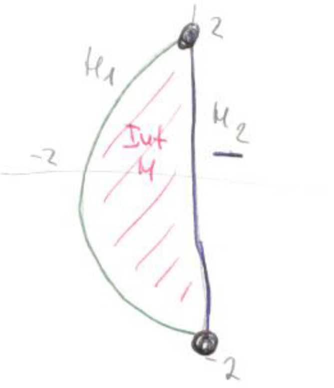
(1) $f(x,y) = -y^2 + x^2 + \frac{4}{3}x^3$ $\qquad M = \{ x^2+y^2 \leq 4, \quad x \leq 0 \}$



- M is bounded (half circle)
- M closed (with its boundary)
- f is continuous (polynomial)

} f attains extrema

(1) Int M $\qquad x^2+y^2<4, \ x<y$

$\frac{\partial f}{\partial x} = 2x + 4x^2 \qquad 2x(1+2x) = 0 \qquad x = 0 \ \vee \ x = -\frac{1}{2}$

$\frac{\partial f}{\partial y} = -2y \quad \to \quad y = 0 \qquad [0,0] \qquad [-\frac{1}{2}, 0]$

$\notin$ Int M

(2) $M_1 \qquad x^2+y^2=4 \quad x<0 \qquad x \in [-2,0), \qquad f,g \in C^\infty(\mathbb{R}^2)$

Lagrange (a) $\nabla g = (2x, 2y) = (0,0) \to \quad [0,0] \notin M_1$

$0^2+0^2 \neq 4$

(b) $\nabla f + \lambda \nabla g = (0,0)$

$2x + 4x^2 + \lambda 2x = 0$

$-2y + \lambda 2y = 0 \quad \to \quad 2y(1-\lambda) = 0$

$x^2+y^2=4$

$y = 0 \qquad\qquad \lambda = 1$

$x^2 = 4 \qquad\qquad 2x + 4x^2 + 2x = 0$

$x = \pm 2 \qquad\qquad 4x(x+1) = 0$

$[-2,0]$ ✓ $\quad [2,0] \notin M$

$x = 0 \qquad\qquad x = -1$

$y^2 = 4 \qquad\qquad y^2 = 3$

$y = \pm 2 \qquad\qquad y = \pm\sqrt{3}$

$[0, \pm 2] \notin M_1 \qquad [-1, \pm\sqrt{3}]$ ✓

M_2 $\qquad x = 0 \quad y \in (-2, 2)$

$f(0,y) = -y^2 \to [0,0]$

③

Suspect points

	$[-\frac{1}{2},0]$	$[0,0]$	$[-2,0]$	$[-1,\sqrt{3}]$	$[-1,-\sqrt{3}]$	$[0,2]$	$[0,-2]$
f:	$+1/12$	0	$-20/3$	$-10/3$	$-10/3$	-4	-4
	↑ glob. max		↑ glob. min				

②

$$\rightarrow \begin{vmatrix} 1 & 2 & 3 & -3 \\ 2 & 1 & 1 & -3 \\ 3 & 0 & 4 & 0 \\ -5 & 3 & 2 & 1 \end{vmatrix} = (-1)^{1+3}\cdot 3 \cdot \begin{vmatrix} 2 & 3 & -3 \\ 1 & 1 & -3 \\ 3 & 2 & 1 \end{vmatrix} + (-1)^{3+3}\cdot 4 \cdot \begin{vmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -5 & 3 & 1 \end{vmatrix}$$

$$= 3\cdot\left(2 + (-27) + (-6) - [-9 - 12 + 3]\right)$$

$$+4\left(1 + 30 - 18 - [15 - 9 + 4]\right) =$$

$$= -39 + 12 = \underline{\underline{-27}}$$

Or

$$\left(\begin{array}{ccc|ccc} -2 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ -2 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 0 & 1 & 0 \\ -2 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right) \sim \begin{array}{c} :2 \\ \\ \\ \end{array}\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right)$$

$$\sim \begin{array}{c} \\ \cdot(-1) \\ :2 \end{array}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & +1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1/2 & 1 & 1/2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & -1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1 & 1/2 \end{array}\right) \rightarrow A^{-1} = \begin{pmatrix} 0 & 1/2 & 0 \\ -1/2 & 0 & 1/2 \\ 1/2 & 1 & 1/2 \end{pmatrix}$$

(3) $$\int \frac{9x^3 - 56x^2 + 17x - 10}{(x-3)(x+2)(3x^2-2x+1)}\,dx$$

$\cdot\ \frac{P}{Q}$ $\deg P = 3 < 4 = \deg Q$ $3x^2-2x+1 \neq 0$

$$\cdot\ = \int \frac{\overset{A}{-2}}{x-3} + \frac{\overset{B}{4}}{x+2} + \frac{\overset{Cx+D}{3x-1}}{3x^2-2x+1}\,dx$$

$$A(x+2)(3x^2-2x+1) + B(x-3)(3x^2-2x+1) + (Cx+D)(x+2)(x-3)$$
$$= 9x^3 - 56x^2 + 17x - 10$$

$x=-2$: $B(-5)(17) = -72 - 224 - 34 - 10$

$\boxed{B = 4}$

$x=3$: $A \cdot 5 \cdot 22 = 243 - 504 + 51 - 10$

$\boxed{A = -2}$

$x=0$: $2\cdot(-2) + 4\cdot(-3) + D\cdot(-6) = -10$

$-6D = 6$ $\boxed{D = -1}$

$x=1$: $(-2)\cdot 3\cdot 2 + 4\cdot(-2)\cdot 2 + (C + (-1))(-6) = 9 - 56 + 17 - 10$

$-12 \quad -16 \quad -6C + 6 \quad = -40$

$C = 3$

$$\int = -2\ln|x-3| + 4\ln|x+2| + \frac{1}{2}\ln|3x^2-2x+1|$$ $x \neq -2$, $x \neq 3$

$$\int \frac{3x-1}{3x^2-2x+1}\,dx = \frac{1}{2}\int \frac{6x-2}{3x^2-2x+1}\,dx = \frac{1}{2}\int \frac{1}{y}\,dy = \frac{1}{2}\ln|y| + C = \frac{1}{2}\ln|3x^2-2x+1| + C$$

$y = 3x^2-2x+1$

$dy = 6x-2\ dx$

(4) $\int_0^{\ln \pi} \sin(e^x)\cos(2e^x)e^x\,dx = \int_1^{\pi} \sin y \cos(2y)\,dy =$

$y = e^x$

$dy = e^x dx$

x	0	$\ln \pi$
y	1	π

$\int \sin y(\cos^2 y - \underbrace{\sin^2 y}_{1-\cos^2 y})\,dy = \int \sin y(-1 + 2\cos^2 y)\,dy =$

$= \underbrace{\int -\sin y\,dy}_{\cos y} + 2\int \cos^2 y \cdot \sin y\,dy = \cos y - \frac{2}{3}\cos^3 y$

$t = \cos y$

$dt = -\sin y\,dy$

$\rightarrow \quad -2\int t^2\,dt \overset{c}{=} -2\frac{t^3}{3} =$

$= -\frac{2}{3}\cos^3 y$

$\int_1^{\pi} \sin y \cos(2y)\,dy = \left[\cos y - \frac{2}{3}\cos^3 y\right]_1^{\pi} = -1 + \frac{2}{3} - \left(\cos 1 - \frac{2}{3}(\cos 1)^3\right)$

$= -\frac{1}{3} - \cos 1 + \frac{2}{3}\cos^3 1$