2nd lesson

https://www2.karlin.mff.cuni.cz/∼kuncova/en/teachMat1.php
kunck6am@natur.cuni.cz

Theory

Definice 1. A *statement* (or proposition) is a sentence which can be declared to be either true or false. (But not both simultaneously.)

Exercises

1. Which sentences are statements?

YES–NO It is raining (right now).

YES–NO Let the sunshine in!

YES–NO We have fish and chips.

YES–NO For every natural number there exists a bigger prime number.

YES–NO $\forall n \in \mathbb{N}\ \exists p : p > n$ and p is prime.

YES–NO Today is Friday or October.

YES–NO What's your favourite animal?

YES–NO Some mammals lay eggs.

YES–NO There exists a mammal, which lays eggs.

YES–NO This sentence is false.

YES–NO $\pi + e$ is irrational number.

2. Negate the following statements:
   (a) All classromm have at least one chair that is broken.
   (b) No classroom has only chairs that are not broken.
   (c) Every student in this class loves dogs or cats.
   (d) Every student in this class loves dogs and cats.
   (e) If a student loves cats, than s/he loves dogs.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Function f is strictly increasing if $f(x) < f(y)$ whenever $x < y$.
   (a) Express the statement using quantifiers.
   (b) Negate the statement.
   (c) Function f is nonincreasing if $f(x) \geq f(y)$ whenever $x < y$. Explain the difference between function, which is not increasing and function, which is nonincreasing. Give examples of such functions.

4. Complete the truth table:

A	B	$\neg A$	$\neg B$	$A \vee B$	$A \wedge B$	$A \implies B$	$A \iff B$
1	1						
1	0						
0	1						
0	0						

5. Let A, B, C be statements. Prove by truth table that following are tautologies:

   (a) $\neg(A \implies B) \Leftrightarrow (A \wedge \neg B)$

   (b) $(A \implies B) \Leftrightarrow (\neg A \vee B)$

   (c) $((A \implies C) \wedge (C \implies B)) \implies (A \implies B)$

6. Let A and B be sets. Use the Venn diagram to show that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

7. Let U be the set of all students of the Charles University. Further, let B be all tudents visiting a Business course, E students visiting an English course and M students visiting a Math course.

   Express by formula and by Venn diagram a set of students taking

   (a) at least one of these courses;

   (b) both Math and English, but not a Business course;

   (c) exactly one course.

8. Let A, B and X be sets. Prove de Morgan's laws:

   (a) $(A \cup B)^c = A^c \cap B^c$,

   (b) $(A \cap B)^c = A^c \cup B^c$.