3rd lesson

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Theory

Conditions	**Defined**	**Not well defined**
$\forall a \in \{-\infty\} \cup \mathbb{R}$	$-\infty + a = a + (-\infty) = -\infty$	$\infty - \infty$
$\forall a \in \{\infty\} \cup \mathbb{R}$	$\infty + a = a + \infty = \infty$	$\frac{0}{0}$
	$-(\infty) = -\infty$ $-(-\infty) = \infty$	$\frac{\infty}{\infty}$
$\forall a \in (0, \infty) \cup \{\infty\}$	$a \cdot \infty = \infty \cdot a = \infty$	$0 \cdot \infty$
$\forall a \in (0, \infty) \cup \{\infty\}$	$a \cdot (-\infty) = -\infty \cdot a = -\infty$	0^0
$\forall a \in (-\infty, 0) \cup \{-\infty\}$	$a \cdot \infty = \infty \cdot a = -\infty$	1^∞
$\forall a \in (-\infty, 0) \cup \{-\infty\}$	$a \cdot (-\infty) = (-\infty) \cdot a = \infty$	∞^0
	$1/\infty = 0$, $1/(-\infty) = 0$	$\frac{1}{0}$

Theorem 1 (Arithemics of limits)**.** Let $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$ be sequences (of real numbers). Further let $\lim_{n\to\infty} a_n = A \in \mathbb{R}^*$ and $\lim_{n\to\infty} b_n = B \in \mathbb{R}^*$. Then

(a) $\lim_{n\to\infty} (a_n + b_n) = A + B$,

(b) $\lim_{n\to\infty} (a_n \cdot b_n) = A \cdot B$,

(c) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}$,

if the right sides are well defined.

Exercises

1. Write down a few first terms of the sequences, sketch the graph, find the limit.

(a) $\lim_{n\to\infty} n$
(b) $\lim_{n\to\infty} n^2$
(c) $\lim_{n\to\infty} \sqrt{n}$
(d) $\lim_{n\to\infty} \frac{1}{n}$
(e) $\lim_{n\to\infty} \frac{1}{n^2}$
(f) $\lim_{n\to\infty} \frac{1}{\sqrt{n}}$
(g) $\lim_{n\to\infty} e^n$
(h) $\lim_{n\to\infty} e^{-n}$
(i) $\lim_{n\to\infty} \ln n$
(j) $\lim_{n\to\infty} \left(\frac{1}{2}\right)^n$
(k) $\lim_{n\to\infty} 2^n$
(l) $\lim_{n\to\infty} n!$

2. Find limits:

(a) $(-1)^n$
(b) $(-1)^n n$
(c) $(-1)^n \frac{1}{n}$
(d) $\cos(\pi n)\sqrt{n}$

3. Find limits:

(a) $\lim_{n\to\infty} -n^8 + 2n^3 - 4$
(b) $\lim_{n\to\infty} \frac{2n^5 + 2n - 7}{n^5 - 6n^2 + 4}$
(c) $\lim_{n\to\infty} \frac{5n^2 + n - 5}{n^3 + 8}$
(d) $\lim_{n\to\infty} \frac{\sqrt[3]{n^2}}{n + 1}$