

Homework 2

Please hand in the solutions per mail to schwarz@karlin.mff.cuni.cz until Saturday the 30th of October.

1. Please provide the right subset of \mathbb{R} such that the following functions are well defined with image in \mathbb{R} . Are they injective? Are they onto \mathbb{R} ?

i $f(x) = \sqrt{x} + \frac{\sin x}{x}$

ii $g(x) = \log(1 + x^{\frac{1}{3}})$

iii $h(x) = e^{x^2 - \tan(x)}$

2. a) Consider the sequence $a_{n+1} = 0.5a_n + 0.5$, $a_0 = 1$ find the limit of this sequence.

b) Find 3 converging subsequence of the sequence $\{\cos(n\pi/2)\}_{n \in \mathbb{N}}$

3. a) Find the domain of definition of the inverse function f^{-1} of the function of $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^{x^2}$. What is the inverse function?

b) Consider $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \sin(x) + \cos(x)$ and $g : (0, \infty) \rightarrow \mathbb{R}$, $g(x) = \log(x)$. For which $x \in \mathbb{R}$ $g \circ f(x)$ is well defined in \mathbb{R} ? $x \in \mathbb{R}$ $f \circ g(x)$ is well defined in \mathbb{R} ?