5th lesson

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Theory

Facts

1. $\lim_{n\to+\infty} \frac{n!}{n^n} = 0$
2. $a > 1$: $\lim_{n\to+\infty} \frac{a^n}{n!} = 0$.
3. $\beta > 0$, $a > 1$: $\lim_{n\to+\infty} \frac{n^\beta}{a^n} = 0$.
4. $\alpha > 0$, $\beta > 0$: $\lim_{n\to+\infty} \frac{\ln^\alpha n}{n^\beta} = 0$.

Hints

$A^2 - B^2 = (A-B)(A+B)$
$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$
$A^n - B^n = (A-B)(A^{n-1} + A^{n-2}B + A^{n-3}B^2 + \ldots + A^2B^{n-3} + AB^{n-2} + B^{n-1})$

$$(A+B)^n = A^n + \binom{n}{1} A^{n-1}B \binom{n}{2} A^{m-2}B^2 + \cdots + B^n$$

Exercises

1. Find limits:

   (a) $\lim\limits_{n\to\infty} \sqrt{n+2} + \sqrt{n}$

   (b) $\lim\limits_{n\to\infty} \sqrt{n+2} - \sqrt{n}$

   (c) $\lim\limits_{n\to\infty} \frac{\sqrt{n^2+1}}{n}$

   (d) $\lim\limits_{n\to\infty} \frac{\sqrt{n-1} - \sqrt{n}}{\sqrt{n^2-3} - \sqrt{(n+2)^2}}$

2. Find limits:

   (a) $\lim\limits_{n\to\infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}}$

   (b) $\lim\limits_{n\to+\infty} \frac{1^n + 2^n + 3^n + 4^n + 5^n}{5,0001^n}$

   (c) $\lim\limits_{n\to\infty} \frac{3^n + n^5 + (n+1)!}{n(n^6 + n!)}$

   (d) $\lim\limits_{n\to\infty} \frac{\ln n + n^3 + \frac{1}{n} + e^n + 5^n}{\ln_{10} n + n^4 + 5^n + n^3 + 4^n}$

   (e) $\lim\limits_{n\to\infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$

Bonus

3. True or false? (Divergent can mean both tending to infinity or oscillating.)

   (a) Let x_n be convergent sequence and y_n be divergent sequence. Decide if sequences a) $x_n + y_n$, b) $x_n y_n$ are divergent.

(b) Let x_n a y_n be divergent sequences. Decide if sequences a) $x_n + y_n$, b) $x_n y_n$ are divergent.

(c) Let $\lim x_n = 0$ a y_n be an arbitrary sequence. Decide if $\lim(x_n y_n) = 0$?

(d) Let $\lim(x_n y_n) = 0$. Decide if either $\lim x_n = 0$ or $\lim y_n = 0$?