10th lesson

https://www2.karlin.mff.cuni.cz/ kuncova/en/teachMat1.php
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Theory

Theorem 1 (Squeeze theorem)**.** Let I be an interval having the point a as a limit point. Let g, f, and h be function defined on I, except possibly at a itself. Suppose that for every x in I not equal to a, we have

$$g(x) \leq f(x) \leq h(x)$$

and also suppose that

$$\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L.$$

Then

$$\lim_{x\to a} f(x) = L.$$

Facts

1. $\beta > 0$, $a > 1$: $\lim_{x\to+\infty} \frac{x^\beta}{a^x} = 0$.
2. $\alpha > 0$, $\beta > 0$: $\lim_{x\to+\infty} \frac{\ln^\alpha x}{x^\beta} = 0$.

Exercises

Set x

1. Find limits:

   (a) $\lim\limits_{x\to5} 10x + 7$

   (b) $\lim\limits_{x\to1} (3x-1)^{10}$

   (c) $\lim\limits_{x\to-1} \dfrac{3x-4}{8x^2+2x-2}$

   (d) $\lim\limits_{x\to\pi} \dfrac{\tan x}{x}$

   (e) $\lim\limits_{x\to\pi} x\cos x$

   (f) $\lim\limits_{x\to\infty} 4 - \dfrac{3}{x^2}$

   (g) $\lim\limits_{x\to3} \ln(2x+6)$

   (h) $\lim\limits_{x\to\infty} \sqrt{x} + \operatorname{arccot} x$

   (i) $\lim\limits_{x\to0+} \dfrac{-\sin x}{\ln x}$

   ∞

2. Find limits:

   (a) $\lim\limits_{x\to\infty} \dfrac{-2x+3}{3x^2+1}$

   (b) $\lim\limits_{x\to\infty} \dfrac{-2x^2+3}{3x^2+1}$

   (c) $\lim\limits_{x\to\infty} \dfrac{-2x^3+3}{3x^2+1}$

   (d) $\lim\limits_{x\to\infty} \dfrac{1}{x^2-x-1}$

   (e) $\lim\limits_{x\to\infty} \dfrac{x}{\sqrt{3x^2+2}}$

   (f) $\lim\limits_{x\to\infty} \dfrac{x-\cos x}{x}$

(g) $\lim_{x\to 0} x^2 \sin\frac{1}{x}$

(h) $\lim_{x\to\infty} \frac{2^x + 3^x}{2^{x+1} + 3^{x+1}}$

(i) $\lim_{x\to\infty} \frac{1^x + 2^x + 3^x + 4^x + 5^x}{5,0001^x}$

(j) $\lim_{x\to\infty} \frac{\ln x + x^3 + \frac{1}{x} + e^x + 5^x}{\ln_{10} x + x^4 + 5^x + x^3 + 4^x}$

(k) $\lim_{x\to\infty} \frac{\sin x}{x}$

(l) $\lim_{x\to\infty} e^{-x}\cos x$

(m) $\lim_{x\to\infty} \frac{x + \sin x}{x - \sin x}$

(n) $\lim_{x\to 0+} x\cos\left(\frac{x+3}{\sqrt{x}-1}\right)$

(o) $\lim_{x\to\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$

(p) $\lim_{x\to\infty} e^x \cos x$

(q) $\lim_{x\to\infty} \frac{x}{\sin x}$

0

3. Find limits:

(a) $\lim_{x\to 1} \frac{x-1}{x^2+x-2}$

(b) $\lim_{x\to -3} \frac{x^2+x-6}{x+3}$

(c) $\lim_{x\to 2} \frac{x^2+3x-4}{x^2-4x+4}$

(d) $\lim_{x\to 0} \frac{1}{\sin x}$

(e) $\lim_{x\to -2} \frac{-4}{x+2}$

(f) $\lim_{x\to 4} \frac{3}{(4-x)^3}$

(g) $\lim_{x\to 3} \frac{2x}{x-3}$

(h) $\lim_{x\to 4} \frac{x^2}{x^2-16}$

(i) $\lim_{x\to -3} \frac{x^2-2x-3}{x^2+6x+9}$

(j) $\lim_{x\to -\infty} \frac{1}{e^x}$

(k) $\lim_{x\to 0} \frac{|2x|}{x}$