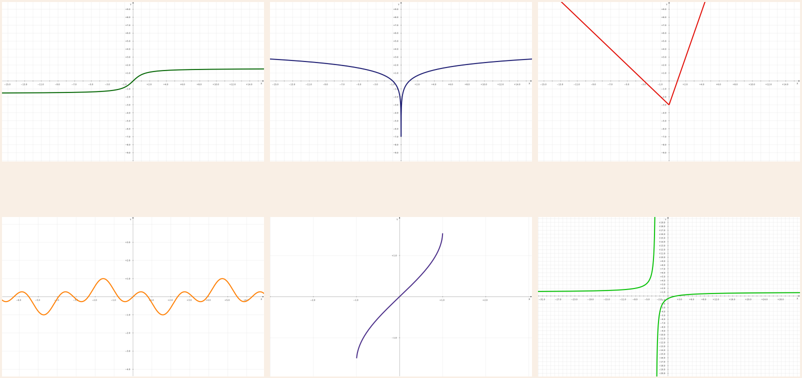
Properties of functions

Kristýna Kuncová

Matematika B2 18/19

Exercise

Find even and odd functions:



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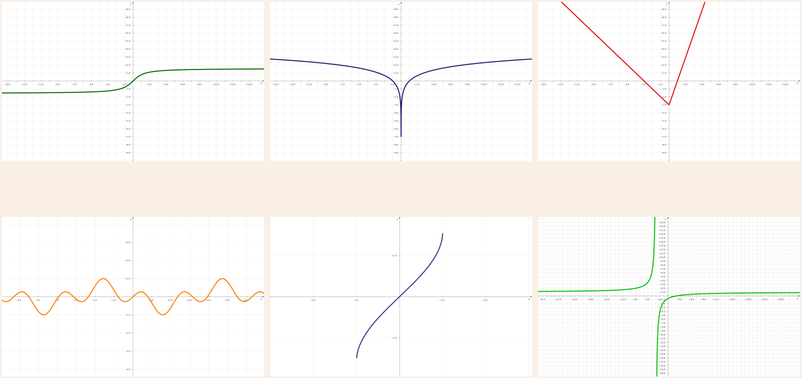

odd: A, D, E (symmetry about the origin)
even: B (symmetry about the y axis)

Definition

Function $f : M \to \mathbb{R}, M \subseteq R$:

- *even*: $\forall x \in M : -x \in M$ a $f(-x) = f(x)$.
- *odd*: $\forall x \in M : -x \in M$ a $f(-x) = -f(x)$.

Exercise

Find even and odd functions:

A $x^3 + 1$

B $x(x^2 + 1)$

C $|x - 2|$

D $e^{x^2} \sin x$

E $|1 + \cos x|$

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even: E

odd: B, D

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D $e^{x^2} \sin x$

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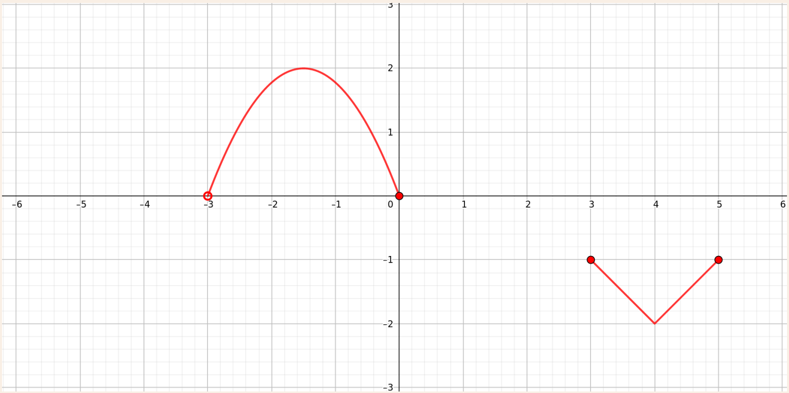
We apply the definition. Even: E, Odd: B, D.
E: Since $\cos x$ is even function, we have $\cos(-x) = \cos x$. Then $f(-x) = |1 + \cos(-x)| = |1 + \cos x| = f(x)$.
B: $f(-x) = -x((-x)^2 + 1) = -x(x^2 + 1) = -f(x)$
D: Since $\sin x$ is odd function, we have $\sin(-x) = -\sin x$. Then $f(-x) = e^{(-x)^2} \sin(-x) = e^{x^2}(-\sin x) = -f(x)$.
The other functions does not fit the definition. Let us try set some different x:
For example A: $f(1) = 1 + 1 = 2$, but $f(-1) = -1 + 1 = 0$. Hence $f(-x) \neq \pm f(x)$, the function is neither even, nor odd.

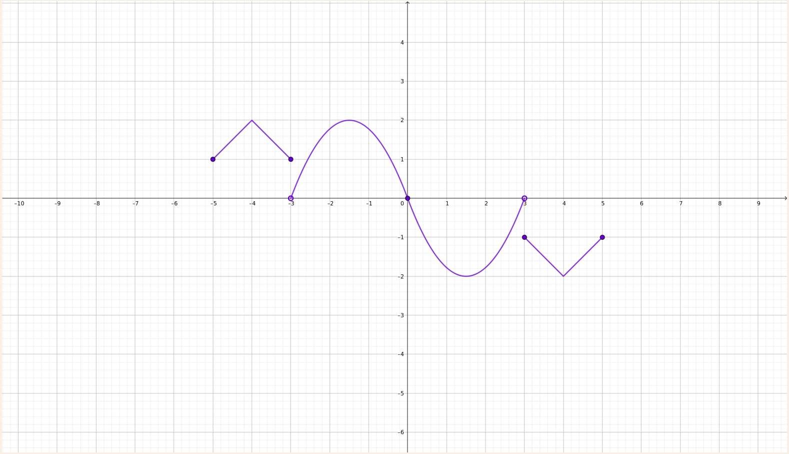
Exercise

Sketch the graph to be odd/even:



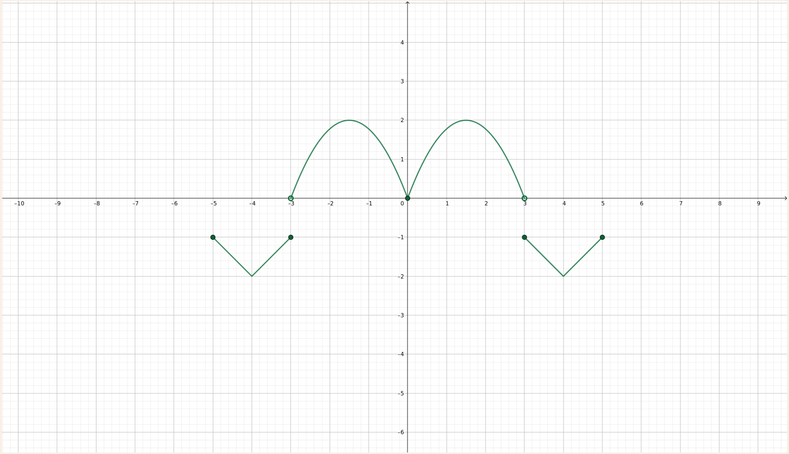
Exercise

Odd:



Exercise

Even:



Question

Complete the table of function values, such that

1. f is an even function
2. g is an odd function
3. h is the composition $g(f(x))$.

x	-3	-2	-1	0	1	2	3
$f(x)$	0	2	2	0			
$g(x)$	0	2	2	0			
$h(x)$							

Source: Calculus: Single and Multivariable, Hughes-Hallet

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Question (True or False?)

1. If a function is even, then it does not have an inverse.
2. If a function is odd, then it does not have an inverse.
3. If $g(x)$ is an even function, then $f(g(x))$ is even for every function $g(x)$.
4. There is a function which is both even and odd.

Question (True or False?)

1. If a function is even, then it does not have an inverse.
2. If a function is odd, then it does not have an inverse.
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False. For example if $f(x) = 0$ only for $x = 0$, then f is even and has inverse $f^{-1} = f$. On the other hand, $\cos(x)$ does not have inverse - it is true for domains larger than one point.
False, for example x^3 has inverse, $\sin x$ has not.
True. Since g is even, we have $g(-x) = g(x)$. Hence $f(g(-x)) = f(g(x))$.
True, $f(x) \equiv 0$.

Source: Calculus: Single and Multivariable, Hughes-Hallet

Definition

We say that $f : M \to \mathbb{R}$, $M \subseteq R$, is *periodic* with a period $p \in \mathbb{R}$, if $\forall x \in M : (x \pm p) \in M$ and $f(x+p) = f(x) = f(x-p)$.

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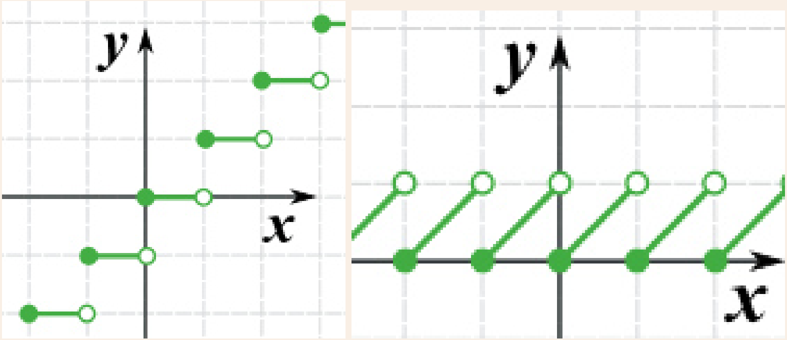
Decide, if the sketched function is periodic:



Caption: https://math.stackexchange.com/questions/582930/how-does-a-piecewise-step-function-work

Inspired by:realisticky.cz

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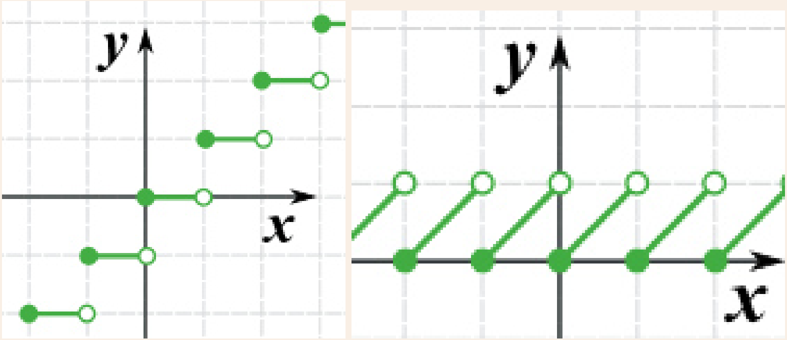

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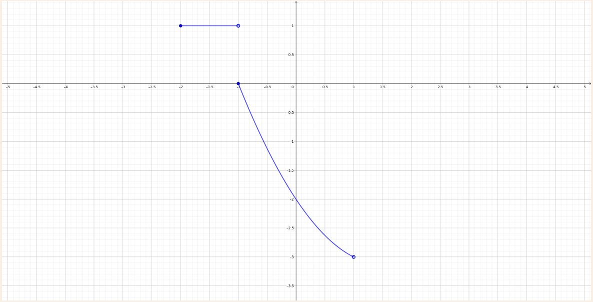
No, Yes.

Exercise

Sketch the graph to be periodic (with smallest possible period):



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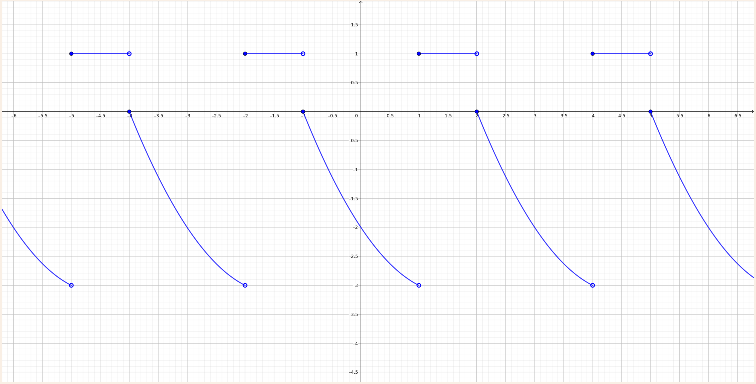

Question (True or False?)

1. If $g(x)$ is a periodic function with period k, then $f(g(x))$ is periodic with period k for every function $f(x)$.
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Question (True or False?)

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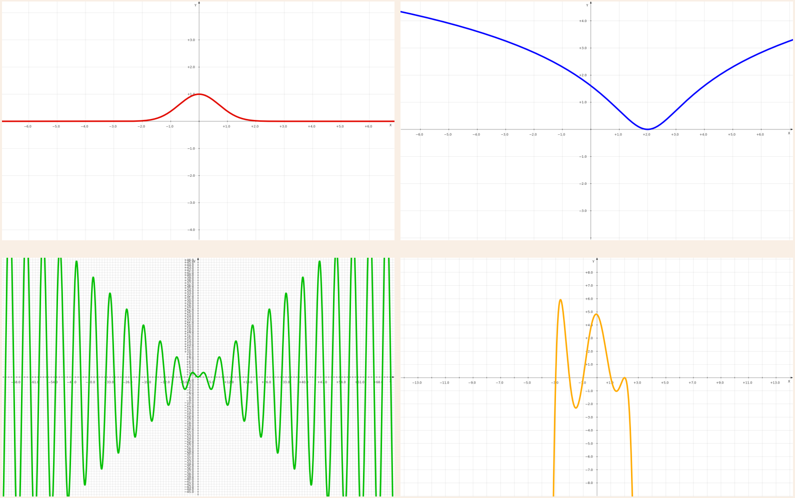
Yes.
No, for example $\sin(x^2)$.

Source: Calculus: Single and Multivariable, Hughes-Hallet

Exercise

Find function which is bounded, bounded from above, bounded from below, unbounded:



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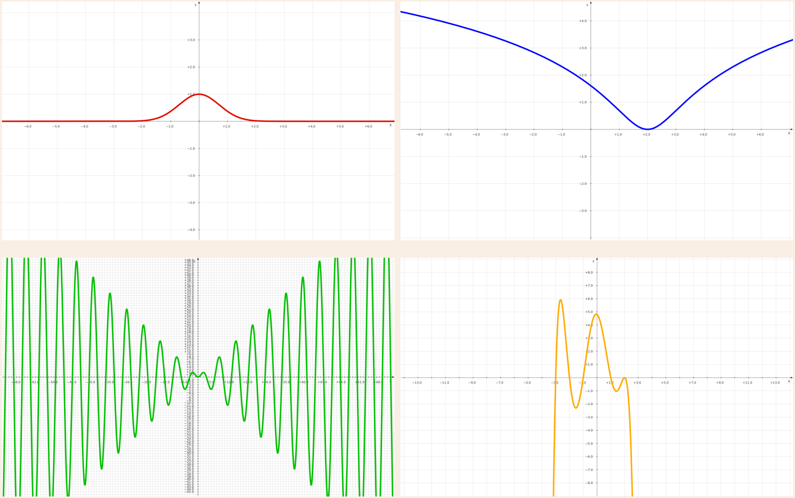
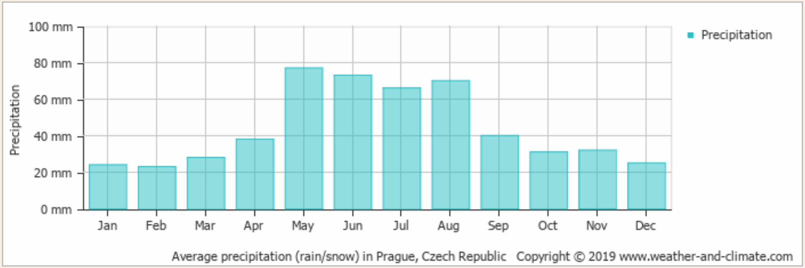


A; B; D; C

Exercise

In the graph there is average precipitation in Prague (per month). Find intervals, where the function is increasing and decreasing



Caption:
https://weather-and-climate.com/average-monthly-Rainfall-Temperature-Sunshine,Prague,Czech-Republic

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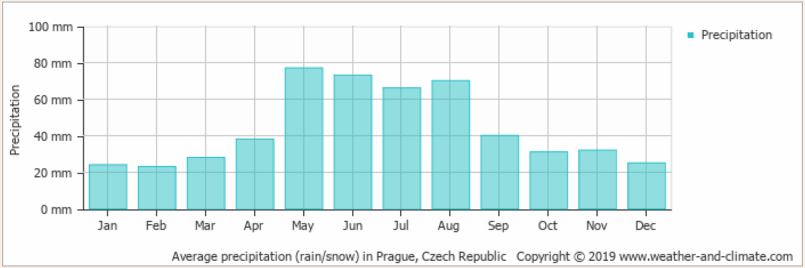
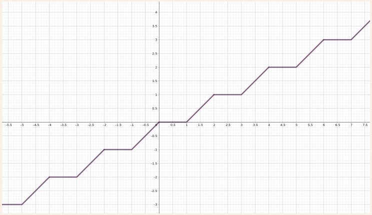


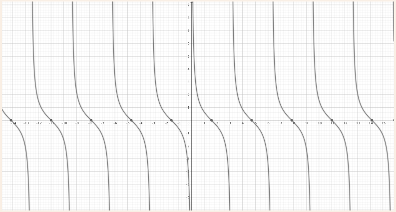
Caption:
https://weather-and-climate.com/average-monthly-Rainfall-Temperature-Sunshine,Prague,Czech-Republic

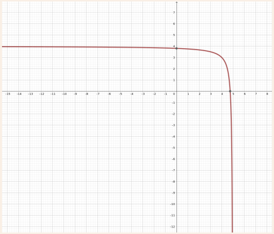
Increasing: February – May, July – August, October – November
Decreasing: January – February, May – July, August – October, November – December

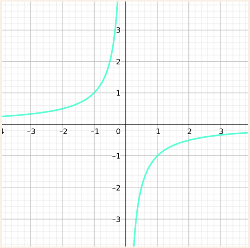
Exercise

What can you say about the monotony of the following functions:









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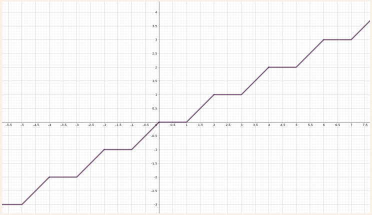
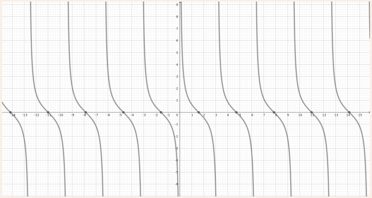


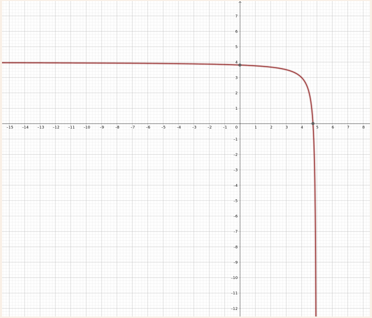
Function is nondecreasing on $\mathbb{R}$.
On intervals $[-1;0]$, $[1;2]$, $[3;4]$ etc. f is increasing.
On $[-2;-1]$, $[0;1]$, $[2;3]$ etc. f is constant = nondecreasing and nonincreasing simultaneously.

Exercise



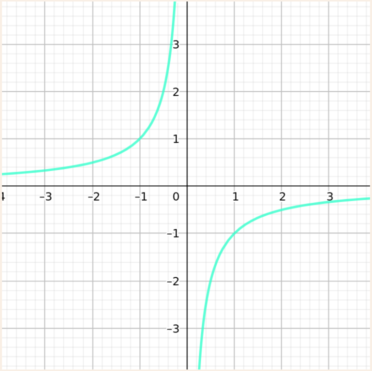
It is cotangens. It is decreasing on intervals $(-\pi; 0)$, $(0; \pi)$ etc. However, f is NOT decreasing on $\mathbb{R}$. Compare for example $f(-1)$ and $f(1/2)$.

Exercise



It is decreasing function on $\mathbb{R}$.

Exercise



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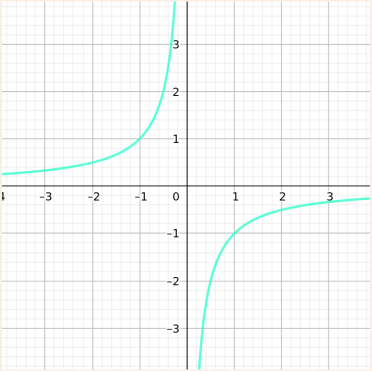
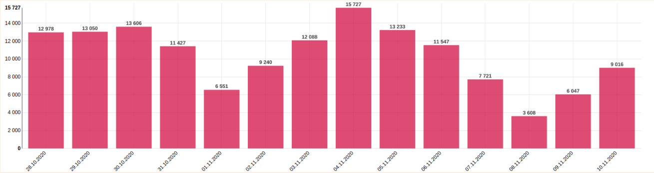


Function is increasing on $(-\infty; 0)$ and on $(0; \infty)$.
However, f is NOT increasing on $(-\infty; 0) \cup (0; \infty)$.

Exercise

In the graph there is daily growth of new cases of COVID for last 14 days. Find local/global maxima and minima.



Caption: https://onemocneni-aktualne.mzcr.cz/covid-19

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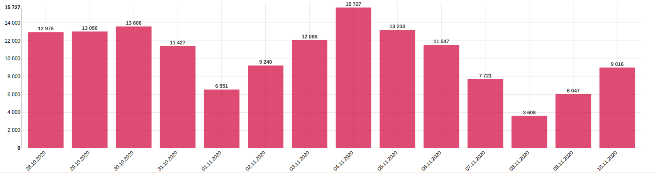
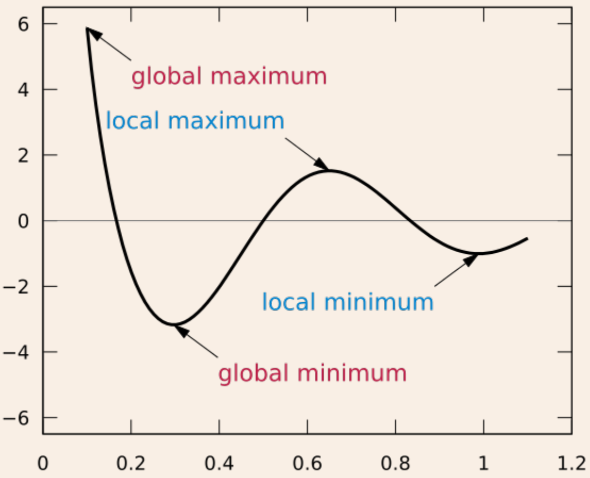


Caption: https://onemocneni-aktualne.mzcr.cz/covid-19

Global maximum: 4.11., global minimum: 8.11.
Local maximum: 30.10., 4.11., 10.11., local minimum 28.10., 1.11., 8.11.

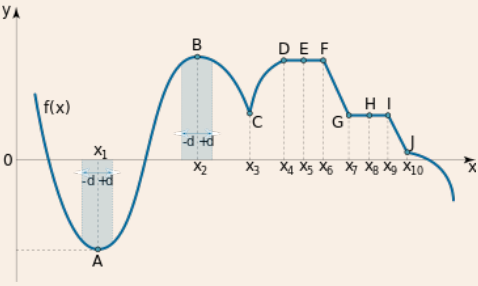
Remark



Caption: https://en.wikipedia.org/wiki/Maxima_and_minima

Remark

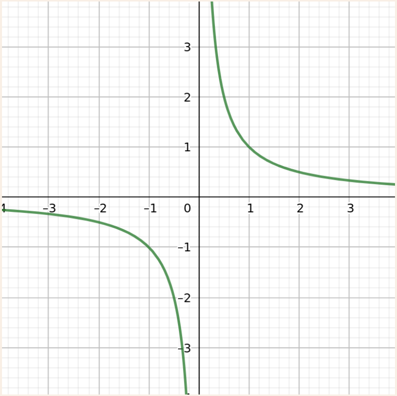
Is there maximum at point x_5?



Caption: https://www.math24.net/local-extrema-functions/

Remark

Find maximum and minimum.



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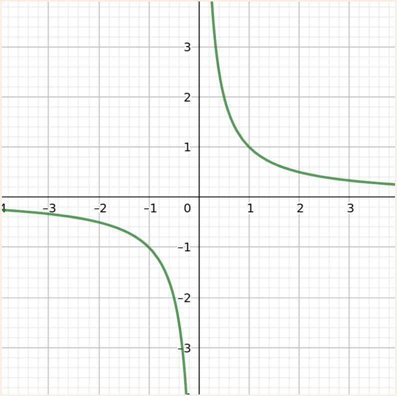

This function does not have any maximum or minimum.