Homework 6

Please hand in the solutions per mail to schwarz@karlin.mff.cuni.cz until Friday the 20th of November.

1. Consider the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = e^x - 10 + sin(\log(1 + x^2))$.

i Is the function continuous?

ii Does the function have a minimum?

iii Is there a point $x \in \mathbb{R}$, such that $f(x) = 0$? (Hint: use the intermediate value theorem).

2. Let $f : [0, 1] \to \mathbb{R}$, $f(x) = x^2$ and $g : [0, 1] \to \mathbb{R}$, $g(x) = e^{-x}$. Is there a value $x \in [0, 1]$ such that $f(x) = g(x)$? (Hint: use the intermediate value theorem).

3. Let $f : \mathbb{R} \to \mathbb{R}$ $f(x) = |x|$ for $x \in [-2, 1]$, $f(x) = 2 - |x|$ for $x \in (1, \infty)$ and $f(x) = 3 - |x|$ for $x \in (-\infty, -1)$.

a) For which $x \in \mathbb{R}$ is f continuous? For which $x \in \mathbb{R}$ is f discontinuous?

b) Find all local maxima and minima of the function f.