0

3. Find limits:

(a) $\lim_{x \to 1} \frac{x-1}{x^2+x-2}$

(b) $\lim_{x \to -3} \frac{x^2+x-6}{x+3}$

(c) $\lim_{x \to 2} \frac{x^2+3x-4}{x^2-4x+4}$

(d) $\lim_{x \to 0} \frac{1}{\sin x}$

(e) $\lim_{x \to -2} \frac{-4}{x+2}$

(f) $\lim_{x \to 4} \frac{3}{(4-x)^3}$

(g) $\lim_{x \to 3} \frac{2x}{x-3}$

(h) $\lim_{x \to 4} \frac{x^2}{x^2-4}$

(i) $\lim_{x \to -3} \frac{x^2-2x-3}{x^2+6x+9}$

(j) $\lim_{x \to -\infty} \frac{1}{e^x}$

(k) $\lim_{x \to 0} \frac{|2x|}{x}$

Of course we must add to (3) the all-important requirement that the limit of the denominator is not 0, that is, $q(a) \neq 0$.

EXAMPLE 7 Using (2) and (3)

Evaluate $\lim_{x \to -1} \dfrac{3x - 4}{8x^2 + 2x - 2}$.

Solution $f(x) = \dfrac{3x - 4}{8x^2 + 2x - 2}$ is a rational function and so if we identify the polynomials $p(x) = 3x - 4$ and $q(x) = 8x^2 + 2x - 2$, then from (2),

$$\lim_{x \to -1} p(x) = p(-1) = -7 \qquad \text{and} \qquad \lim_{x \to -1} q(x) = q(-1) = 4.$$

Since $q(-1) \neq 0$ it follows from (3) that

$$\lim_{x \to -1} \frac{3x - 4}{8x^2 + 2x - 2} = \frac{p(-1)}{q(-1)} = \frac{-7}{4} = -\frac{7}{4}. \qquad ■$$

You should not get the impression that we can *always* find a limit of a function by substituting the number *a directly into the function.*

EXAMPLE 8 Using Theorem 2.2.3

Evaluate $\lim_{x \to 1} \dfrac{x - 1}{x^2 + x - 2}$.

Solution The function in this limit is rational, but if we substitute $x = 1$ into the function we see that this limit has the indeterminate form $0/0$. However, by simplifying *first*, we can then apply Theorem 2.2.3(*iii*):

$$\begin{aligned} \lim_{x \to 1} \frac{x - 1}{x^2 + x - 2} &= \lim_{x \to 1} \frac{x - 1}{(x - 1)(x + 2)} \qquad \leftarrow \text{cancellation is valid provided that } x \neq 1 \\ &= \lim_{x \to 1} \frac{1}{x + 2} \\ &= \frac{\lim_{x \to 1} 1}{\lim_{x \to 1} (x + 2)} = \frac{1}{3}. \end{aligned} \qquad ■$$

◀ If a limit of a rational function has the indeterminate form $0/0$ as $x \to a$, then by the Factor Theorem of algebra $x - a$ must be a factor of both the numerator and the denominator. Factor those quantities and cancel the factor $x - a$.

Sometimes you can tell at a glance when *a limit does not exist*.

Theorem 2.2.5 A Limit That Does Not Exist

Let $\lim_{x \to a} f(x) = L_1 \neq 0$ and $\lim_{x \to a} g(x) = 0$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

does not exist.

PROOF We will give an indirect proof of this result based on Theorem 2.2.3. Suppose $\lim_{x \to a} f(x) = L_1 \neq 0$ and $\lim_{x \to a} g(x) = 0$ and suppose further that $\lim_{x \to a} (f(x)/g(x))$ exists and equals L_2. Then

$$\begin{aligned} L_1 = \lim_{x \to a} f(x) &= \lim_{x \to a} \left(g(x) \cdot \frac{f(x)}{g(x)} \right), \qquad g(x) \neq 0, \\ &= \left(\lim_{x \to a} g(x) \right) \left(\lim_{x \to a} \frac{f(x)}{g(x)} \right) = 0 \cdot L_2 = 0. \end{aligned}$$

By contradicting the assumption that $L_1 \neq 0$, we have proved the theorem. ■

11.2 Techniques for Evaluating Limits

What you should learn

- Use the dividing out technique to evaluate limits of functions.
- Use the rationalizing technique to evaluate limits of functions.
- Use technology to approximate limits of functions graphically and numerically.
- Evaluate one-sided limits of functions.
- Evaluate limits of difference quotients from calculus.

Why you should learn it

Many definitions in calculus involve the limit of a function. For instance, in Exercises 69 and 70 on page 768, the definition of the velocity of a free-falling object at any instant in time involves finding the limit of a position function.

Dividing Out Technique

In Section 11.1, you studied several types of functions whose limits can be evaluated by direct substitution. In this section, you will study several techniques for evaluating limits of functions for which direct substitution fails.

Suppose you were asked to find the following limit.

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

Direct substitution fails because -3 is a zero of the denominator. By using a table, however, it appears that the limit of the function as x approaches -3 is -5.

x	-3.01	-3.001	-3.0001	-3	-2.9999	-2.999	-2.99
$\frac{x^2+x-6}{x+3}$	-5.01	-5.001	-5.0001	?	-4.9999	-4.999	-4.99

Another way to find the limit of this function is shown in Example 1.

Example 1 Dividing Out Technique

Find the limit.

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

Solution

Begin by factoring the numerator and dividing out any common factors.

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3} = \lim_{x \to -3} \frac{(x-2)(x+3)}{x+3} \quad \text{Factor numerator.}$$

$$= \lim_{x \to -3} \frac{(x-2)\cancel{(x+3)}}{\cancel{x+3}} \quad \text{Divide out common factor.}$$

$$= \lim_{x \to -3} (x-2) \quad \text{Simplify.}$$

$$= -3 - 2 \quad \text{Direct substitution}$$

$$= -5 \quad \text{Simplify.}$$

✓CHECKPOINT Now try Exercise 11.

This procedure for evaluating a limit is called the **dividing out technique.** The validity of this technique stems from the fact that when two functions agree at all but a single number c, they must have identical limit behavior at $x = c$. In Example 1, the functions given by

$$f(x) = \frac{x^2 + x - 6}{x + 3} \quad \text{and} \quad g(x) = x - 2$$

agree at all values of x other than

$$x = -3.$$

So, you can use $g(x)$ to find the limit of $f(x)$.

EXAMPLE 10. Evaluate infinite limit

$$\lim_{x\to 2} \frac{x^2+3x-4}{x^2-4x+4}$$

Factoring and sign analysis:

$$= \lim_{x\to 2} \frac{(x+4)(x-1)}{(x-2)^2} = \frac{(6)\cdot(1)}{(0^+)} = +\infty$$

EXAMPLE 11. Evaluate infinite limit

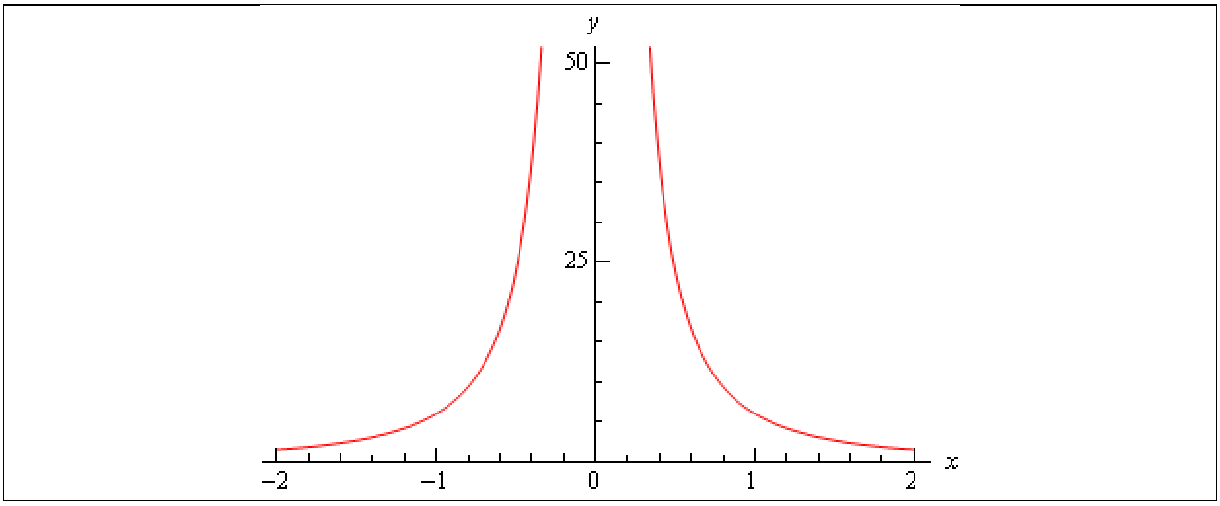
$$\lim_{x \to 0} \frac{1}{\sin x}$$

Sign analysis for one-sided limits:

$$\lim_{x \to 0^+} \frac{1}{\sin x} = \frac{1}{(0^+)} = +\infty$$

$$\lim_{x \to 0^-} \frac{1}{\sin x} = \frac{1}{(0^-)} = -\infty$$

Limit at 0 does not exist



With this next example we'll move away from just an *x* in the denominator, but as we'll see in the next couple of examples they work pretty much the same way.

3e

Example 3 Evaluate each of the following limits.

$$\lim_{x \to -2^+} \frac{-4}{x+2} \qquad \lim_{x \to -2^-} \frac{-4}{x+2} \qquad \lim_{x \to -2} \frac{-4}{x+2}$$

Solution

Let's again start with the right-hand limit. With the right-hand limit we know that we have,

$$x > -2 \qquad \Rightarrow \qquad x+2 > 0$$

Also, as *x* gets closer and closer to -2 then $x+2$ will be getting closer and closer to zero, while staying positive as noted above. So, for the right-hand limit, we'll have a negative constant divided by an increasingly small positive number. The result will be an increasingly large and negative number. So, it looks like the right-hand limit will be negative infinity.

For the left-hand limit we have,

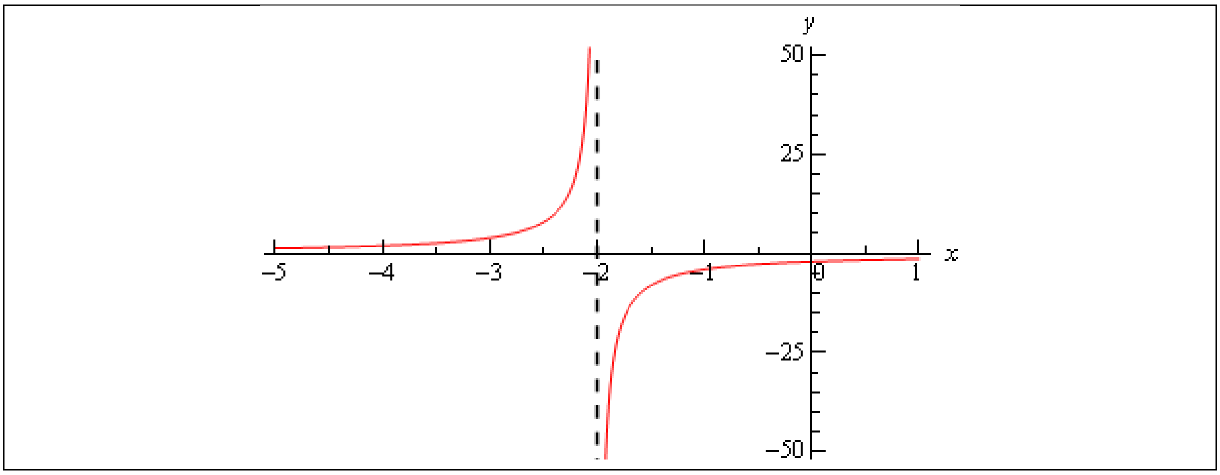
$$x < -2 \qquad \Rightarrow \qquad x+2 < 0$$

and $x+2$ will get closer and closer to zero (and be negative) as *x* gets closer and closer to -2. In this case then we'll have a negative constant divided by an increasingly small negative number. The result will then be an increasingly large positive number and so it looks like the left-hand limit will be positive infinity.

Finally, since two one sided limits are not the same the normal limit won't exist.

Here are the official answers for this example as well as a quick graph of the function for verification purposes.

$$\lim_{x \to -2^+} \frac{-4}{x+2} = -\infty \qquad \lim_{x \to -2^-} \frac{-4}{x+2} = \infty \qquad \lim_{x \to -2} \frac{-4}{x+2} \text{ doesn't exist}$$



At this point we should briefly acknowledge the idea of vertical asymptotes. Each of the three previous graphs have had one. Recall from an Algebra class that a vertical asymptote is a vertical line (the dashed line at $x = -2$ in the previous example) in which the graph will go towards infinity and/or minus infinity on one or both sides of the line.

In an Algebra class they are a little difficult to define other than to say pretty much what we just said. Now that we have infinite limits under our belt we can easily define a vertical asymptote as follows,

Definition

The function *f(x)* will have a vertical asymptote at $x = a$ if we have any of the following limits at $x = a$.

$$\lim_{x \to a^-} f(x) = \pm\infty \qquad \lim_{x \to a^+} f(x) = \pm\infty \qquad \lim_{x \to a} f(x) = \pm\infty$$

Note that it only requires one of the above limits for a function to have a vertical asymptote at $x = a$.

Using this definition we can see that the first two examples had vertical asymptotes at $x = 0$ while the third example had a vertical asymptote at $x = -2$.

We aren't really going to do a lot with vertical asymptotes here but wanted to mention them at this point since we'd reached a good point to do that.

Let's now take a look at a couple more examples of infinite limits that can cause some problems on occasion.

Example 4 Evaluate each of the following limits.

$$\lim_{x \to 4^+} \frac{3}{(4-x)^3} \qquad \lim_{x \to 4^-} \frac{3}{(4-x)^3} \qquad \lim_{x \to 4} \frac{3}{(4-x)^3}$$

Solution

Let's start with the right-hand limit. For this limit we have,

$$x > 4 \qquad \Rightarrow \qquad 4 - x < 0 \qquad \Rightarrow \qquad (4-x)^3 < 0$$

also, $4 - x \to 0$ as $x \to 4$. So, we have a positive constant divided by an increasingly small negative number. The results will be an increasingly large negative number and so it looks like the right-hand limit will be negative infinity.

For the left-handed limit we have,

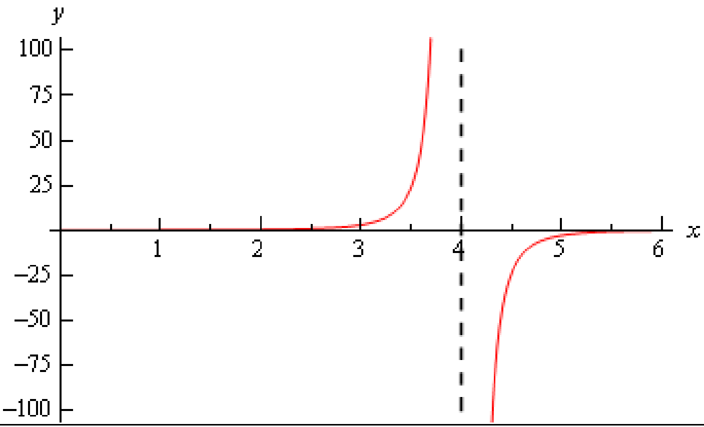
$$x < 4 \qquad \Rightarrow \qquad 4 - x > 0 \qquad \Rightarrow \qquad (4-x)^3 > 0$$

and we still have, $4 - x \to 0$ as $x \to 4$. In this case we have a positive constant divided by an increasingly small positive number. The results will be an increasingly large positive number and so it looks like the left-hand limit will be positive infinity.

The normal limit will not exist since the two one-sided limits are not the same. The official answers to this example are then,

$$\lim_{x \to 4^+} \frac{3}{(4-x)^3} = -\infty \qquad \lim_{x \to 4^-} \frac{3}{(4-x)^3} = \infty \qquad \lim_{x \to 4} \frac{3}{(4-x)^3} \text{ doesn't exist}$$

Here is a quick sketch to verify our limits.



All the examples to this point have had a constant in the numerator and we should probably take a quick look at an example that doesn't have a constant in the numerator.

Example 5 Evaluate each of the following limits.

$$\lim_{x \to 3^+} \frac{2x}{x-3} \qquad \lim_{x \to 3^-} \frac{2x}{x-3} \qquad \lim_{x \to 3} \frac{2x}{x-3}$$

Solution

Let's take a look at the right-handed limit first. For this limit we'll have,

$$x > 3 \qquad \Rightarrow \qquad x - 3 > 0$$

The main difference here with this example is the behavior of the numerator as we let *x* get closer and closer to 3. In this case we have the following behavior for both the numerator and denominator.

$$x - 3 \to 0 \quad \text{and} \quad 2x \to 6 \quad \text{as} \quad x \to 3$$

So, as we let *x* get closer and closer to 3 (always staying on the right of course) the numerator, while not a constant, is getting closer and closer to a positive constant while the denominator is getting closer and closer to zero and will be positive since we are on the right side.

This means that we'll have a numerator that is getting closer and closer to a non-zero and positive constant divided by an increasingly smaller positive number and so the result should be an increasingly larger positive number. The right-hand limit should then be positive infinity.

For the left-hand limit we'll have,

$$x < 3 \qquad \Rightarrow \qquad x - 3 < 0$$

As with the right-hand limit we'll have the following behaviors for the numerator and the denominator,

$$x - 3 \to 0 \quad \text{and} \quad 2x \to 6 \quad \text{as} \quad x \to 3$$

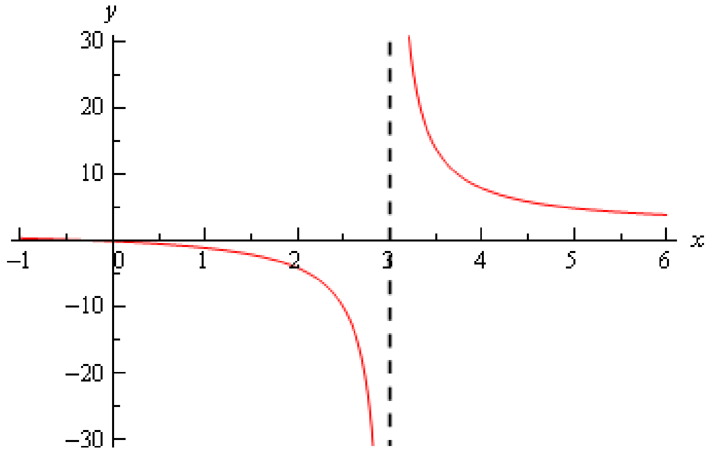
The main difference in this case is that the denominator will now be negative. So, we'll have a numerator that is approaching a positive, non-zero constant divided by an increasingly small negative number. The result will be an increasingly large and negative number.

The formal answers for this example are then,

$$\lim_{x \to 3^+} \frac{2x}{x-3} = \infty \qquad \lim_{x \to 3^-} \frac{2x}{x-3} = -\infty \qquad \lim_{x \to 3} \frac{2x}{x-3} \quad \text{doesn't exist}$$

As with most of the examples in this section the normal limit does not exist since the two one-sided limits are not the same.

Here's a quick graph to verify our limits.



So far all we've done is look at limits of rational expressions, let's do a couple of quick examples with some different functions.

Limit examples

Example 1

3e

Evaluate

$$\lim_{x \to 4} \frac{x^2}{x^2 - 4}$$

If we try direct substitution, we end up with "$\frac{16}{0}$" (i.e., a non-zero constant over zero), so we'll get either $+\infty$ or $-\infty$ as we approach 4. We then need to check left- and right-hand limits to see which one it is, and to make sure the limits are equal from both sides.

- Left-hand limit:

$$\lim_{x \to 4^-} \frac{x^2}{(x-4)(x+4)}$$

  As $x \to 4^-$, the function is negative since $\frac{(+)^2}{(-)(+)} = (-)$, so the left-hand limit is $-\infty$.

- Right-hand limit:

$$\lim_{x \to 4^+} \frac{x^2}{(x-4)(x+4)}$$

  As $x \to 4^+$, the function is positive since $\frac{(+)^2}{(+)(+)} = (+)$, so the right-hand limit is $+\infty$.

Since the left- and right-hand limits are not equal,

$$\lim_{x \to 4} \frac{x^2}{x^2 - 4} \text{ DNE}$$

Example 2

3i

Evaluate

$$\lim_{x \to -3} \frac{x^2 - 2x - 3}{x^2 + 6x + 9}$$

If we try direct substitution, we end up with "$\frac{12}{0}$", so we'll get either $+\infty$ or $-\infty$ as we approach -3. As in the last example, we need to check left- and right-hand limits to see which one it is, and to make sure the limits are equal from both sides.

- Left-hand limit:

$$\lim_{x\to -3^-} \frac{(x-3)(x+1)}{(x+3)^2}$$

As $x \to -3^-$, the function is positive since $\frac{(-)(-)}{(-)^2} = \frac{(+)}{(+)} = (+)$, so the left-hand limit is $+\infty$.

- Right-hand limit:

$$\lim_{x\to -3^+} \frac{(x-3)(x+1)}{(x+3)^2}$$

As $x \to -3^+$, the function is positive since $\frac{(-)(-)}{(+)^2} = \frac{(+)}{(+)} = (+)$, so the right-hand limit is also $+\infty$.

Since the left- and right-hand limits are the same,

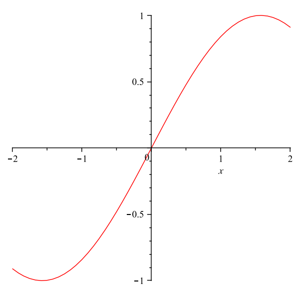
$$\lim_{x\to 4} \frac{x^2-2x-3}{x^2+6x+9} = \infty$$

Example 3

Evaluate

$$\lim_{x\to 0^+} \frac{2}{\sin(x)}$$

First of all, we note that direct substitution fails (we get "$\frac{2}{0}$"). There are a couple of different ways we can look at this problem. For either one, we observe that as $x \to 0^+$, $\sin(x)$ also goes to zero from values greater than zero (i.e., $\sin(x) \to 0^+$): So, $\displaystyle\lim_{x\to 0^+} \frac{2}{\sin(x)}$ is either $+\infty$



or $-\infty$. From what we observed above, we know the function will be $\frac{(+)}{(+)} = (+)$, so the limit is $+\infty$.

The other way we can approach this is to replace $\sin(x)$ with another variable that goes to the same value as $\sin(x)$ when we take the limit. Since $\sin(x) \to 0^+$ as $x \to 0^+$, then

$$\lim_{x\to 0^+} \frac{2}{\sin(x)} = \lim_{t\to 0^+} \frac{2}{t} \quad (\text{ which still} = \infty).$$

To show this one formally, we first note that as $x \to \infty$, then

$$x^2 \to \infty$$

as well, so

$$-x^2 \to -\infty$$

and

$$3 - x^2 \to -\infty$$

also. So, we can replace the "$3 - x^2$" in the exponent with another variable (say, t) that goes to $-\infty$ without changing the limit, i.e.,

$$\lim_{x \to \infty} e^{3-x^2} = \lim_{t \to -\infty} e^t \quad (= 0 \text{ by properties mentioned in class}).$$

Example 7

Evaluate

$$\lim_{x \to -\infty} \frac{1}{e^x}$$

In this example, we first rewrite the limit as

$$\lim_{x \to -\infty} e^{-x},$$

which is $+\infty$ from properties mentioned in class.

Example 6 – *Evaluating One-Sided Limits*

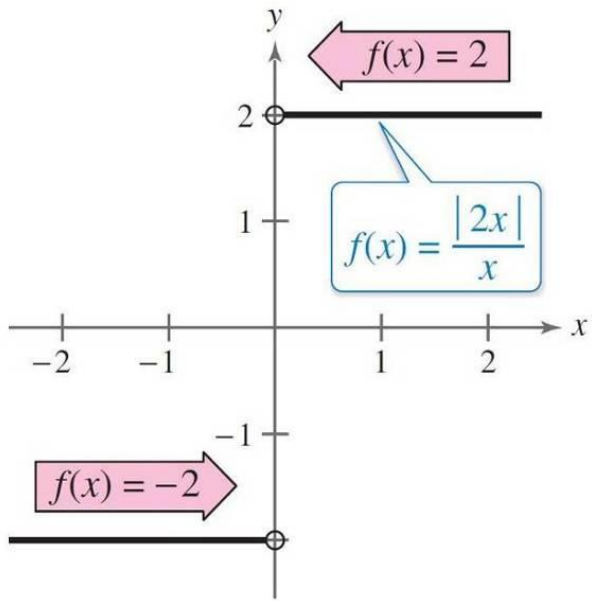
32

Find the limit as $x \to 0$ from the left and the limit as $x \to 0$ from the right for

$$f(x) = \frac{|2x|}{x}.$$

Solution:

From the graph of *f*, shown in the figure, you can see that $f(x) = -2$ for all $x < 0$.



Example 6 – *Solution*

cont'd

So, the limit from the left is

$$\lim_{x \to 0^-} \frac{|2x|}{x} = -2.$$

Limit from the left

Because $f(x) = 2$ for all $x > 0$, the limit from the right is

$$\lim_{x \to 0^+} \frac{|2x|}{x} = 2.$$

Limit from the right

11th lesson

https://www2.karlin.mff.cuni.cz/ kuncova/en/teachMat1.php
kunck6am@natur.cuni.cz

Exercises

1. Find limits:

(a) $\lim\limits_{x\to 1} \dfrac{x-1}{x^2+x-2}$

(b) $\lim\limits_{x\to -3} \dfrac{x^2+x-6}{x+3}$

(c) $\lim\limits_{x\to 2} \dfrac{x^2+3x-4}{x^2-4x+4}$

(d) $\lim\limits_{x\to 0} \dfrac{1}{\sin x}$

(e) $\lim\limits_{x\to -2} \dfrac{-4}{x+2}$

(f) $\lim\limits_{x\to 4} \dfrac{3}{(4-x)^3}$

(g) $\lim\limits_{x\to 3} \dfrac{2x}{x-3}$

(h) $\lim\limits_{x\to 4} \dfrac{x^2}{x^2-16}$

(i) $\lim\limits_{x\to -3} \dfrac{x^2-2x-3}{x^2+6x+9}$

(j) $\lim\limits_{x\to -\infty} \dfrac{1}{e^x}$

(k) $\lim\limits_{x\to 0} \dfrac{|2x|}{x}$

2. Find limits:

(a) $\lim\limits_{x\to -1} e^{x^2+3}$

Solution: Let's start with the inner function

$$\lim_{x\to -1} x^2+3 = 4.$$

Now the outer function

$$\lim_{y\to 4} e^y = e^4$$

Together we have

$$\lim_{x\to -1} e^{x^2+3} = e^4$$

(b) $\lim\limits_{x\to \frac{\sqrt{\pi}}{2}} \tan x^2$

Solution: Inner function:

$$\lim_{x\to \frac{\sqrt{\pi}}{2}} x^2 = \frac{\pi}{4}$$

Outer function:

$$\lim_{y\to \frac{\pi}{4}} \tan y = 1$$

Together

$$\lim_{x\to \frac{\sqrt{\pi}}{2}} \tan x^2 = 1$$

(c) $\lim_{x\to\infty} \ln \frac{x-1}{2+x}$
Solution: Inner function:

$$\lim_{x\to\infty} \frac{x-1}{2+x} = \lim_{x\to\infty} \frac{x}{x} \cdot \frac{1-\frac{1}{x}}{\frac{2}{x}+1} = \frac{1-0}{0+1} = 1.$$

Outer function:

$$\lim_{y\to 1} \ln y = 0$$

Together:

$$\lim_{x\to\infty} \ln \frac{x-1}{2+x} = 0$$

(d) $\lim_{x\to\infty} \cos \frac{1}{x}$
Solution: Inner function:

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

Outer function:

$$\lim_{y\to 0} \cos y = 1$$

Together:

$$\lim_{x\to\infty} \cos \frac{1}{x} = 1$$

(e) $\lim_{x\to -1} \arctan \frac{3x-2}{(x+1)^2}$
Solution: Inner function:

$$\lim_{x\to -1} \frac{3x-2}{(x+1)^2} = \frac{-5}{0+} = -\infty$$

Outer function:

$$\lim_{x\to -\infty} \arctan y = -\frac{\pi}{2}$$

Together:

$$\lim_{x\to -1} \arctan \frac{3x-2}{(x+1)^2} = -\frac{\pi}{2}$$

(f) $\lim_{x\to\infty} 2^{\sqrt{x^2+x}-\sqrt{x^2-x}}$
Solution: Inner function:

$$\lim_{x\to\infty} \sqrt{x^2+x} - \sqrt{x^2-x} = \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}} = \lim_{x\to\infty} \frac{x}{x} \cdot \frac{2}{\sqrt{1+\frac{1}{x}}+\sqrt{1-\frac{1}{x}}} = 1$$

Outer function:

$$\lim_{y\to 1} 2^y = 2^1 = 2$$

Together:

$$\lim_{x\to\infty} 2^{\sqrt{x^2+x}-\sqrt{x^2-x}} = 2$$

(g) $\lim_{x\to 2+} e^{1+\ln(x-2)}$
Solution: Inner function:

$$\lim_{x\to 2+} 1+\ln(x-2) = 1-\infty = -\infty$$

Outer function:

$$\lim_{y\to -\infty} e^y = 0$$

Together:

$$\lim_{x\to 2+} e^{1+\ln(x-2)} = 0$$

(h) $\lim_{x\to\infty} \cot\left(\frac{e^x-\cos x}{2e^x+\sin x}\right)$
Solution: Inner function:

$$\lim_{x\to\infty} \frac{e^x-\cos x}{2e^x+\sin x} = \lim_{x\to\infty} \frac{e^x}{e^x}\cdot\frac{1-\frac{\cos x}{e^x}}{2+\frac{\sin x}{e^x}} = \frac{1-0}{2+0} = \frac{1}{2}.$$

Outer function:

$$\lim_{y\to\frac{1}{2}} \cot y = \cot\frac{1}{2}$$

Together:

$$\lim_{x\to\infty} \cot\left(\frac{e^x-\cos x}{2e^x+\sin x}\right) = \cot\frac{1}{2}$$

(i) $\lim_{x\to 2} \operatorname{arccot} \frac{x^2-3x+2}{x^3-4x^2+4x}$
Solution: Inner function:

$$\lim_{x\to 2} \frac{x^2-3x+2}{x^3-4x^2+4x} = \lim_{x\to 2} \frac{(x-2)(x-1)}{x(x-2)(x-2)} = \lim_{x\to 2} \frac{(x-1)}{x(x-2)}$$

We need to find left and right limit:

$$\lim_{x\to 2+} \frac{(x-1)}{x(x-2)} = \frac{1}{2\cdot 0+} = \infty.$$

$$\lim_{x\to 2-} \frac{(x-1)}{x(x-2)} = \frac{1}{2\cdot 0-} = -\infty.$$

Outer function:

$$\lim_{x\to\infty} \operatorname{arccot}\ y = 0$$

$$\lim_{x\to-\infty} \operatorname{arccot}\ y = \pi$$

Together:

$$\lim_{x\to 2+} \operatorname{arccot} \frac{x^2-3x+2}{x^3-4x^2+4x} = 0$$

$$\lim_{x\to 2-} \operatorname{arccot} \frac{x^2-3x+2}{x^3-4x^2+4x} = \pi$$

and hence $\lim_{x\to 2} \operatorname{arccot} \frac{x^2-3x+2}{x^3-4x^2+4x}$ does not exists.

(j) $\lim\limits_{x\to\infty} \arcsin \ln \dfrac{e^{x+1}-2^x}{e^x+2^x}$

Solution: Inner function:

$$\lim_{x\to\infty} \frac{e^{x+1}-2^x}{e^x+2^x} \lim_{x\to\infty} \frac{e^x}{e^x} \cdot \frac{e^1-\left(\frac{2}{e}\right)^x}{1+\left(\frac{2}{e}\right)^x} = \frac{e+0}{1-0} = e- .$$

Middle function:

$$\lim_{y\to e-} \ln y = 1-$$

$$\lim_{z\to 1-} \arcsin z = \frac{\pi}{2}$$

Together:

$$\lim_{x\to\infty} \arcsin \ln \frac{e^{x+1}-2^x}{e^x+2^x} = \frac{\pi}{2}$$