Mathematics II - List of concept questions

21/22

Exercise (2D)

Sketch the following points and connect them.

$$(14,5),(13,2),(12,0),(13,-3),(10,-1),(4,-2),(3,-4),$$

$$(1,-3),(-4,-3),(-6,-2),(-6,-7),(-8,-5),(-9,-2),$$

$$(-13,-1),(-11,0),(-14,1),(-12,2),(-9,3),(-4,3),(-2,7),$$

$$(0,3),(3,2),(9,1),(14,5).$$

`https://mathcrush.com/geometry_worksheets/`

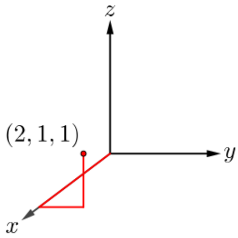
Exercise (3D)

`https://www.geogebra.org/classic/ydu8a7t7`

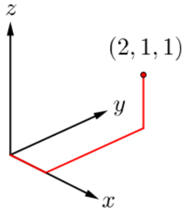
Exercise

Which picture(s) plots the point $(2, 1, 1)$ correctly?

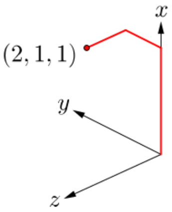
A.



B.



C.



https://www.cpp.edu/conceptests/question-library/mat214.shtml

Exercise

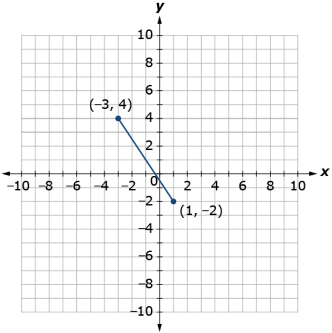
Find

A $(1, 2, 3, 4) + (-2, 0, 3, -1)$

B $-2(1, 2, 3, 4)$

Exercise

Find the distance of the points



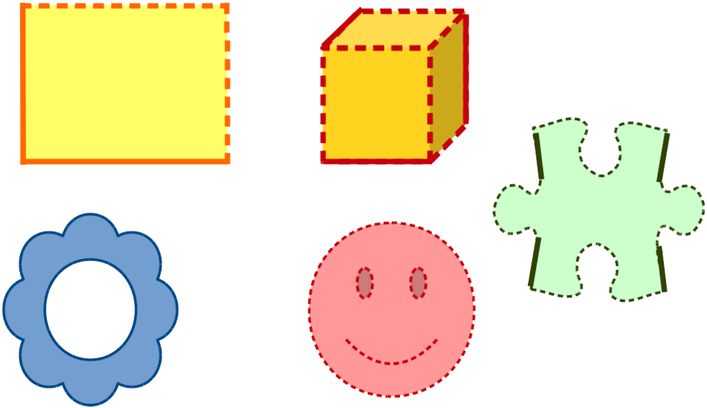
A https://www.summitlearning.org/guest/focusareas/862919

B $(1, -2, 3), (0, -3, -2)$

C $(-1, 0, 3, 2), (1, -1, 2, -3)$

Exercise

Find the interior



Exercise

Decide, if the set is closed or open, find the interior, the boundary, the closure.

$$M = \{[x, y] \in \mathbb{R}^2 : 1 < x \leq 2, 3 \leq y \leq 5\}.$$

Exercise

Find

$$\lim_{j\to\infty}\left(\frac{1}{j}, \frac{2j+1}{j}\right)$$

Find the limits of

$x^j = \left(1 + \frac{1}{j}, 3 - \frac{2}{j^2}, e^{-j}\right)$

$x^j = \left((-1)^j, \arctan(j^3)\right)$

Exercise

Decide, if the sets are closed or open (or nothing)

1. $(0, 1)$ in $\mathbb{R}$
2. $(0, \infty)$ in $\mathbb{R}$
3. $(-\infty, 2]$ in $\mathbb{R}$
4. $x^2 + y^2 < 4$ in $\mathbb{R}^2$
5. $x^2 + y^2 \geq 2$ in $\mathbb{R}^2$

Exercise

Find bounded sets

- A $x \in [-1, 3], 0 < y \leq 100$
- B $x^2 + y^2 + z^2 \leq 5$
- C $|x + y| < 6$

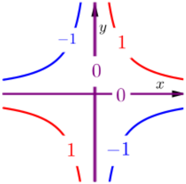
Exercise

Find compact sets

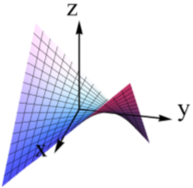
1. $(0, 1)$
2. $[1, 2] \times [-1, -3]$
3. $1 < x^2 + (y - 3)^2 + z^2 \leq 4$
4. $xyz \leq 1$

Exercise

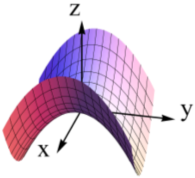
Find the graph for the contourlines



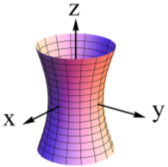
A.



B.



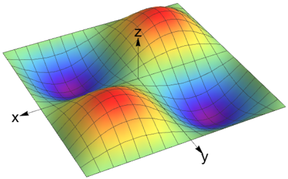
C.

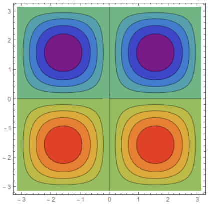


http://www.cpp.edu/~conceptests/question-library/mat214.shtml

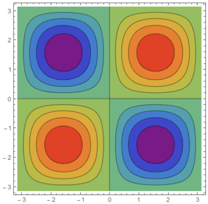
Exercise

Find the contourlines for the graph.

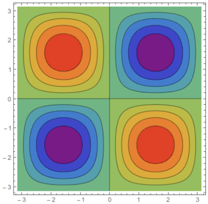




(a) A



(b) B



(c) C

Exercise

Connect the contourlines and the functions

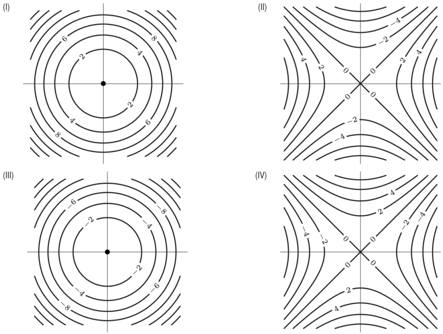


Figure: Hughes Hallett et al c 2009, John Wiley & Sons

A $-x^2 + y^2$

B $x^2 - y^2$

C $-x^2 - y^2$

D $x^2 + y^2$

Exercise

1. $\lim_{(x,y)\to(2,-1)} x^2 - 2xy + 3y^2 - 4x + 3y - 6$
2. $\lim_{(x,y)\to(2,-1)} \frac{2x+3y}{4x-3y}$
3. $\lim_{(x,y)\to(0,0)} \frac{x^2+xy}{x+y}$

In the table there are values of a function $f(x,y)$. Does there exist the limit

$$\lim_{(x,y)\to(0,0)} f(x,y)?$$

$x \backslash y$	-1.0	-0.5	-0.2	0	0.2	0.5	1.0
-1.0	0.00	0.60	0.92	1.00	0.92	0.60	0.00
-0.5	-0.60	0.00	0.72	1.00	0.72	0.00	-0.6
-0.2	-0.92	-0.72	0.00	1.00	0.00	-0.72	-0.92
0	-1.00	-1.00	-1.00		-1.00	-1.00	-1.00
0.2	-0.92	-0.72	0.00	1.00	0.00	-0.72	-0.92
0.5	-0.60	0.00	0.72	1.00	0.72	0.00	-0.6
1.0	0.00	0.60	0.92	1.00	0.92	0.60	0.00

https://www.cpp.edu/conceptests/question-library/mat214.shtml

Exercise

$$\lim_{(x,y)\to(4,1)} \sqrt{\frac{x^2 - 3xy}{x + y}}$$

Exercise

Where is continuous $f(x, y) = \cos \frac{x}{y}$?

A Everywhere except at the origin

B Everywhere except along the x-axis.

C Everywhere except along the y-axis.

D Everywhere except along the line $y = x$.

Exercise

Where is continuous $f(x, y) = \operatorname{sgn} xy$?

A Everywhere except along the axes.

B Everywhere except along the x-axis.

C Everywhere except at the origin.

D Everywhere except along the line $y = x$.

Exercise

Find continuous functions (at $\mathbb{R}^2$)

- A $\ln(x^2 + y^2 + 1)$
- B $\frac{x-y}{e^{xy}}$
- C $\frac{\sqrt{y-1}}{x^2}$
- D $\sin(2x) + x\cot(x^3 + 2y)$
- E $\operatorname{sgn}(x^4 + y^4)$

Exercise

Find $\frac{\partial f}{\partial x}$, if $f(x,y) = x^3 + 3x^2y - 5x - 7y^3 + y - 5$

A $\dfrac{\partial f}{\partial x} = 3x^2 + 6xy - 5 - 7y^3 + y$

B $\dfrac{\partial f}{\partial x} = 3x^2 + 6xy - 5$

C $\dfrac{\partial f}{\partial x} = x^3 + 3 - 21y^2 + 1 - 5$

D $\dfrac{\partial f}{\partial x} = 3x^2 - 21y^2 + 1$

Find $\frac{\partial f}{\partial y}$, if $f(x,y) = x^2 \ln(x^2y)$

A $\dfrac{\partial f}{\partial y} = \dfrac{2x}{y}$

B $\dfrac{\partial f}{\partial y} = \dfrac{1}{y}$

C $\dfrac{\partial f}{\partial y} = \dfrac{x^2}{y}$

D $\dfrac{\partial f}{\partial y} = \dfrac{1}{x^2y}$

According to: `https://www.wiley.com/college/hugheshallett/0470089148/conceptests/concept.pdf`

Exercise

The values of a function $f(x, y)$ are in the table. Which statement is most accurate?
(In the left columnt there is x, in the the first row there is y.)

$x \backslash y$	0	1	2	3
0	3	5	7	9
1	2	4	6	8
2	1	3	5	7
3	0	2	4	6

A $\frac{\partial f}{\partial x}(1, 2) \approx -1$

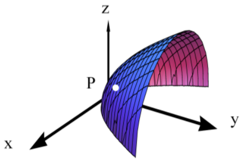
B $\frac{\partial f}{\partial y}(1, 2) \approx 2$

C $\frac{\partial f}{\partial x}(3, 2) \approx 1$

D $\frac{\partial f}{\partial y}(3, 2) \approx 4$

https://www.cpp.edu/conceptests/question-library/mat214.shtml

Exercise



A $\frac{\partial f}{\partial x} > 0, \frac{\partial f}{\partial y} > 0$

B $\frac{\partial f}{\partial x} < 0, \frac{\partial f}{\partial y} > 0$

C $\frac{\partial f}{\partial x} > 0, \frac{\partial f}{\partial y} < 0$

D $\frac{\partial f}{\partial x} < 0, \frac{\partial f}{\partial y} < 0$

`https://www.cpp.edu/conceptests/question-library/mat214.shtml`

Exercise (True or false?)

1. Let $f(x, y, z) = x^2 + z + 3$. Then the partial derivative $\frac{\partial f}{\partial y}$ is not defined, because there is no y in the function.
2. Is there a function $f(x, y)$ such that $\frac{\partial f}{\partial y} = 3y^2$ and $\frac{\partial f}{\partial x} = 3x^2$?

Exercise

Find a function, which is not constant, but
$\frac{\partial f}{\partial x} = 0$ for every x.

Exercise

Find functions, which are $\mathcal{C}^1(\mathbb{R}^2)$.

A e^{xy}

B $\sqrt[3]{x^2 + y^2}$

C $\frac{\sin(x-2y)}{2+x^2+y^2}$

D $\ln \frac{y}{x}$

Exercise

Find the tangent plane of a function $f(x, y) = xy$ at the point $(2, 3)$.

A $z - 6 = x(x - 2) + y(y - 3)$

B $z - 6 = y(x - 2) + x(y - 3)$

C $z - 6 = 2(x - 2) + 3(y - 3)$

D $z - 6 = 3(x - 2) + 2(y - 3)$

Exercise

Find the tangent plane of a function $f(x, y, z, u) = \ln(xy + z^2 - u)$ at the point $a = (1, 0, 2, 3)$.

Exercise

Let $h(u,v) = \sin x \cos y$, where $x = (u-v)^2$ and $y = u^2 - v^2$. Find $\partial h/\partial u$ a $\partial h/\partial v$.

Exercise

Let $h(u,v) = xy$, where $x = u \cos v$ and $y = u \sin v$. Then for $\partial h/\partial v$ we have

A $\dfrac{\partial h}{\partial v} = 0$

B $\dfrac{\partial h}{\partial v} = u^2 \cos(2v)$

C $\dfrac{\partial h}{\partial v} = -u^3 \sin^2 v \cos v + u^3 \sin v \cos^2 v$

D Something else.

Exercise

Let $f(x, y)$ satisfies the Chain rule theorem assumptions. Show, that a function $h(u, v, w) = \frac{uv}{w} \ln u + uf\left(\frac{v}{u}, \frac{w}{u}\right)$, where $x = \frac{v}{u}$,$y = \frac{w}{u}$ satisfies the following condition

$$u\frac{\partial h}{\partial u} + v\frac{\partial h}{\partial v} + w\frac{\partial h}{\partial w} = h + \frac{uv}{w}.$$

Exercise

Which condition for the Implicit function theorem is NOT satisfied?

- A $x^2 + y^3 = 4$ at $(2, 0)$
- B $y - \frac{1}{2}\sin y = x$ at (π, π)
- C $\sin(xy) + x^2 + y^2 = 1$ at $(0, 3)$
- D $|x| + e^{x+y} = 1$ at $(0, 0)$

Exercise

Find the gradient of $f(x, y, z) = y\cos^3(x^2 z)$ at the point $[2, 1, 0]$:

A $(1/5, 0, 1/5)$

B $(0, 0, 1/5)$

C $(0, 1, 0)$

D $(1, 0, 1/2)$

Exercise

The bicyclist is on a trip up the hill, which can be described as $f(x,y) = 25 - 2x^2 - 4y^2$. When she is at the point $[1, 1, 19]$, it starts to rain, so she decides to go down the hill as steeply as possible (so that she is down quickly). In what direction will she start her decline?

A $(-4x; -8y)$

B $(4x; 8y)$

C $(-4; -8)$

D $(4; 8)$

Exercise

1. Consider the points A, B, C, D, E. Find the critical points.
2. Which of these points are probably points of
   2.1 local maximum,
   2.2 local minimum,
   2.3 saddle poi

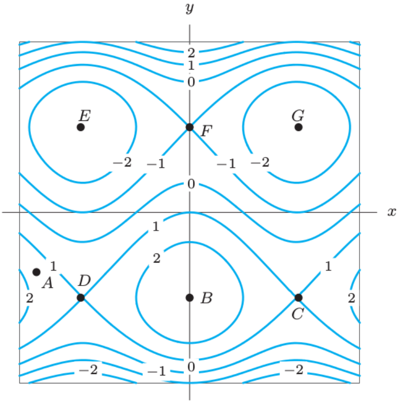


Figure: Calculus, 6th Edition; Hughes-Hallett, Gleason, McCallum et al.

Exercise

Find the second partial derivatives of the function $f(x, y) = x^2 + xy + y^2$.

Exercise

Find $\frac{\partial^2 f}{\partial x \partial y}$, if $f(x, y) = e^{xy}$

A e^{xy}

B ye^{xy}

C $x^2 e^{xy}$

D $e^{xy}(xy + 1)$

Exercise

Find $\frac{\partial^2 f}{\partial y \partial x}$, if $f(x, y) = e^{xy}$

A e^{xy}

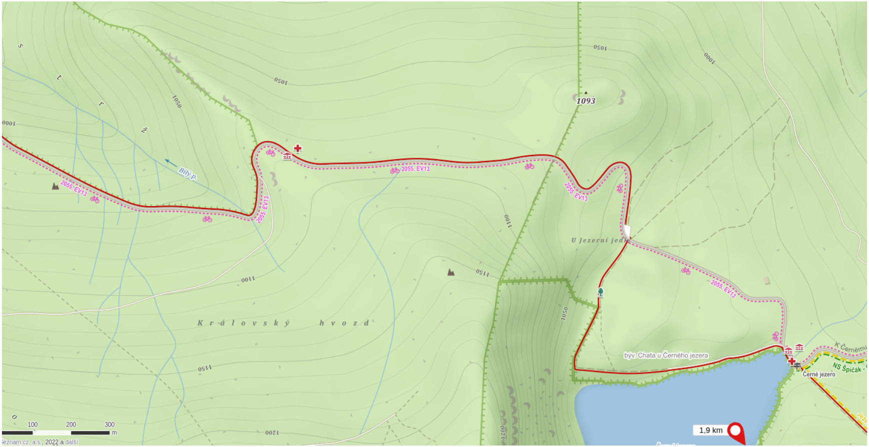
B ye^{xy}

C $x^2 e^{xy}$

D $e^{xy}(xy + 1)$

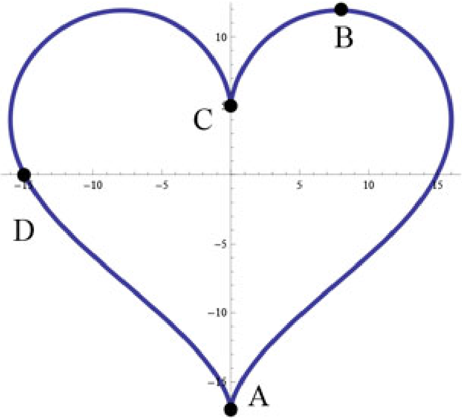
Exercise

You follow the red route. Where is the highest point of your trip?



Exercise

Where is the minimum and maximum of the function $f(x, y) = y$ along the



curve?

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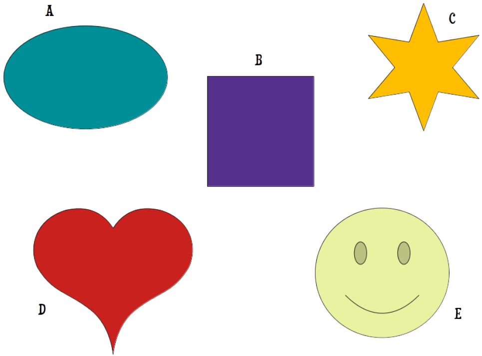
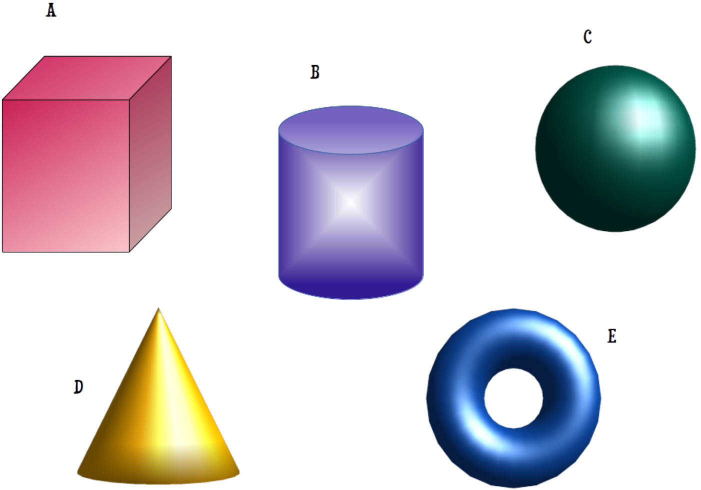
Exercise

Find convex sets



Exercise

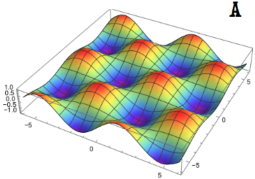
Find convex sets

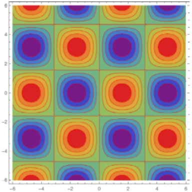


Exercise

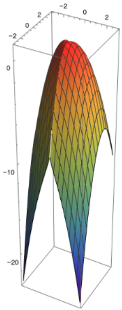
Find quasiconcave functions:

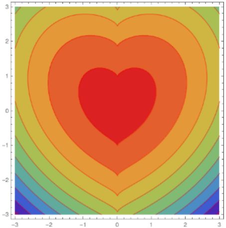
A





C





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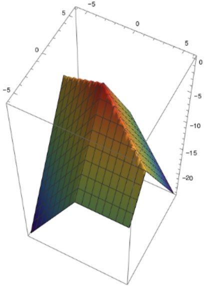
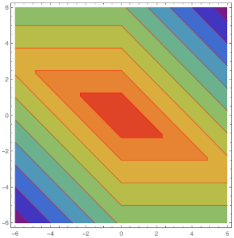




Exercise

Decide if the following functions are convex or concave on $\mathbb{R}^2$.

- A $f(x, y) = x^2 + y^2$
- B $f(x, y) = -x^4 - y^4$
- C $f(x, y) = -x^2 + y^2$

Exercise

Find the type of the matrix

$$\begin{pmatrix} 6 & 11 & -2 \\ 23 & 31 & 5 \end{pmatrix}$$

A 2x3 B 3x2 C 6

Exercise

Are **A** and **B** equal?

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 4 \\ -1 & -2 & 5 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 4 & 0 & 4 \\ 1 & 2 & 3 \\ -1 & -2 & 5 \end{pmatrix}$$

Exercise

Let

$$A = \begin{pmatrix} 4 & 6 \\ 20 & 24 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix}.$$

Find $A + B$

A 71

B
$$\begin{pmatrix} 6 & 9 \\ 7 & 11 \end{pmatrix}$$

C
$$\begin{pmatrix} 6 & 11 \\ 23 & 31 \end{pmatrix}$$

D
$$\begin{pmatrix} 26 & 62 \\ 112 & 268 \end{pmatrix}$$

E
$$\begin{pmatrix} 4 & 6 & 2 & 5 \\ 20 & 24 & 3 & 7 \end{pmatrix}$$

Exercise

Let

$$A = \begin{pmatrix} 4 & 6 \\ 20 & 7 \end{pmatrix}$$

Find $5A$

A

$$\begin{pmatrix} 9 & 6 \\ 20 & 7 \end{pmatrix}$$

B

$$\begin{pmatrix} 9 & 11 \\ 25 & 12 \end{pmatrix}$$

C

$$\begin{pmatrix} 20 & 6 \\ 20 & 7 \end{pmatrix}$$

D

$$\begin{pmatrix} 20 & 30 \\ 100 & 35 \end{pmatrix}$$

Exercise

Find **AB**, if

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix}$$

A $\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$

B $\begin{pmatrix} 0 & -2 \\ 2 & 5 \end{pmatrix}$

C $\begin{pmatrix} 0 & 0 \\ -6 & 2 \end{pmatrix}$

D something else

E **AB** is not well defined

Exercise

Find **AB**, if

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

A $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

B $\begin{pmatrix} 10 & 7 \end{pmatrix}$

C $\begin{pmatrix} 8 & 4 \\ -3 & -2 \end{pmatrix}$

D $\begin{pmatrix} 7 \\ 10 \end{pmatrix}$

E **AB** is not well defined

Exercise

Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Find

1. AI
2. IA

Exercise

Let

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 \\ 3 & -3 \end{pmatrix}$$

Find

1. AB
2. BA

Exercise

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{pmatrix}.$$

Find $\mathbf{A}^T$?

A

$$A^T = \begin{pmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{pmatrix}$$

B

$$A^T = \begin{pmatrix} 2 & 0 & -2 \\ 3 & -1 & 0 \\ 1 & 3 & 4 \end{pmatrix}$$

C

$$A^T = \begin{pmatrix} -2 & 0 & 4 \\ 0 & -1 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

D

$$A^T = \begin{pmatrix} 1 & 3 & 4 \\ 3 & -1 & 0 \\ 2 & 0-2 & \end{pmatrix}$$

Exercise

Let

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 0 \\ 5 & -1 \end{pmatrix}$$

Find

1. $(AB)^T$
2. $A^T B^T$
3. $B^T A^T$

Exercise

Let $\mathbf{A}$ and $\mathbf{B}$ are matrices of the type 2×3. Which of these operations are NOT well defined?

A $\mathbf{A} + \mathbf{B}$

B $\mathbf{A}^T\mathbf{B}$

C $\mathbf{BA}$

D $\mathbf{AB}^T$

E $\mathbf{AB}$

Exercise

We want to multiply matrices $\mathbf{A} \times \mathbf{B}$. We need:

A $\mathbf{A}$ and $\mathbf{B}$ needs to have the same number of rows.

B $\mathbf{A}$ and $\mathbf{B}$ needs to have the same number of columns.

C the number of rows of $\mathbf{A}$ needs to be the same as the number of columns of $\mathbf{B}$

D the number of columns of $\mathbf{A}$ needs to be the same as the number of rows of $\mathbf{B}$

Exercise

Let $\mathbf{A}$ is a matrix of the type 2×3 and $\mathbf{B}$ is of the type 3×6. Find the type of $\mathbf{AB}$:

- A 2x6
- B 6x2
- C 3x3
- D 2x3
- E 3x6

Exercise (True or False?)

Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices of the same dimension. Then

$$(\mathbf{A} + \mathbf{B}) \times (\mathbf{A} + \mathbf{B}) = \mathbf{A}^2 + 2\mathbf{AB} + \mathbf{B}^2.$$

Exercise

Let

$$\mathbf{A} = \begin{pmatrix} 0 & 4 \\ 2 & 0 \end{pmatrix}$$

Find $\mathbf{A}^{-1}$

A

$$\begin{pmatrix} 0 & 4 \\ 2 & 0 \end{pmatrix}$$

B

$$\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$$

C

$$\begin{pmatrix} 0 & 1/4 \\ 1/2 & 0 \end{pmatrix}$$

D

$$\begin{pmatrix} 0 & 1/2 \\ 1/4 & 0 \end{pmatrix}$$

Exercise

Find the determinant of

$$\begin{pmatrix} 5 & 4 \\ 1 & 3 \end{pmatrix}$$

- A 4
- B 11
- C 15
- D 19

Exercise

Find the determinant of

$$\begin{pmatrix} 5 & 2 & -1 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

- A 0
- B 6
- C 15
- D 22

Exercise

We have

$$\det\begin{pmatrix} -1 & 15 & 16 \\ 2 & 5 & 4 \\ 2 & 3 & 5 \end{pmatrix} = -107.$$

Find

$$\det\begin{pmatrix} 2 & 5 & 4 \\ 2 & 3 & 5 \\ -1 & 15 & 16 \end{pmatrix}?$$

A -107

B 107

C something else

Exercise

We have

$$\det \begin{pmatrix} -2 & 1 & 3 \\ 2 & 0 & 4 \\ 1 & 3 & 1 \end{pmatrix} = 44.$$

Find

$$\det \begin{pmatrix} -2 & 1 & 3 \\ 0 & 1 & 7 \\ 1 & 3 & 1 \end{pmatrix}?$$

A 44

B -44

C 88

D something else

Exercise

We have

$$\det\begin{pmatrix} -2 & 1 & 3 \\ 2 & 0 & 4 \\ 1 & 3 & 1 \end{pmatrix} = 44.$$

Find

$$\det\begin{pmatrix} -2 & 1 & 3 \\ 2 & 0 & 4 \\ 0 & 7 & 5 \end{pmatrix}?$$

A 44

B -44

C 88

D 22

E something else

Exercise

Let $\mathbf{A}$ be a matrix of type $(2x2)$. Find $\det(5\mathbf{A})$.

- A $5 \det \mathbf{A}$
- B $10 \det \mathbf{A}$
- C $25 \det \mathbf{A}$
- D something else

Exercise

Let $\det \mathbf{A} = 3$. Find $\det \mathbf{A}^{-1}$.

A 1/3

B 3

C 9

D hard to say.

Exercise

We have

$$\det\begin{pmatrix}-2 & 1 & 3\\ 2 & 0 & 4\\ 1 & 3 & 1\end{pmatrix} = 44.$$

Find

$$\det\begin{pmatrix}-2 & 2 & 1\\ 1 & 0 & 3\\ 3 & 4 & 1\end{pmatrix}?$$

A 44

B 1/44

C 88

D 22

E -44

Exercise

Which of the following matrices do NOT have inverse matrix?

A
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

B
$$\begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix}$$

C
$$\begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}$$

D
$$\begin{pmatrix} 0 & 4 \\ 2 & 0 \end{pmatrix}$$

E All of them have inverse matrix.

Exercise

Let $u = (1, 2, 4)$ and $v = (-2, 0, 5)$. Then $2u - 3v$ is

A $(-4, 4, 23)$

B $(8, 4, -7)$

C $(8, 4, 23)$

D $(7, 6, 2)$

Exercise

Express $z = (-5, 3, 6)$ as the linear combination of $x = (1, -1, 4)$ and $y = (-3, 2, 6)$.

A $-5x$

B $-2x + y$

C $x + 2y$

D $2x + y$

E impossible

Exercise

Express w as the linear combination of u and v.

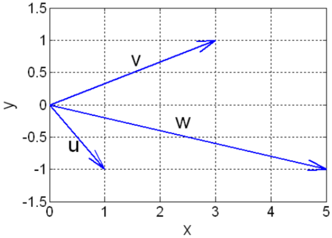


Figure: `https://www.chegg.com/homework-help/questions-and-answers/write-vector-w-linear-combination-u-v-q55559120`

A $w = 2u + v$

B $w = u + v$

C $w = -u + v$

D $w = u - v$

E w cannot be written like that.

Exercise

Which of the following vector can be written as the linear combination of vectors $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$?

A $(0, 2, 0)$

B $(-3, 0, 1)$

C $(0.4, 3.7, -1.5)$

Exercise

Describe the set of all linear combinations of vectors $(2, 4, 6)$ and $(-1, -2, -3)$?

A point B line C vector D plane E space

Exercise

Describe the set of all linear combinations of vectors $(1, 2, 0)$ and $(-1, 1, 0)$?

A point B line C vector D plane E space

Exercise

The vectors $(1, 0, 0)$, $(0, 0, 2)$, $(3, 0, 4)$ are

- A linearly dependent
- B linearly independent

Exercise

Find the rank of the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Exercise

Let

$$\mathbf{A} = \begin{pmatrix} 5 & 4 & -8 & 1 \\ 1 & 3 & 4 & 8 \\ 0 & 2 & 1 & 3 \\ -1 & -2 & 4 & 1 \end{pmatrix}.$$

After the transformation we get

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Find the rank of $\mathbf{A}$:

A 0

B 1

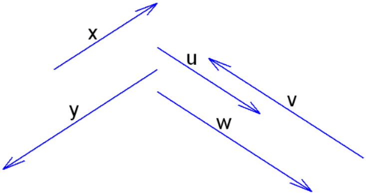
C 2

D 3

E 4

Exercise

We made a matrix from the vectors x, y, u, v and w. Find rank of this matrix.



http://mathquest.carroll.edu/libraries/FHMW.student.edition.pdf

A 1

B 2

C 3

D 4

E 5

Exercise

Decide about definiteness of the following matrices:

$$\begin{pmatrix} -2 & 0 \\ 0 & -5 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 8 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise

Which of this matrices can NOT be negative semidefinite?

A

$$\begin{pmatrix} 5 & 1 & -4 \\ 3 & 9 & 4 \\ 1 & 2 & -5 \end{pmatrix}$$

B

$$\begin{pmatrix} -1 & 0 & 8 \\ 3 & -2 & 1 \\ 1 & 0 & -2 \end{pmatrix}$$

C

$$\begin{pmatrix} -1 & 2 & 4 \\ -3 & 0 & 3 \\ -11 & 6 & -5 \end{pmatrix}$$

Exercise

Find the Hessian matrix of the function $x^3 + y^4 + 3x^2$ at the point $[-2, 0]$.

Exercise

Connect the functions in the left column with their antiderivatives on the right.

1. 0	A $-\cos x$
2. 1	B $\sin x$
3. x	C x
4. $\cos x$	D 1
5. $\sin x$	E $\frac{x^2}{2}$

Exercise

Find $\int e^x \, \mathrm{d}x$:

A e^x

B $-e^x$

C $e^x + 3$

D $e^x + e^\pi$

E $2e^x + 2$

Exercise

Find $\int x \sin x$.

- A $F = \sin x + x \cos x$
- B $F = \sin x - x \cos x$
- C $F = x \sin x + \cos x$

Exercise

Find F. You know that $F = \int 3x^2 + 2x \, \mathrm{d}x$ and $F(0) = 1$.

Exercise (True or false?)

A If $f'(x) = g'(x)$, then $f(x) = g(x)$ (for all x).

B If $\int f(x) = \int g(x)$, then $f(x) = g(x)$ (for all x).

http://www.math.cornell.edu/~GoodQuestions/GQbysection_pdfversion.pdf

Exercise

Which of the following functions definitely have primitive function?

A $\frac{1}{x}, x \in \mathbb{R}$

B $\arctan x^2, x \in \mathbb{R}$

C $\ln x, x \in (0, \infty)$

D $\frac{x^2}{x^3+1}, x \in \mathbb{R}$

E $\cot x, x \in (0, \pi)$

Exercise

Find integrals, which should be solved by integration by parts

A $\int xe^{x^2}\,\mathrm{d}x$

B $\int x\cos x\,\mathrm{d}x$

C $\int 1\ln x\,\mathrm{d}x$

D $\int \frac{x}{\ln x}\,\mathrm{d}x$

E $\int \sin x\ln x^2\,\mathrm{d}x$

Exercise

By parts or by substitution?

A $\displaystyle\int \arcsin x \, \mathrm{d}x$

B $\displaystyle\int \frac{x}{1+x^2} \, \mathrm{d}x$

C $\displaystyle\int (x^2-3) \ln x \, \mathrm{d}x$

D $\displaystyle\int \frac{1}{x \ln x} \, \mathrm{d}x$

E $\displaystyle\int x^2 \cos 2x \, \mathrm{d}x$

https://learningapps.org/display?v=pgeigqe6j21

Exercise

True or false?

A $\int kf = k \int f$

B $\int f + g = \int f + \int g$

C $\int f - g = \int f - \int g$

D $\int f \cdot g = \int f \cdot \int g$

E $\int f / g = \int f / \int g$

Exercise

Find a mistake.

1. $$\int \frac{3x^2+1}{2x}\,\mathrm{d}x = \frac{x^3+x}{x^2}+c$$

2. $\forall a \in \mathbb{R}$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1}+c$$

Calculus: Single and Multivariable, 6th Edition, Deborah Hughes-Hallett and col.

Exercise

Find rational functions.

A $\frac{3x-4+x^4}{x^2-2x+1}$

B x^6+5

C $\frac{x^5-8x+2}{3}$

D $\frac{\sqrt{2+5}}{1+\sqrt[3]{x^3-8}}$

E $\frac{(3x-4)(2x+5)}{(x-1)(x^2+2)}$

Exercise

Find the multiplicity of $\lambda = -2$ of the polynomial $P(x) = (x^2 + x - 2)(x + 2)^3$.

A -2 B 1 C 2 D 3 E 4

Exercise

Use the Riemann sums and estimate the integral

$$\int_0^{15} f(x)\,\mathrm{d}x.$$

Check the table for some values of f:

x	0	3	6	9	12	15
$f(x)$	50	48	44	36	24	8

Table: Applied Calculus, 6th Edition, Deborah Hughes-Hallett and col.

Exercise (True – False)

A Let f be a function. Then $\int_0^2 f(x)\,\mathrm{d}x \leq \int_0^3 f(x)\,\mathrm{d}x$.

B If $\int_2^6 g(x)\,\mathrm{d}x \leq \int_2^6 f(x)\,\mathrm{d}x$, then $g(x) \leq f(x)$ for all $2 \leq x \leq 6$.

Exercise

Let f be an odd function such that $\int_{-2}^{0} f(x)\,\mathrm{d}x = 4$. Find

1. $\int_{0}^{2} f(x)\,\mathrm{d}x$
2. $\int_{-2}^{2} f(x)\,\mathrm{d}x$

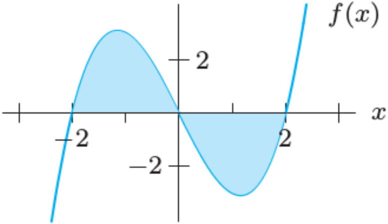


Figure: Applied Calculus, 6th Edition, Deborah Hughes-Hallett and col.

Exercise

Decide, if the integrals are

A $\displaystyle\int_{-\pi}^{0} \sin x \, \mathrm{d}x$

B $\displaystyle\int_{0}^{\pi} \cos x \, \mathrm{d}x$

C $\displaystyle\int_{-\pi}^{\pi} \sin x \, \mathrm{d}x$

D $\displaystyle\int_{-\pi/2}^{\pi/2} \cos x \, \mathrm{d}x$

E $\displaystyle\int_{0}^{2\pi} e^{-x} \sin x \, \mathrm{d}x$

1. positive
2. 0
3. negative