Continuity

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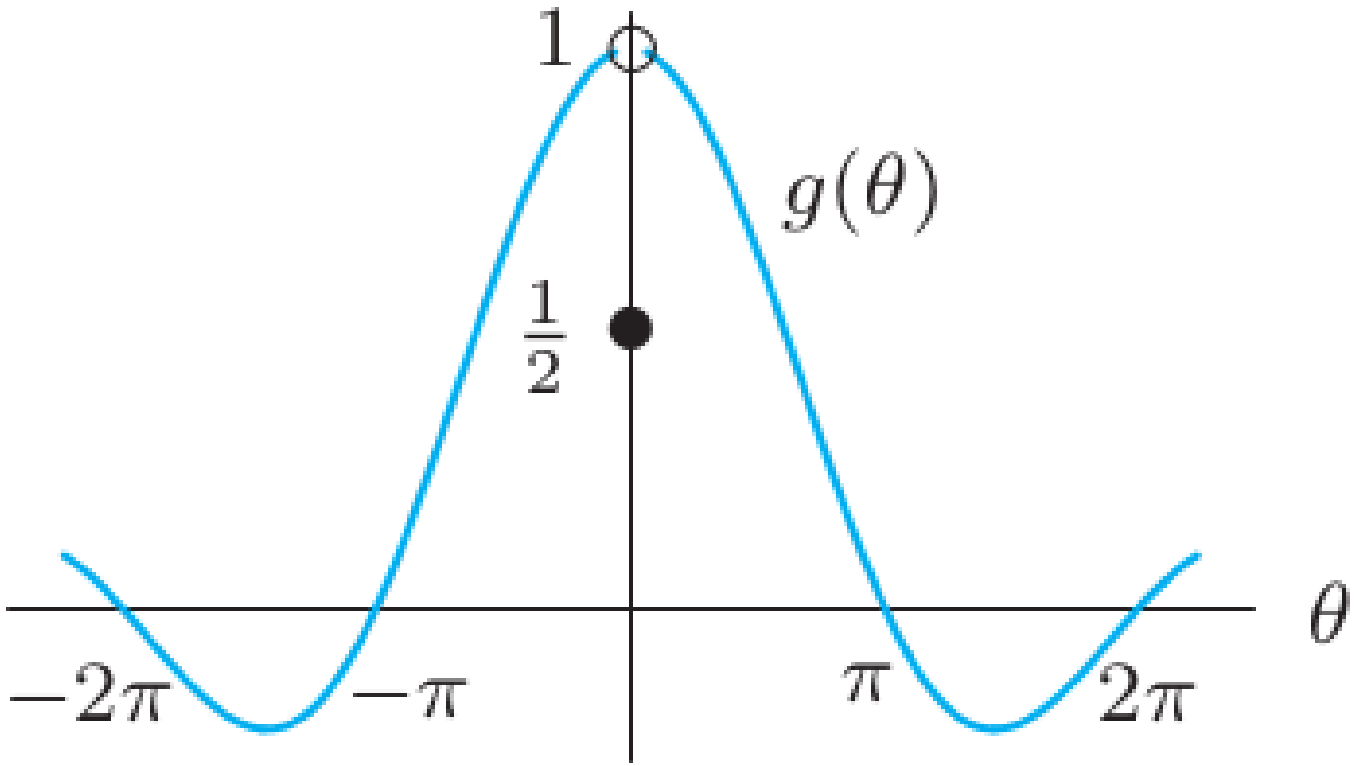
Definition

Let $f : M \to \mathbb{R}$, $M \subset \mathbb{R}$, $a \in M$. We say that f is *continuous at* a,aif

$$\lim_{x \to a} f(x) = f(a).$$



Caption: Calculus: Single and Multivariable, Hughes-Hallet

Question

Sketch a graph of function, which

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Find functions continuous on $\mathbb{R}$:

A $x^3 + \sin(4 - x)$

B $\frac{e^x}{2+x}$

C $\frac{2+x}{e^x}$

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Is not.

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Sketch a graph of the function such that:

1. f is continuous
2. $f(0) = 2$
3. f is decreasing for $0 \leq x \leq 3$
4. f is increasing for $3 < x \leq 5$
5. f is decreasing for $x > 5$
6. $f \to 9$ as $x \to \infty$

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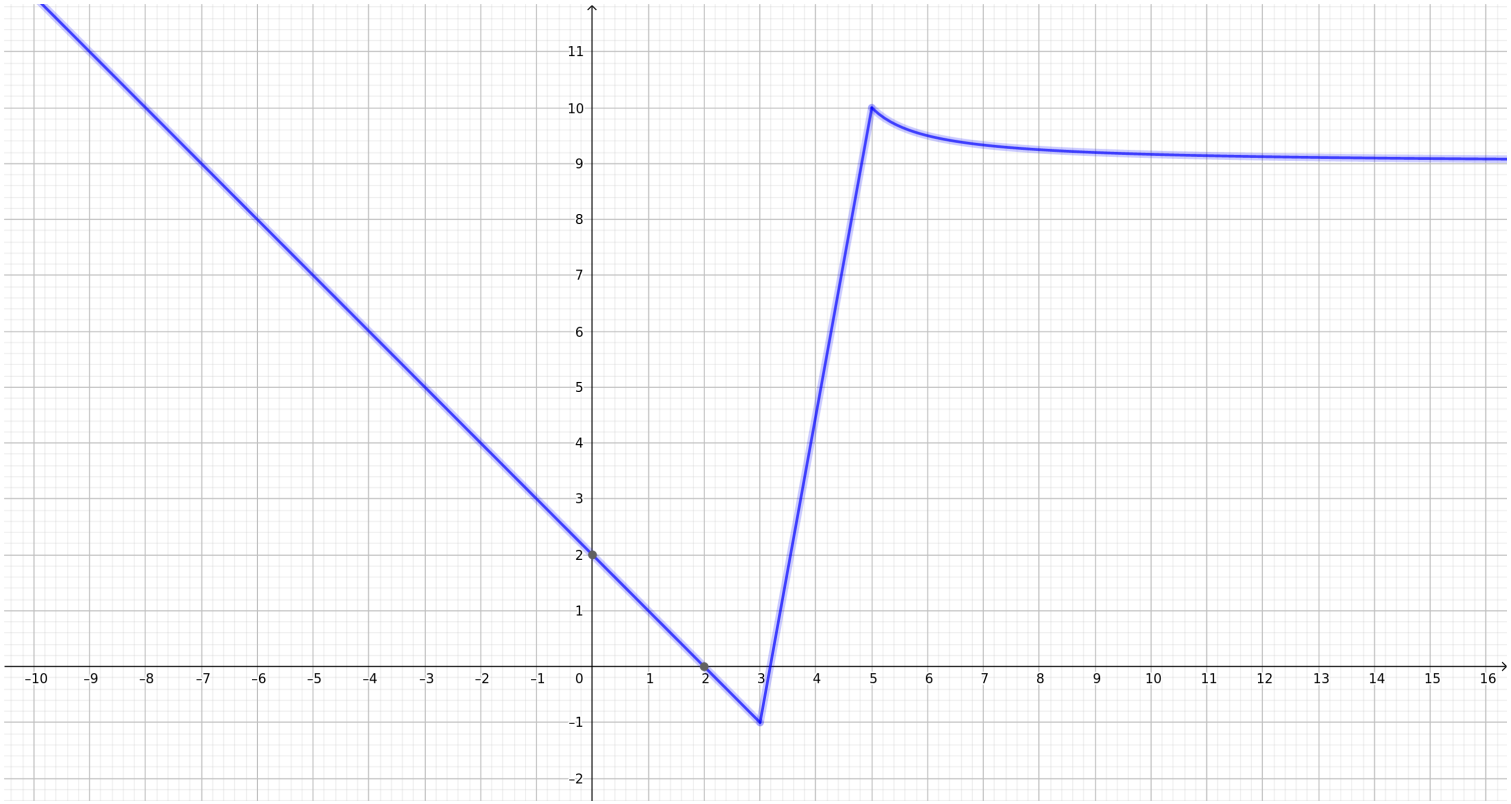
11
10
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−1
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−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

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no solution:(

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