9th lesson

https://www2.karlin.mff.cuni.cz/ kuncova/en/teachMat1.php
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Exercises

Graphs

1. Sketch a graph and find limits:

(a) $\lim_{x\to\infty} 42 = 42$

(b) $\lim_{x\to\infty} x^2 = \infty$

(c) $\lim_{x\to\infty} \frac{1}{x} = 0$

(d) $\lim_{x\to\infty} \sqrt{x} = \infty$

(e) $\lim_{x\to-\infty} \sqrt{x^3} \quad \nexists$

(f) $\lim_{x\to\infty} \ln x = \infty$

(g) $\lim_{x\to\infty} e^x = \infty$

(h) $\lim_{x\to-\infty} e^x = 0$

(i) $\lim_{x\to\infty} e^{-x} = 0$

(j) $\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$

(k) $\lim_{x\to\infty} \operatorname{arccot} x = 0$

(l) $\lim_{x\to\infty} \sin x \quad \nexists$

(m) $\lim_{x\to0+} \ln x = -\infty$

(n) $\lim_{x\to0+} \frac{1}{x} = \infty$

(o) $\lim_{x\to0-} \frac{1}{x} = -\infty$

(p) $\lim_{x\to0} \frac{1}{x^2} = \infty$

(q) $\lim_{x\to1-} \arcsin x = \frac{\pi}{2}$

(r) $\lim_{x\to\frac{\pi}{2}} \tan x \quad \nexists$

(s) $\lim_{x\to-2} \ln x \quad \nexists$

Set x

2. Find limits:

(a) $\lim_{x\to5} 10x + 7 = 57$

(b) $\lim_{x\to1} (3x-1)^{10} = 2^{10}$

(c) $\lim_{x\to-1} \frac{3x-4}{8x^2+2x-2} = \frac{-7}{4}$

(d) $\lim_{x\to\pi} \frac{\tan x}{x} = 0$

(e) $\lim_{x\to\pi} x\cos x = -\pi$

(f) $\lim_{x\to\infty} 4 - \frac{3}{x^2} = 4$

(g) $\lim_{x\to3} \ln(2x+6) = \ln(12)$

(h) $\lim_{x\to\infty} \sqrt{x} + \operatorname{arccot} x = \infty$

(i) $\lim_{x\to0+} \frac{-\sin x}{\ln x} = 0$

3. Find

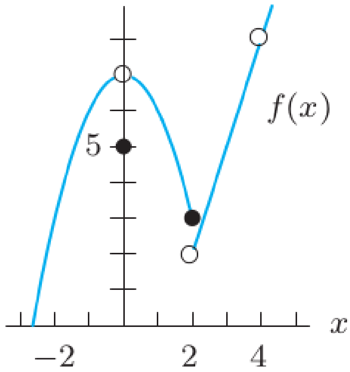
   (a) $\lim_{x\to 0} f(x) = 7$

   (b) $\lim_{x\to 2} f(x) \nexists$

   (c) $\lim_{x\to 2+} f(x) = 2$

   (d) $\lim_{x\to 2-} f(x) 3$

   (e) $\lim_{x\to 4} f(x) = 8$

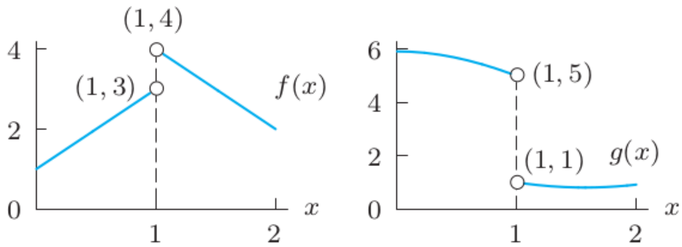


4. Find

   (a) $\lim_{x\to 1-} f(x) + g(x) = 8$

   (b) $\lim_{x\to 1+} f(x) + 2g(x) = 6$

   (c) $\lim_{x\to 1-} f(x)g(x) = 15$



5. Find a function with the following properties (just sketch a graph). Both conditions must be valid simultaneously.

   (a) $\lim_{x\to\infty} f(x) = -\infty$

   $\lim_{x\to -\infty} f(x) = -\infty$

   Solution: $-x^2$

   (b) $\lim_{x\to\infty} f(x) = 1$

   $\lim_{x\to -\infty} f(x) = \infty$

   Solution: $1 + e^{-x}$

   (c) $\lim_{x\to\infty} f(x) = 2$

   $\lim_{x\to -1} f(x) = \infty$

   Solution: $2 + \frac{1}{(x+1)^2}$

6. Find a function (just sketch a graph), which is continuous on $\mathbb{R} \setminus \{5\}$.

   Solution: $\frac{1}{x-5}$

7. Find a function (just sketch a graph), which is increasing, but not continuous on (whole) interval $[0, 5]$.

   Solution: $x + \lfloor x \rfloor$

8.

False Let f be function, sucht that f is continuous on $[0,10]$, $f(0)=0$, $f(10)=100$. Then f is nonnegative on $[0,10]$.

False Let $P(x)$ and $Q(x)$ be polynomials (hence they are continuous). Then $P(x)/Q(x)$ is also continuous.

9. For which x is this function continuous?

$$f(x)=\begin{cases}\sin x & x\in(-\infty,-1]\\ -x^2 & x\in(-1,0)\\ 1 & x=0\\ \sqrt{x} & x\in(0,4)\\ 6-x & x\in[4,\infty)\end{cases}$$

A $x=-1$
B $x=0$
C $x=2$
D $x=4$
E $x=\infty$

C, D

10. Find $k\in\mathbb{R}$, such that the following functions are continuous on $\mathbb{R}$.

(a)

$$f(x)=\begin{cases}kx, & x<1,\\ x+3, & 1\le x\end{cases}$$

$k=4$

(b)

$$f(x)=\begin{cases}k\cos x, & x<\pi,\\ 3\pi-x, & \pi\le x\end{cases}$$

$k=-2\pi$

(c)

$$f(x)=\begin{cases}x+k, & x<5,\\ kx, & 5\le x\end{cases}$$

$k=5/4$

11. Which of the following functions are continuous (consider $f(t)$, where t denotes time):

(a) Amount of gas in Your car tank on the way from Lisbon to Helsinki.
(b) The age of the oldest person in Czech Republic.
(c) Number of students visiting lectures during semester.

12. Bacteria population (in thousands) in Your Petri dish can be described with the following function (time t is in months):

$$P(t) = \begin{cases} e^{kt}, 0 \leq t \leq 12, \\ 100, t > 12. \end{cases}$$

(a) How many bacteria do You have at the beginning?
Solution: 1 000

(b) What can You tell about k?
$k = \frac{\ln 100}{12}$

(c) Can You describe the situation with Your own words? How is the population changing? Why?

Source for almost all today's excercises: Calculus: Single and Multivariable, Hughes-Hallet