Exam - sample

The exam consists of written and oral part. Their dates will be in SIS. Please, enroll to both parts of the exam. And let us know, if you have some special needs (the sooner the better).

Written part

The written part comprises 4 questions:

1. find extrema over a set (extrema + Lagrange multipliers) - 7 points;
2. compute inverse/determinant of a matrix - 5 points;
3. integral with partial fractions - 7 points;
4. definite integral with per partes and substitution - 6 points.

Students have 120 minutes and can use any literature (notes, tables, textbooks...), but no technical devices (phone, calculator, watches...).

Example of the written part

1. (7 points) Find global maximum and minimum of the function f on the set M.

   $$f(x,y) = -y^2 + x^2 + \frac{4}{3}x^3, \qquad M = \{[x,y] \in \mathbb{R}^2; x^2 + y^2 \leq 4, x \leq 0\}.$$

2. (5 points) Compute the determinant of the following matrix

   $$\begin{pmatrix} 1 & 2 & 3 & -3 \\ 2 & 1 & 1 & -3 \\ 3 & 0 & 4 & 0 \\ -5 & 3 & 2 & 1 \end{pmatrix}$$

   or

   Find the inverse matrix of $\mathbf{A} = \begin{pmatrix} -2 & -1 & 1 \\ 2 & 0 & 0 \\ -2 & 1 & 1 \end{pmatrix}$

3. (7 points) Find the primitive function (do not forget on conditions):

   $$\int \frac{9x^3 - 56x^2 + 17x - 10}{(x-3)(x+2)(3x^2 - 2x + 1)} \, \mathrm{d}x$$

4. (6 points) Compute the definite integral

   $$\int_0^{\ln \pi} \sin(e^x) \cos(2e^x) e^x \, \mathrm{d}x$$

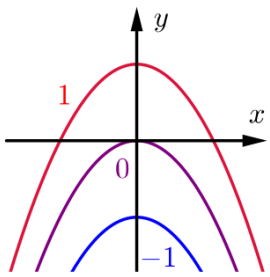
Oral part

The oral part tests knowledge of definitions, theorems, some proofs (those from the lecture) and concept questions (similar to those in the lecture).

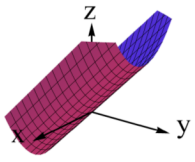
The students have (almost arbitrary) time for preparation, but can not use any literature or electronic devices.

Example of the oral part

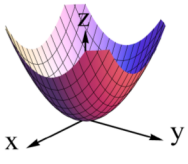
1. (2x2 points) Definition of: **inverse matrix**, **open set**.
2. (2.5 points) Statement of the theorem: **Implicit function theorem**.
3. (2.5+7 points) Statement and proof of the theorem: **Linearity of a primitive function**.
4. (6x1.5 points) Answer concept questions:
   (a) Find bounded sets
      i. $x \in [-1, 3]$, $0 < y \leq 100$
      ii. $x^2 + y^2 + z^2 \leq 5$
      iii. $|x + y| < 6$
   (b) Find the suitable graph for given contour lines:



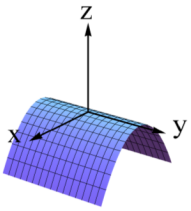
A.



B.

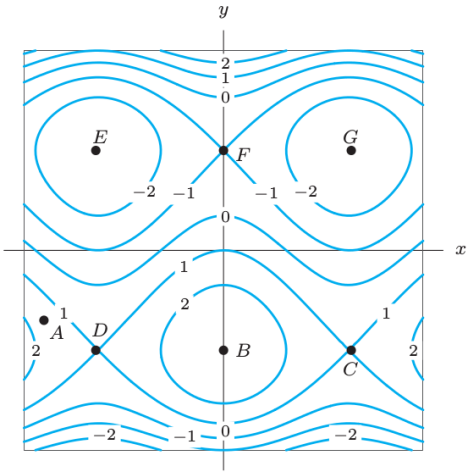


C.



http://www.cpp.edu/~conceptests/question-library/mat214.shtml

(c) i. Find the critical points.
ii. Find the points of
A. local minimum,
B. local maximum,
C. saddle?



Source 1: Calculus, 6th Edition; Hughes-Hallett, Gleason, McCallum et al.

(d) Let matrices A and B be invertible. Let $AB = C$. Find C^{-1}:

i. $A^{-1}B^{-1}$
ii. $B^{-1}A^{-1}$
iii. AB^{-1}
iv. BA^{-1}
v. C does not have to be invertible.

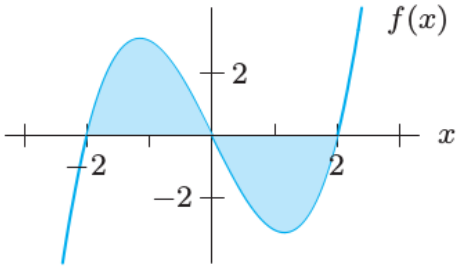
(e) Decide, if it is better to use per partes or substitution

i. $\int \arcsin x \, \mathrm{d}x$
ii. $\int \frac{x}{1+x^2} \, \mathrm{d}x$
iii. $\int (x^2 - 3) \ln x \, \mathrm{d}x$
iv. $\int \frac{1}{x \ln x} \, \mathrm{d}x$
v. $\int x^2 \cos 2x \, \mathrm{d}x$

(f) There is an odd function on the picture. If you now that $\int_{-2}^{0} f(x)\,\mathrm{d}x = 4$, find

i. $\int_{0}^{2} f(x)\,\mathrm{d}x$

ii. $\int_{-2}^{2} f(x)\,\mathrm{d}x$



Source 2: Calculus: Single and Multivariable, Hughes-Hallet