$$\lim_{n\to\infty} \frac{\sqrt[3]{n^3+6n} - \sqrt[3]{n^3+7}}{\underset{A}{\sqrt{n^2+4}} - \underset{B}{\sqrt{n^2+1}}} \qquad \frac{\sqrt[3]{\infty} - \sqrt[3]{\infty}}{\sqrt{\infty} - \sqrt{\infty}}$$

$$\frac{A+B}{A+B}$$

$$= \lim \frac{\sqrt[3]{n^3+6n} - \sqrt[3]{n^3+7}}{\sqrt{n^2+4} - \sqrt{n^2+1}} \cdot \frac{\sqrt{n^2+4} + \sqrt{n^2+1}}{\sqrt{n^2+4} + \sqrt{n^2+1}}$$

$$A \quad \infty - \infty \quad B$$

$$(A-B)(A+B) = A^2 - B^2$$

$$= \lim \frac{\sqrt[3]{n^3+6n} - \sqrt[3]{n^3+7}}{\underbrace{\cancel{n^2}+4 - \cancel{n^2} - 1}_{3}} \cdot \frac{\sqrt{n^2+4} + \sqrt{n^2+1}}{1}$$

$$A^3 - B^3 = (A-B)(A^2+AB+B^2)$$

$$= \lim_{n\to\infty} \frac{\overset{A}{\sqrt[3]{n^3+6n}} - \overset{B}{\sqrt[3]{n^3+7}}}{3} \cdot \left(\sqrt{n^2+4} + \sqrt{n^2+1}\right) \cdot$$

$$\cdot \frac{\left(\sqrt[3]{n^3+6n}\right)^2 + \sqrt[3]{n^3+6n}\,\sqrt[3]{n^3+7} + \left(\sqrt[3]{n^3+7}\right)^2}{\left(\sqrt[3]{n^3+6n}\right)^2 + \sqrt[3]{n^3+6n}\,\sqrt[3]{n^3+7} + \left(\sqrt[3]{n^3+7}\right)^2}$$

$A-$

$$\underset{A^2}{\left(\sqrt[3]{n^3+6n}\right)^2} \underset{+}{+} \underset{A}{\sqrt[3]{n^3+6n}}\,\underset{B}{\sqrt[3]{n^3+7}} \underset{+B^2}{+ \left(\sqrt[3]{n^3+7}\right)^2}$$

$$\lim \frac{\sqrt{n^2+4} + \sqrt{n^2+1}}{3} \cdot \frac{\cancel{n^3}+6n - (\cancel{n^3}+7)}{\left(\sqrt[3]{n^3+6n}\right)^2 + \sqrt[3]{n^3+6n}\,\sqrt[3]{n^3+7} + \left(\sqrt[3]{n^3+7}\right)^2}$$

$$= \lim_{n\to\infty} \frac{\overset{n^1}{6n-7}}{3} \cdot \frac{\overset{n^1}{\sqrt{n^2+4}} + \overset{n^1}{\sqrt{n^2+1}}}{\left(\sqrt[3]{n^3+6n}\right)^2 + \underbrace{\sqrt[3]{n^3+6n}}_{n}\,\underbrace{\sqrt[3]{n^3+7}}_{n} + \left(\sqrt[3]{n^3+7}\right)^2}$$

$$\sqrt{n^2} = n^1 \qquad \left(\sqrt[3]{n^3}\right)^2 = n^2 \qquad \sqrt[3]{n^3}\,\sqrt[3]{n^3} = n^2$$

$$\frac{\infty \cdot (\infty + \infty)}{\infty + \infty \cdot \infty + \infty} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{\cancel{n}\left(6 - \frac{7}{n}\right)}{3} \cdot \frac{\cancel{n}\left(\sqrt{1+\frac{4}{n^2}} + \sqrt{1+\frac{1}{n^2}}\right)}{\cancel{n^2}\left(\left(\sqrt[3]{1+\frac{6}{n^2}}\right)^2 + \sqrt[3]{1+\frac{6}{n^2}}\,\sqrt[3]{1+\frac{7}{n^3}} + \left(\sqrt[3]{1+\frac{7}{n^3}}\right)^2\right)}$$

$$\sqrt{n^2+4} = \sqrt{n^2\left(1+\frac{4}{n^2}\right)} = n\sqrt{1+\frac{4}{n^2}}$$

$$\left(\sqrt[3]{n^3+6n}\right)^2 = \left(\sqrt[3]{n^3\left(1+\frac{6}{n^2}\right)}\right)^2 = \left(\sqrt[3]{n^3}\,\sqrt[3]{1+\frac{6}{n^2}}\right)^2$$
$$= n^2\left(\sqrt[3]{1+\frac{6}{n^2}}\right)^2$$

$$\frac{6-0}{3} \cdot \frac{\sqrt{1+0} + \sqrt{1+0}}{\left(\sqrt[3]{1+0}\right)^2 + \sqrt[3]{1+0}\,\sqrt[3]{1+0} + \left(\sqrt[3]{1+0}\right)^2}$$

$$= \frac{6 \cdot 2}{3 \cdot (1+1+1)} = \frac{12}{9} = \frac{4}{3}$$