

## Homework 8

Please hand in the solutions per mail to [schwarz@karlin.mff.cuni.cz](mailto:schwarz@karlin.mff.cuni.cz) until Sunday the 6th of December.

1. Consider the function  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = e^x$  for  $x > 0$  and  $f(x) = x$  for  $x \leq 0$ .
  - i Is the function continuous (at 0)?
  - ii Is the function differentiable (at 0)? (Use difference quotients.)
  - iii Does the function possess a global minimum or maximum in  $\mathbb{R}$ ?
2. Consider  $\arccos : [-1, 1] \rightarrow \mathbb{R}$  as the inverse function of  $\cos : [0, \pi] \rightarrow [-1, 1]$ .
  - i Prove that  $\arccos'(x) = \frac{-1}{\sqrt{1-x^2}}$  for  $x \in (-1, 1)$ .
  - ii  $\arccos(1) = 0$  and  $\arccos(-1) = \pi$ . Does the function possess a left/right derivative at  $-1$  and  $1$ ?
3. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = |x|$  in  $[-1, 1]$ ,  $f(x) = x^2$  for  $x > 1$ .
  - i For which  $x \in [-1, \infty)$  is the function differentiable?
  - ii Calculate its derivative at the points where it exists?
  - iii Does this function possess a global minimum?