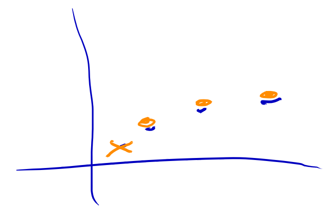
$a_1 = 0$ $\qquad \boxed{a_n = \frac{a_{n-1} + 3}{4}}$ $\qquad \boxed{L = 1}$

$a_2 = \frac{3}{4}$ $\qquad a_2 = \frac{a_1+3}{4} = \frac{0+3}{4}$

$a_3 = \frac{15}{16}$ $\qquad a_3 = \frac{a_2 + 3}{4} = \frac{\frac{3}{4} + 3}{4} = \frac{15}{16}$

Assumed $\exists$

$\lim a_n = L$ $\qquad \lim a_{n+1} = L$



$$\lim \frac{a_n+3}{4} = L$$

$$\frac{\lim (a_n + 3)}{\lim 4} = L$$

$$\frac{\lim a_n + \lim 3}{\lim 4} = L$$

$$\frac{\lim a_n + 3}{4} = L$$

$$\frac{L+3}{4} = L \qquad L + 3 = 4L$$

$$3 = 3L$$

$$\boxed{1 = L}$$

bounded

\+ monotone $\rightarrow$ has lim $\underline{\underline{\mathbb{R}}}$

Guess

$a_n \le 1$ $\qquad a_1 = 0 \le 1$ ✓

$a_n \le 1$ $\qquad a_{n+1} \overset{?}{=} \frac{a_n+3}{4} \le \frac{1+3}{4} = 1$ ✓

bounded :) $\qquad a_1 < 1$ ✓ $\quad a_2 < 1 \quad a_3 < 1 \ldots\ldots$

0 $\quad$ 3/4 $\quad$ 15/16

increasing $\qquad a_n \overset{?}{\le} a_{n+1}$ $\qquad a_1 \le a_2$ ✓

assume $\qquad a_n \le a_{n+1}$

want $\qquad \boxed{a_{n+1} \overset{?}{\le} a_{n+2}}$

$+3 \rightarrow a_n + 3 \le a_{n+1} + 3$

$:4 \rightarrow \frac{a_n+3}{4} \le \frac{a_{n+1}+3}{4}$

$\rightarrow \underline{a_{n+1} \le a_{n+2}}$ ✓

increasing

$1, 1, 2, 3, 5, 8, 13 \ldots$

Fibonacci

$a_1 \quad a_2$

$a_1 = 1 \quad a_2 = 1$

$a_3 = a_1 + a_2$

$a_4 = a_3 + a_4$

$a_5 = a_4 + a_3$

$a_n = a_{n-1} + a_{n-2}$