion
22. Definiteness and convexity (Theorem 23)
23. Sufficient condition of the existence of local extremum

Integrals

Definitions

1. antiderivative of f on I
2. rational function
3. root of multiplicity k
4. partition of the interval
5. refinement of the partition
6. Riemann integral (including definition of $\underline{\int_a^b} f$, $\overline{\int_a^b} f$)

Theorems

1. Uniqueness of an antiderivative **Proof**.
2. Existence of an antiderivative
3. Linearity of antiderivatives **Proof**.
4. substitution **Proof**.

5. integration by parts **Proof**.
6. fundamental theorem of algebra
7. polynomial division
8. factorisation into monomials
9. roots of a polynomial with real coefficients
10. Newton-Leibniz formula
11. integration by parts for definite integral **Proof**.
12. substitution for definite integral
13. Properties of Riemann integral (Theorem 14+15).
14. Linearity of Riemann integral (Theorem 16).
15. Arrangement and the Riemann integral (Theorem 17).
16. Continuity and Riemann integral (Theorem 18).
17. Continuity and Riemann integral (Theorem 18).
18. fundamental theorem of calculus (Theorem 19).