4th lesson

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Theory

Conditions	**Defined**	**Not well defined**
$\forall a \in \{-\infty\} \cup \mathbb{R}$	$-\infty + a = a + (-\infty) = -\infty$	$\infty - \infty$
$\forall a \in \{\infty\} \cup \mathbb{R}$	$\infty + a = a + \infty = \infty$	$\frac{0}{0}$
	$-(\infty) = -\infty \; -(-\infty) = \infty$	$\frac{\infty}{\infty}$
$\forall a \in (0, \infty) \cup \{\infty\}$	$a \cdot \infty = \infty \cdot a = \infty$	$0 \cdot \infty$
$\forall a \in (0, \infty) \cup \{\infty\}$	$a \cdot (-\infty) = -\infty \cdot a = -\infty$	0^0
$\forall a \in (-\infty, 0) \cup \{-\infty\}$	$a \cdot \infty = \infty \cdot a = -\infty$	1^∞
$\forall a \in (-\infty, 0) \cup \{-\infty\}$	$a \cdot (-\infty) = (-\infty) \cdot a = \infty$	∞^0
	$1/\infty = 0$, $1/(-\infty) = 0$	$\frac{1}{0}$

Theorem 1 (Arithemics of limits)**.** Let $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ be sequences (of real numbers). Further let $\lim_{n\to\infty} a_n = A \in \mathbb{R}^*$ and $\lim_{n\to\infty} b_n = B \in \mathbb{R}^*$. Then

(a) $\lim_{n\to\infty} (a_n + b_n) = A + B$,

(b) $\lim_{n\to\infty} (a_n \cdot b_n) = A \cdot B$,

(c) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}$,

if the right sides are well defined.

Hinty

$$A^2 - B^2 = (A - B)(A + B)$$
$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$
$$A^n - B^n = (A - B)(A^{n-1} + A^{n-2}B + A^{n-3}B^2 + \ldots + A^2B^{n-3} + AB^{n-2} + B^{n-1})$$
$$(A + B)^n = A^n + \binom{n}{1}A^{n-1}B\binom{n}{2}A^{m-2}B^2 + \cdots + B^n$$

Exercises

1. Find limits:

   (a) $(-1)^n$ (b) $(-1)^n n$ (c) $(-1)^n \frac{1}{n}$ (d) $\cos(\pi n)\sqrt{n}$

2. Find limits:

   (a) $\lim_{n\to\infty} -n^8 + 2n^3 - 4$

   (b) $\lim_{n\to\infty} \frac{2n^5 + 2n - 7}{n^5 - 6n^2 + 4}$

   (c) $\lim_{n\to\infty} \frac{5n^2 + n - 5}{n^3 + 8}$

   (d) $\lim_{n\to\infty} \frac{\sqrt[3]{n^2}}{n+1}$

3. Find limits:

   (a) $\lim_{n\to\infty} \sqrt{n+2} + \sqrt{n}$

   (b) $\lim_{n\to\infty} \sqrt{n+2} - \sqrt{n}$

   (c) $\lim_{n\to\infty} \frac{\sqrt{n^2+1}}{n}$

   (d) $\lim_{n\to\infty} \frac{\sqrt{n-1} - \sqrt{n}}{\sqrt{n^2-3} - \sqrt{(n+2)^2}}$

Bonus

4. Find limits:

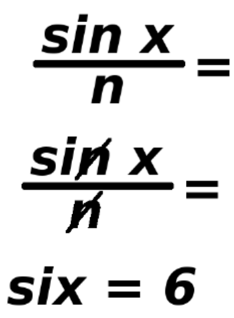
   (a) $\lim_{n\to\infty} \frac{(n+4)^{100} - (n+3)^{100}}{(n+2)^{100} - n^{100}}$

   (b) $\lim_{n\to\infty} \sqrt[3]{n+1} - \sqrt[3]{n}$

   (c) $\lim_{n\to\infty} \sqrt{n}(\sqrt{n+1} - \sqrt{n})$

   (d) $\lim_{n\to\infty} \sqrt[3]{n^3+1} - \sqrt{n^2+1}$

   (e) $\lim_{n\to\infty} \frac{\sqrt[3]{n^2+7} - \sqrt[3]{n^2+1}}{\sqrt[3]{n^2+6} - \sqrt[3]{n^2}}$



Zdroj 1: http://laughtingjoke.blogspot.com/2010/04/sin-x.html