6th lesson

https://www2.karlin.mff.cuni.cz/ kuncova/en/teachMat1.php
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Theory

Definition 1.

Theorem 2. Let $\{b_k\}$ be a subsequence of $\{a_n\}$. If $\lim_{n\to\infty} a_n = A \in \mathbb{R}$, then also $\lim_{k\to\infty} b_k = A$.

Facts

1. $\lim_{n\to+\infty} \frac{n!}{n^n} = 0$
2. $a > 1$: $\lim_{n\to+\infty} \frac{a^n}{n!} = 0$.
3. $\beta > 0$, $a > 1$: $\lim_{n\to+\infty} \frac{n^\beta}{a^n} = 0$.
4. $\alpha > 0$, $\beta > 0$: $\lim_{n\to+\infty} \frac{\ln^\alpha n}{n^\beta} = 0$.

Exercises

1. Find limits:

   (a) $\displaystyle\lim_{n\to\infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}}$

   (b) $\displaystyle\lim_{n\to+\infty} \frac{1^n + 2^n + 3^n + 4^n + 5^n}{5,0001^n}$

   (c) $\displaystyle\lim_{n\to\infty} \frac{3^n + n^5 + (n+1)!}{n(n^6 + n!)}$

   (d) $\displaystyle\lim_{n\to\infty} \frac{\ln n + n^3 + \frac{1}{n} + e^n + 5^n}{\ln_{10} n + n^4 + 5^n + n^3 + 4^n}$

   (e) $\displaystyle\lim_{n\to\infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$

2. Find infimum, minimum, maximum and supremum in $\mathbb{R}$:

   (a) $\mathbb{N}$

   (b) $(0;2]$

   (c) $(0;1) \cap \mathbb{Q}$

   (d) $\{x \in \mathbb{Z}; x \geq -\sqrt{6}\}$

   (e) $\{(-1)^n\sqrt{n}; n \in \mathbb{N}\}$

   (f) $\{\arctan x; x \in \mathbb{R}\}$

   (g) $\left\{1 - \dfrac{1}{n}; n \in \mathbb{N}\right\}$

   (h) $\left\{\dfrac{1 + (-1)^n}{2}; n \in \mathbb{N}\right\}$

   (i) $\left\{\cos \dfrac{n\pi}{2}; n \in \mathbb{N}\right\}$

   (j) $\{(-1)^n n; n \in \mathbb{N}\}$

3. Find lim sup and lim inf of the following sequences. Can we find also limits?

   (a) $x_n = 1 - \dfrac{1}{n}$

   (b) $x_n = (-1)^{n-1}\left(2 + \dfrac{3}{n}\right)$

   (c) $x_n = \dfrac{(-1)^n}{n} + \dfrac{1 + (-1)^n}{2}$

   (d) $x_n = 1 + \dfrac{n}{n+1}\cos\dfrac{n\pi}{2}$

   (e) $x_n = 1 + 2(-1)^{n+1} + 3 \cdot (-1)^{\frac{n(n-1)}{2}}$

   (f) $x_n = (-1)^n n$

   (g) $x_n = -n[2 + (-1)^n]$