Continuity

Kristýna Kuncová



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Definition

Let $f : M \to \mathbb{R}, M \subset \mathbb{R}, a \in M$. We say that f is *continuous at a*,aif

 $\lim_{x \to a} f(x) = f(a).$



Caption: Calculus: Single and Multivariable, Hughes-Hallet

Sketch a graph of function, which

- 1. is continuous on \mathbb{R}
- 2. is continuous on $\mathbb{R} \setminus \{-3\}$
- 3. has discontinuity at infinitely many points.

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Image: A mathematical states of the state

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Question

Find functions continuous on \mathbb{R} :

A
$$x^{3} + \sin(4 - x)$$

B $\frac{e^{x}}{2 + x}$
C $\frac{2 + x}{e^{x}}$
D $\cos(e^{\sqrt{x}})$

$$E \ln(2+x^2)$$

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E $\ln(2+x^2)$

Is the following function continuous?

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

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Is the following function continuous?

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Is not.

Source: Calculus: Single and Multivariable, Hughes-Hallet

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Sketch a graph of the function such that:

- 1. f is continuous
- **2.** f(0) = 2
- 3. *f* is decreasing for $0 \le x \le 3$
- 4. *f* is increasing for $3 < x \le 5$
- 5. *f* is decreasing for x > 5

6. $f \rightarrow 9$ as $x \rightarrow \infty$

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Find $k \in \mathbb{R}$, such that f is continuous on \mathbb{R} .

1.

$$f(x) = \begin{cases} kx, & x < 3, \\ 5, & 3 \le x \end{cases}$$

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Image: A mathematical states and a mathem

Find $k \in \mathbb{R}$, such that f is continuous on \mathbb{R} .

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k = 5/3

Find $k \in \mathbb{R}$, such that f is continuous on \mathbb{R} .

 $f(x) = \begin{cases} kx, & x < 3, \\ 5, & 3 \le x \end{cases}$

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 $k = \frac{e^6 - 1}{2}$

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3.

2.

$$f(x) = \begin{cases} k \cos x, & x < \pi, \\ k + x, & \pi \le x \end{cases}$$

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Find $k \in \mathbb{R}$, such that *f* is continuous on \mathbb{R} . 1. $f(x) = \begin{cases} kx, & x < 3, \\ 5, & 3 \le x \end{cases}$ k = 5/32. $f(x) = \begin{cases} \ln(kx+1), & 0 < x < 2, \\ x+4, & 2 \le x \end{cases}$ $k = \frac{e^{6}-1}{2}$ 3.

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 $k = -\frac{\pi}{2}$

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