19th lesson

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Theory

Theorem 1 (Necessary condition for a local extremum)**.** Suppose that a function f has a local extremum at $x_0 \in \mathbb{R}$. If $f'(x_0)$ exists, then $f'(x_0) = 0$.

Theorem 2 (Sign of the derivative and monotonicity)**.** Let I be an interval and f is continuous on I. Let f has a derivative at every inner point of I (denoted by $\operatorname{Int} I$).

(i) if $f'(x) > 0$ for every $x \in \operatorname{Int} I$, then f is **increasing** on I;

(ii) if $f'(x) \geq 0$ for every $x \in \operatorname{Int} I$, then f is **non-decreasing** on I;

(iii) if $f'(x) < 0$ for every $x \in \operatorname{Int} I$, then f is **decreasing** on I;

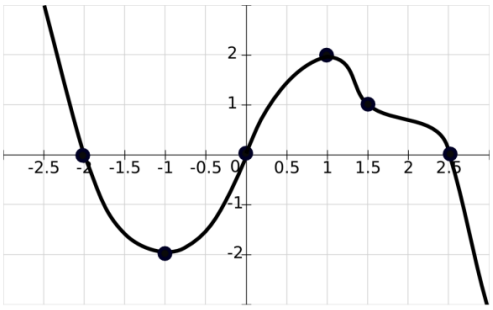
(iv) if $f'(x) \leq 0$ for every $x \in \operatorname{Int} I$, then f is **non-increasing** on I.

Exercises

Find the (local) maxima/minima. Find the intervals of monotonicity.

1. (a) $f(x) = 2x^3 - 3x^2 - 12x + 12$
   (b) $f(x) = 2x^3 + 9x^2 - 108x + 30$
   (c) $f(x) = \dfrac{x^2 + 4}{2x}$
   (d) $f(x) = 2x^3 - x^4$
   (e) $f(x) = e^x(x^2 - x - 5)$
   (f) $f(x) = (x - 2)^{2/3} + 1$
   (g) $f(x) = x^2 e^{3x}$
   (h) $f(x) = \ln(1 - \ln x)$
   (i) $f(x) = x - \sin x$
   (j) $f(x) = \dfrac{2x}{x^2 + 4}$
   (k) $f(x) = \sqrt[3]{x} e^{x/6}$

2. The graph is a derivative of a function. Find the intervals, where the function is increasing or decreasing, find extrema.



Source 1: https://liavas.net/courses/calc1/files/Inc_dec_1st_der_test.pdf

3. You would like to make a rectangular fence for sheeps. One side of the fence will be along your house - so you do not need to build a fence on this edge. But You have only 80 m of fencing. Find the largest possible area of the sheep fence (and find the length of rectangular sides).

Source 2: https://www.cbr.com/shaun-the-sheep-best-worst-episodes-imdb/

4. You plant potatoes. On July 1 You have 80 quintals of potatoes, but the amount of poptatoes increases: 1 quintal per day. On July 1 You obtain 2 dollars per 1 quintal, then the price decrease by 2 cents per quintal every day.

   When it is the best time to harvest your potatoes?