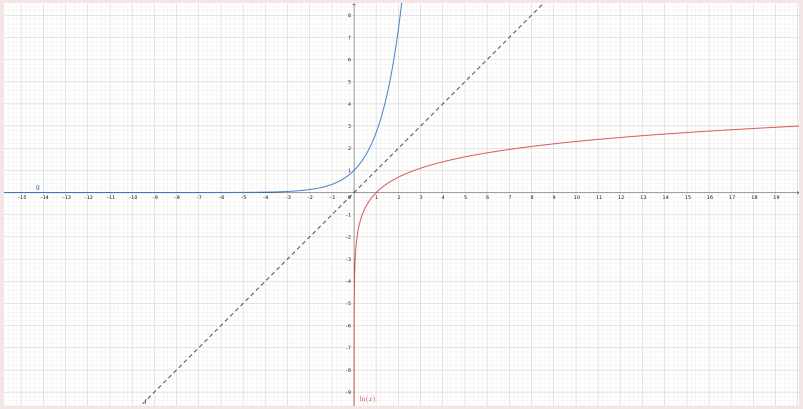
Inverse trigonometric functions

Kristýna Kuncová

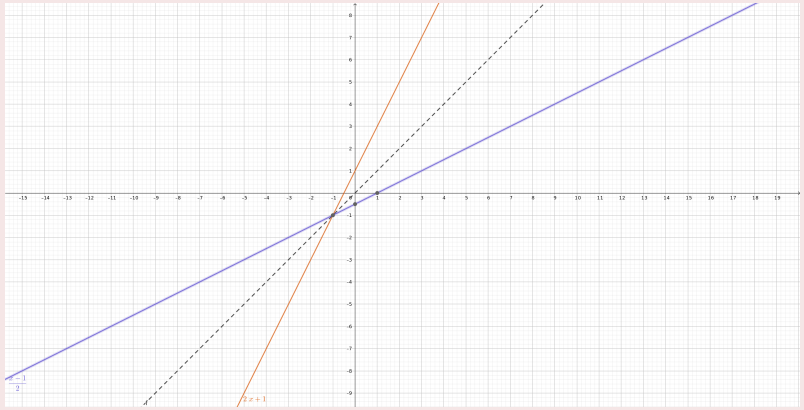
Inverse functions

Example

e^x vs $\ln x$

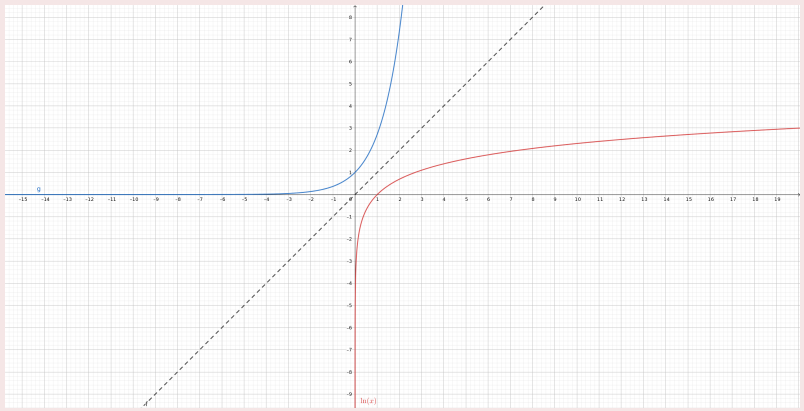


$2x + 1$ vs $\frac{x-1}{2}$

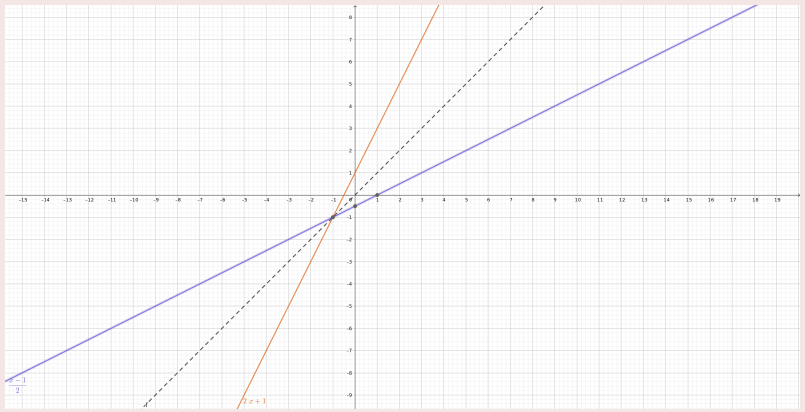


Example

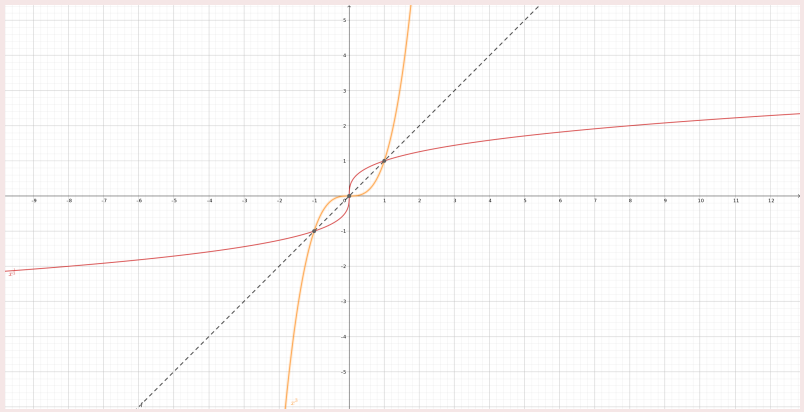
e^x vs $\ln x$



$2x + 1$ vs $\frac{x-1}{2}$



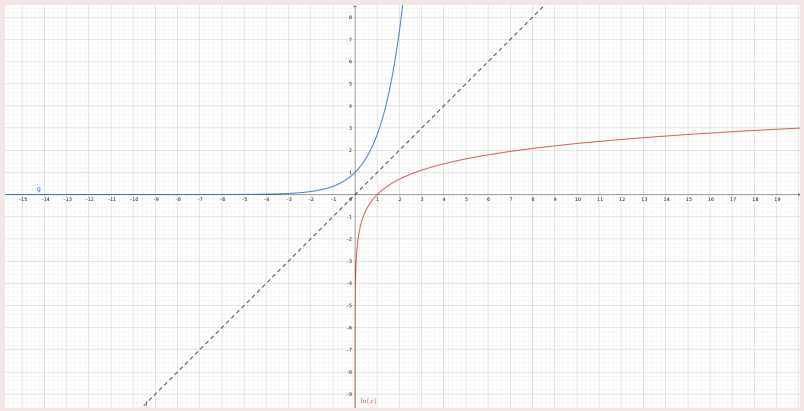
x^3 vs $\sqrt[3]{x}$



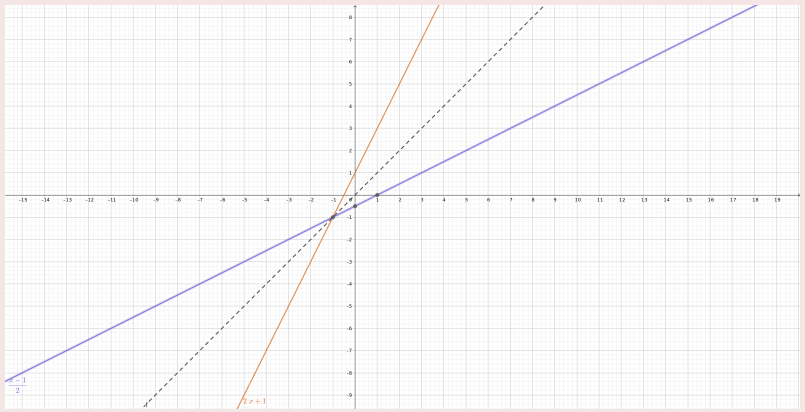
Inverse functions

Example

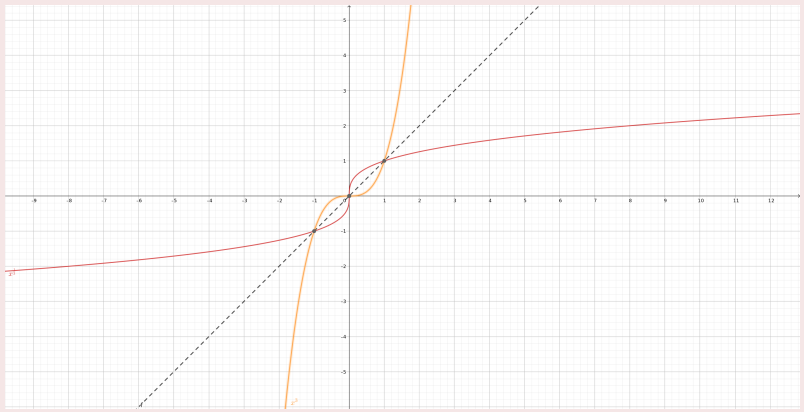
e^x vs $\ln x$



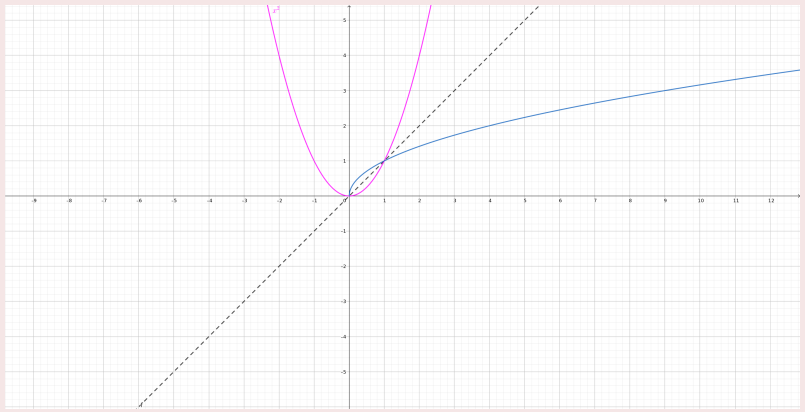
$2x + 1$ vs $\frac{x-1}{2}$

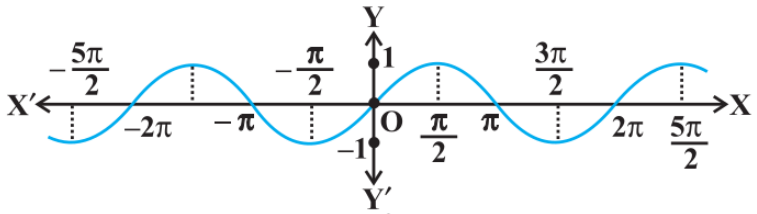


x^3 vs $\sqrt[3]{x}$



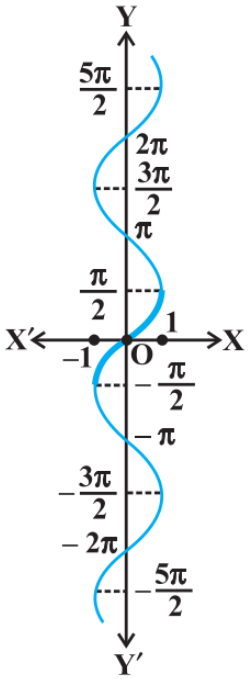
x^2 vs $\sqrt{x}$





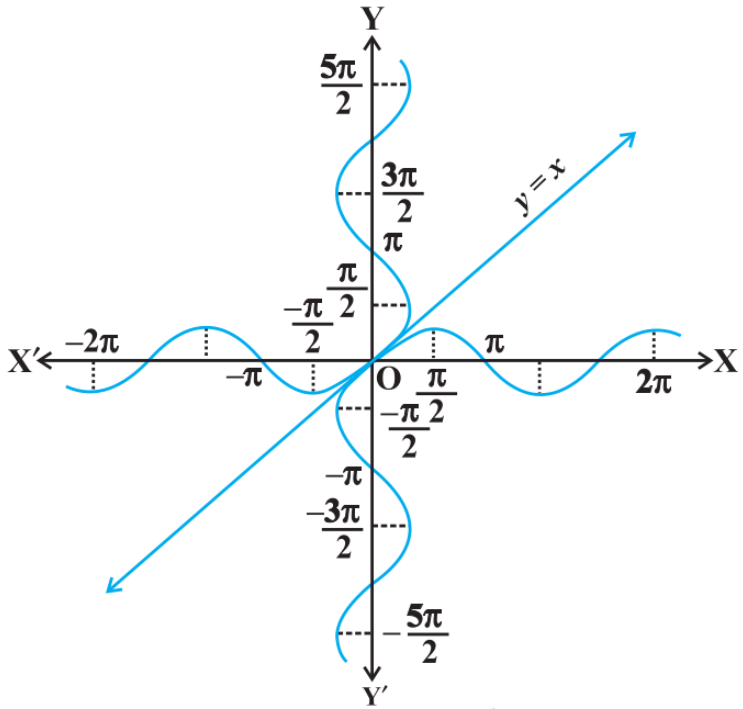
$y = \sin x$

Fig 2.1 (i)



$y = \sin^{-1} x$

Fig 2.1 (ii)

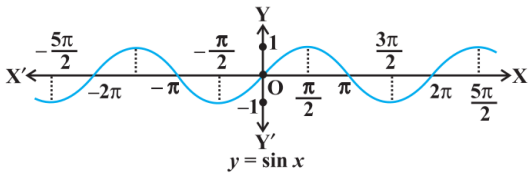


$y = \sin x$ and $y = \sin^{-1} x$

Fig 2.1 (iii)

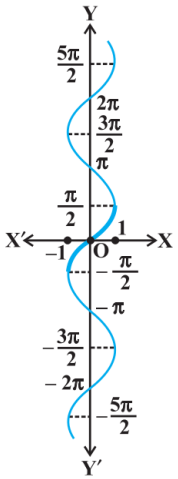
Caption: http://ncert.nic.in/ncerts/l/lemh102.pdf

Arcsin



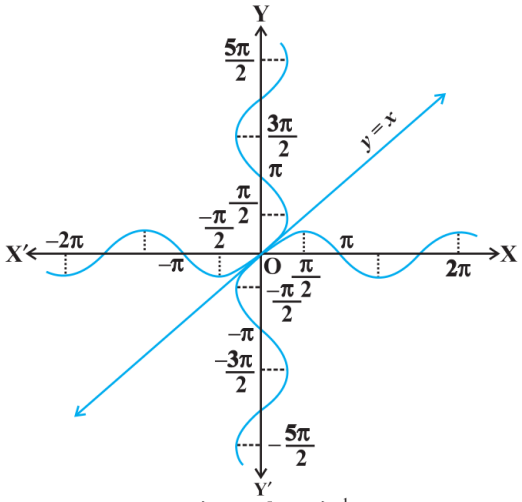
$y = \sin x$

Fig 2.1 (i)



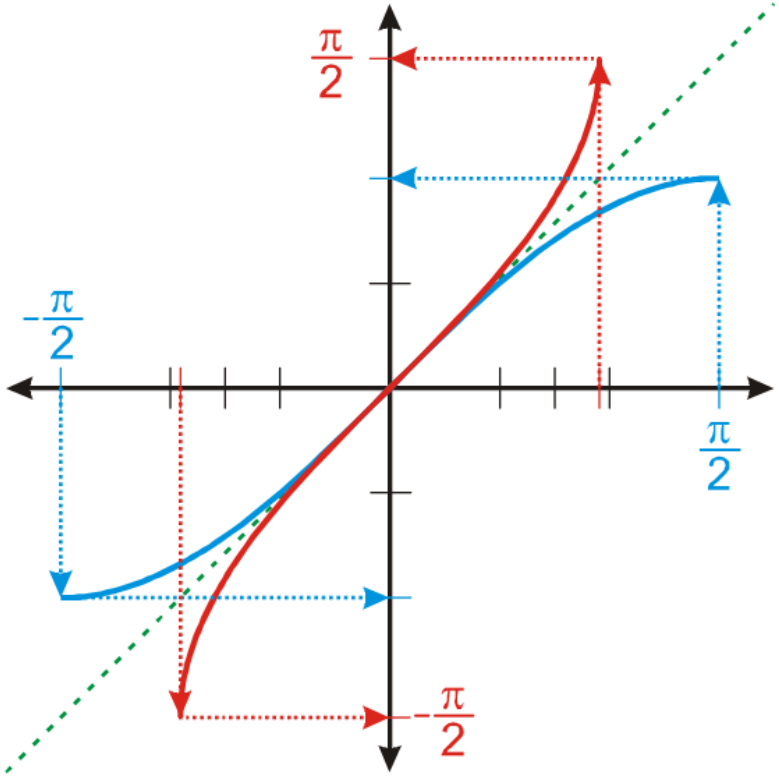
$y = \sin^{-1} x$

Fig 2.1 (ii)



$y = \sin x$ and $y = \sin^{-1} x$

Fig 2.1 (iii)



Caption: http://www.realisticky.cz/ucebnice/01%20Matematika%20S%C5%A0/04%20Goniometrie/02%20Goniometrick%C3%A9%20funkce/16%20Funkce%20arkus%20sinus.pdf

Arcsin - exercise

Question

Find $\arcsin \frac{1}{2}$. (Which angle α should we take to obtain $\sin \alpha = \frac{1}{2}$?)

A 0

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $\frac{\pi}{3}$

Arcsin - exercise

Question

Find $\arcsin \frac{1}{2}$. (Which angle α should we take to obtain $\sin \alpha = \frac{1}{2}$?)

A 0

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $\frac{\pi}{3}$

B

Arcsin - exercise

Question

Find arcsin $\frac{1}{2}$. (Which angle α should we take to obtain $\sin\alpha = \frac{1}{2}$?)

A 0

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $\frac{\pi}{3}$

B

Question

Find arcsin $-\frac{\sqrt{3}}{2}$?

A $\frac{\pi}{3}$

B $-\frac{\pi}{3}$

C $\frac{5\pi}{3}$

D $\frac{4\pi}{3}$

Arcsin - exercise

Question

Find arcsin $\frac{1}{2}$. (Which angle α should we take to obtain $\sin\alpha = \frac{1}{2}$?)

A 0

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $\frac{\pi}{3}$

B

Question

Find arcsin $-\frac{\sqrt{3}}{2}$?

A $\frac{\pi}{3}$

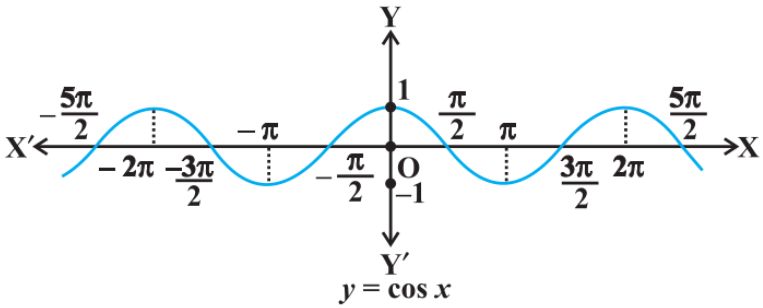
B $-\frac{\pi}{3}$

C $\frac{5\pi}{3}$

D $\frac{4\pi}{3}$

B

Arccos



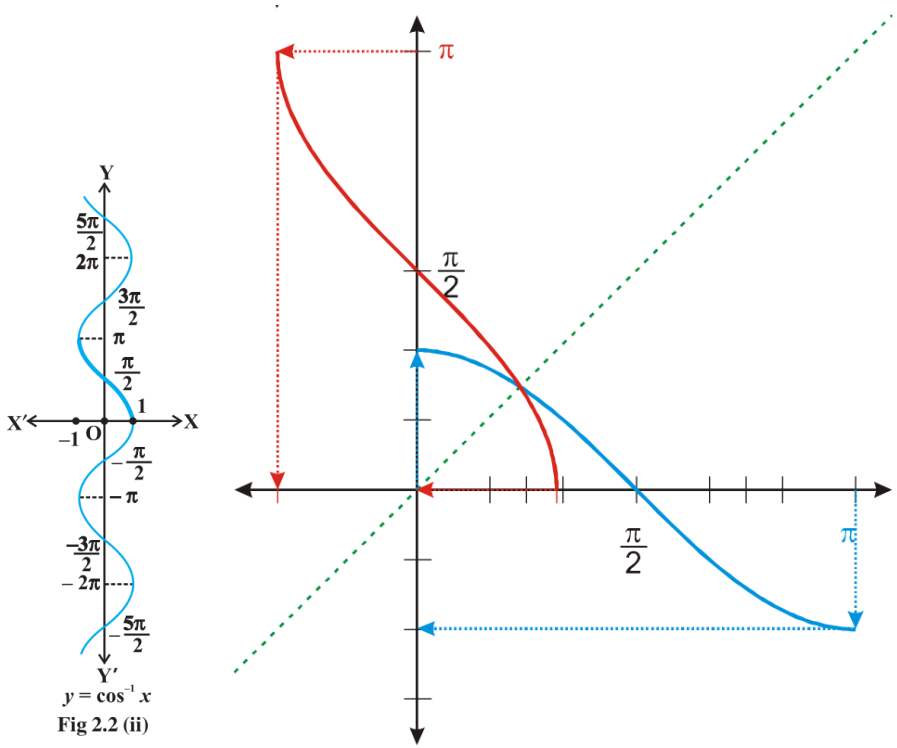


Fig 2.2 (ii)

Caption: http://www.realisticky.cz/ucebnice/01%20Matematika%20S%C5%A0/04%20Goniometrie/02%20Goniometrick%C3%A9%20funkce/17%20Cyklometrick%C3%A9%20funkce.pdf

Arccos - exercise

Question

Find arccos $\frac{\sqrt{2}}{2}$? (Which angle α should we take to obtain $\cos\alpha = \frac{\sqrt{2}}{2}$?)

A 1

B $\frac{\pi}{2}$

C $\frac{\pi}{4}$

D $\frac{3\pi}{4}$

Arccos - exercise

Question

Find arccos $\frac{\sqrt{2}}{2}$? (Which angle α should we take to obtain $\cos\alpha = \frac{\sqrt{2}}{2}$?)

A 1

B $\frac{\pi}{2}$

C $\frac{\pi}{4}$

D $\frac{3\pi}{4}$

C

Arccos - exercise

Question

Find arccos $\frac{\sqrt{2}}{2}$? (Which angle α should we take to obtain $\cos\alpha = \frac{\sqrt{2}}{2}$?)

A 1

B $\frac{\pi}{2}$

C $\frac{\pi}{4}$

D $\frac{3\pi}{4}$

C

Question

Find arccos $-\frac{\sqrt{2}}{2}$?

A $\frac{\pi}{4}$

B $-\frac{\pi}{4}$

C $\frac{5\pi}{4}$

D $\frac{3\pi}{4}$

Arccos - exercise

Question

Find arccos $\frac{\sqrt{2}}{2}$? (Which angle α should we take to obtain $\cos\alpha = \frac{\sqrt{2}}{2}$?)

A 1

B $\frac{\pi}{2}$

C $\frac{\pi}{4}$

D $\frac{3\pi}{4}$

C

Question

Find arccos $-\frac{\sqrt{2}}{2}$?

A $\frac{\pi}{4}$

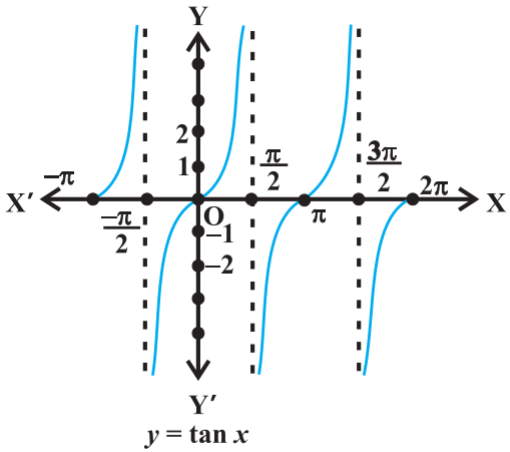
B $-\frac{\pi}{4}$

C $\frac{5\pi}{4}$

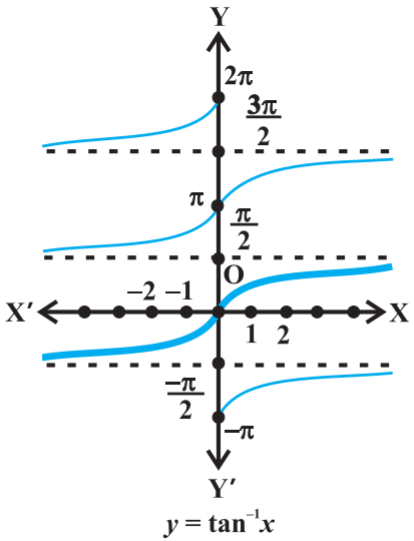
D $\frac{3\pi}{4}$

D

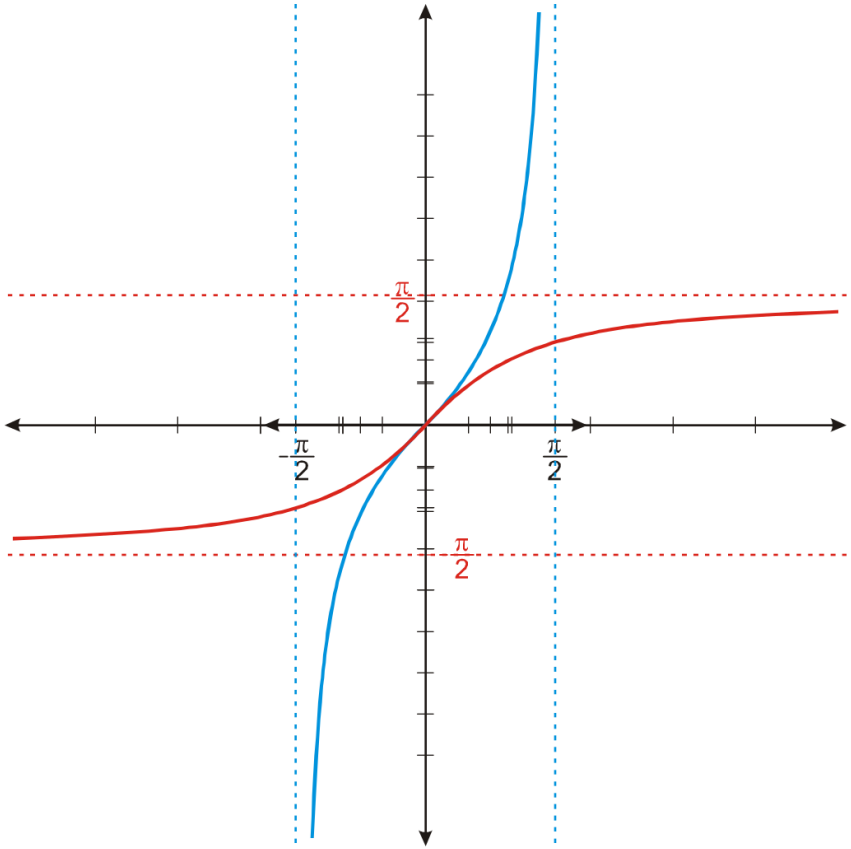
Arctan



$y = \tan x$



$y = \tan^{-1} x$



Arctan- exercise

Question

Find arctan 1? (Which angle α should we take to obtain $\tan \alpha = 1$?)

A 0

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $-\frac{\pi}{3}$

Arctan- exercise

Question

Find arctan 1? (Which angle α should we take to obtain $\tan \alpha = 1$?)

A 0

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $-\frac{\pi}{3}$

C

Arctan- exercise

Question

Find arctan 1? (Which angle α should we take to obtain $\tan \alpha = 1$?)

A 0

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $-\frac{\pi}{3}$

C

Question

Find arctan $-\sqrt{3}$?

A 0

B $-\frac{\pi}{3}$

C $\frac{\pi}{3}$

D $\frac{2\pi}{3}$

E $\frac{4\pi}{3}$

Arctan- exercise

Question

Find arctan 1? (Which angle α should we take to obtain $\tan \alpha = 1$?)

A 0

B $\frac{\pi}{6}$

C $\frac{\pi}{4}$

D $-\frac{\pi}{3}$

C

Question

Find arctan $-\sqrt{3}$?

A 0

B $-\frac{\pi}{3}$

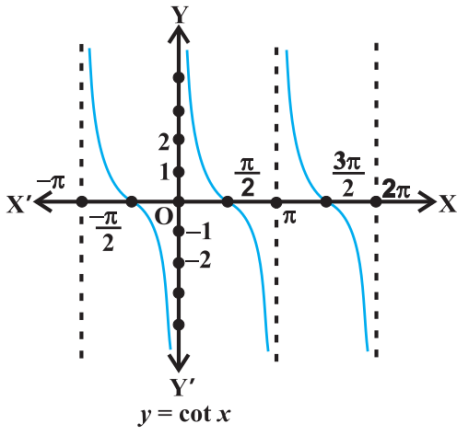
C $\frac{\pi}{3}$

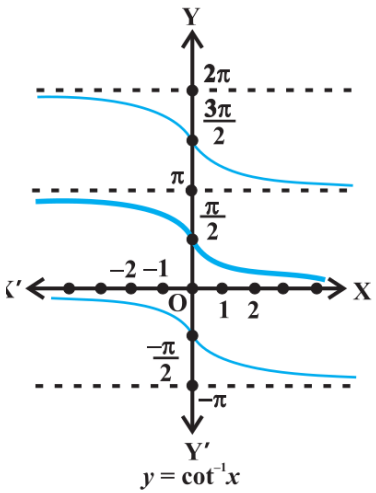
D $\frac{2\pi}{3}$

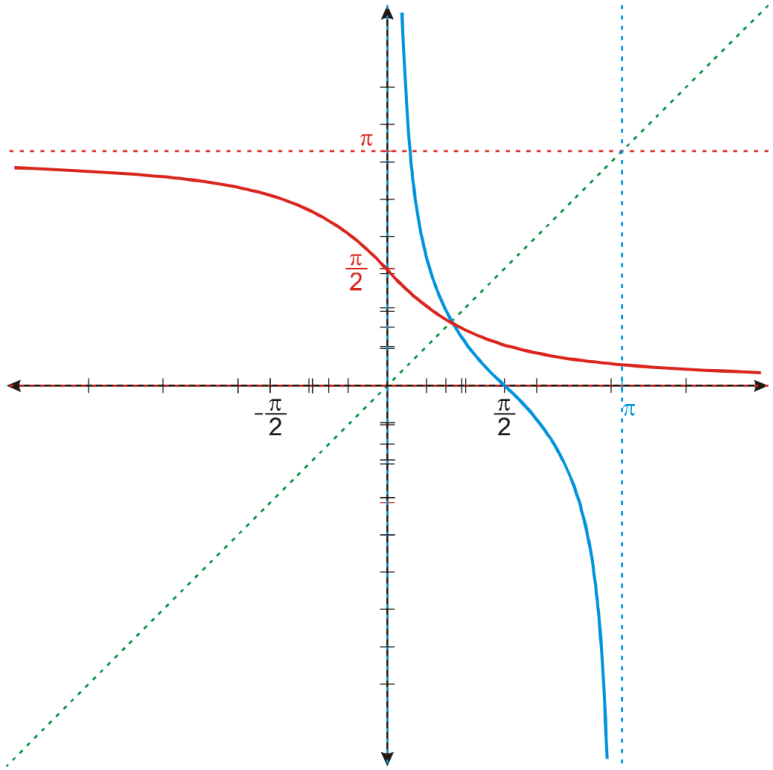
E $\frac{4\pi}{3}$

B

Arccot


Y
2
1
X′
−π
−π/2
O
−1
−2
π/2
π
3π/2
2π
X
Y′
y = cot x


Y
2π
3π/2
π
π/2
−2 −1
X′
O
1 2
X
−π/2
−π
Y′
y = cot⁻¹x


π
π/2
−π/2
π/2
π

Arccot - exercise

Question

Find arccot 0? (Which angle α should we take to obtain $\cot \alpha = 0$?)

A 0
B 1
C $-\frac{\pi}{2}$
D $\frac{\pi}{2}$
E does not exist

Arccot - exercise

Question

Find arccot 0? (Which angle α should we take to obtain $\cot \alpha = 0$?)

A 0
B 1
C $-\frac{\pi}{2}$
D $\frac{\pi}{2}$
E does not exist

D

Arccot - exercise

Question

Find arccot 0? (Which angle α should we take to obtain $\cot \alpha = 0$?)

A 0
B 1
C $-\frac{\pi}{2}$
D $\frac{\pi}{2}$
E does not exist

D

Question

Find arccot -1?

A $\frac{\pi}{4}$
B $-\frac{\pi}{4}$
C $-\frac{3\pi}{4}$
D $\frac{3\pi}{4}$

Arccot - exercise

Question

Find arccot 0? (Which angle α should we take to obtain $\cot \alpha = 0$?)

A 0
B 1
C $-\frac{\pi}{2}$
D $\frac{\pi}{2}$
E does not exist

D

Question

Find arccot -1?

A $\frac{\pi}{4}$
B $-\frac{\pi}{4}$
C $-\frac{3\pi}{4}$
D $\frac{3\pi}{4}$

D

Question

A $\mathbb{R}$

B $[-\frac{\pi}{2}; \frac{\pi}{2}]$

C $(-\frac{\pi}{2}; \frac{\pi}{2})$

D $[0; \pi]$

E $[-1; 1]$

Find the domain of arcsin x?

Domains

Question

A $\mathbb{R}$

B $[-\frac{\pi}{2}; \frac{\pi}{2}]$

C $(-\frac{\pi}{2}; \frac{\pi}{2})$

D $[0; \pi]$

E $[-1; 1]$

Find the domain of arcsin x? E

Domains

Question

A $\mathbb{R}$
B $[-\frac{\pi}{2}; \frac{\pi}{2}]$
C $(-\frac{\pi}{2}; \frac{\pi}{2})$
D $[0; \pi]$
E $[-1; 1]$

Find the domain of $\arcsin x$? E
Find the domain of $\arccos x$?

Domains

Question

A $\mathbb{R}$
B $[-\frac{\pi}{2}; \frac{\pi}{2}]$
C $(-\frac{\pi}{2}; \frac{\pi}{2})$
D $[0; \pi]$
E $[-1; 1]$

Find the domain of arcsin x? E
Find the domain of arccos x? E

Domains

Question

A $\mathbb{R}$
B $[-\frac{\pi}{2}; \frac{\pi}{2}]$
C $(-\frac{\pi}{2}; \frac{\pi}{2})$
D $[0; \pi]$
E $[-1; 1]$

Find the domain of arcsin x? E
Find the domain of arccos x? E
Find the domain of arctan x?

Domains

Question

A $\mathbb{R}$
B $[-\frac{\pi}{2}; \frac{\pi}{2}]$
C $(-\frac{\pi}{2}; \frac{\pi}{2})$
D $[0; \pi]$
E $[-1; 1]$

Find the domain of arcsin x? E
Find the domain of arccos x? E
Find the domain of arctan x? A

Domains

Question

A $\mathbb{R}$
B $[-\frac{\pi}{2}; \frac{\pi}{2}]$
C $(-\frac{\pi}{2}; \frac{\pi}{2})$
D $[0; \pi]$
E $[-1; 1]$

Find the domain of arcsin x? E
Find the domain of arccos x? E
Find the domain of arctan x? A
Find the domain of arccot x?

Domains

Question

A $\mathbb{R}$
B $[-\frac{\pi}{2}; \frac{\pi}{2}]$
C $(-\frac{\pi}{2}; \frac{\pi}{2})$
D $[0; \pi]$
E $[-1; 1]$

Find the domain of arcsin x? E
Find the domain of arccos x? E
Find the domain of arctan x? A
Find the domain of arccot x? A

Range

Question

A $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

B $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

C $[0; \pi]$

D $(0; \pi)$

E $[-1; 1]$

Find the range of arcsin x?

Range

Question

A $[-\frac{\pi}{2}; \frac{\pi}{2}]$

B $(-\frac{\pi}{2}; \frac{\pi}{2})$

C $[0; \pi]$

D $(0; \pi)$

E $[-1; 1]$

Find the range of arcsin x? A

Range

Question

A $[-\frac{\pi}{2}; \frac{\pi}{2}]$

B $(-\frac{\pi}{2}; \frac{\pi}{2})$

C $[0; \pi]$

D $(0; \pi)$

E $[-1; 1]$

Find the range of arcsin x? A

Find the range of arccos x?

Range

Question

A $[-\frac{\pi}{2}; \frac{\pi}{2}]$
B $(-\frac{\pi}{2}; \frac{\pi}{2})$
C $[0; \pi]$
D $(0; \pi)$
E $[-1; 1]$

Find the range of arcsin x? A
Find the range of arccos x? C

Range

Question

A $[-\frac{\pi}{2}; \frac{\pi}{2}]$
B $(-\frac{\pi}{2}; \frac{\pi}{2})$
C $[0; \pi]$
D $(0; \pi)$
E $[-1; 1]$

Find the range of arcsin x? A
Find the range of arccos x? C
Find the range of arctan x?

Range

Question

A $[-\frac{\pi}{2}; \frac{\pi}{2}]$
B $(-\frac{\pi}{2}; \frac{\pi}{2})$
C $[0; \pi]$
D $(0; \pi)$
E $[-1; 1]$

Find the range of arcsin x? A
Find the range of arccos x? C
Find the range of arctan x? B

Range

Question

A $[-\frac{\pi}{2}; \frac{\pi}{2}]$
B $(-\frac{\pi}{2}; \frac{\pi}{2})$
C $[0; \pi]$
D $(0; \pi)$
E $[-1; 1]$

Find the range of arcsin x? A
Find the range of arccos x? C
Find the range of arctan x? B
Find the range of arccot x?

Range

Question

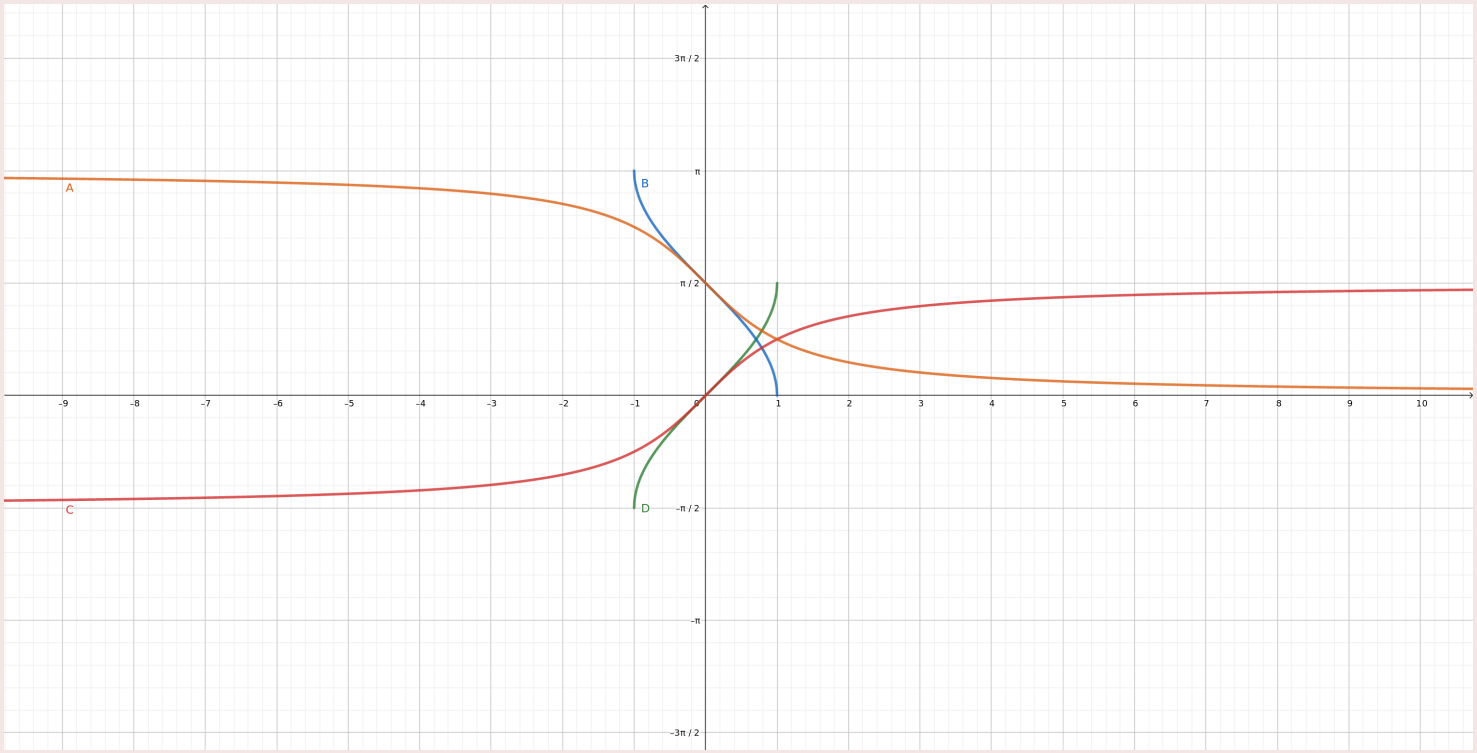
A $[-\frac{\pi}{2}; \frac{\pi}{2}]$
B $(-\frac{\pi}{2}; \frac{\pi}{2})$
C $[0; \pi]$
D $(0; \pi)$
E $[-1; 1]$

Find the range of arcsin x? A
Find the range of arccos x? C
Find the range of arctan x? B
Find the range of arccot x? D

Question

Find graphs of

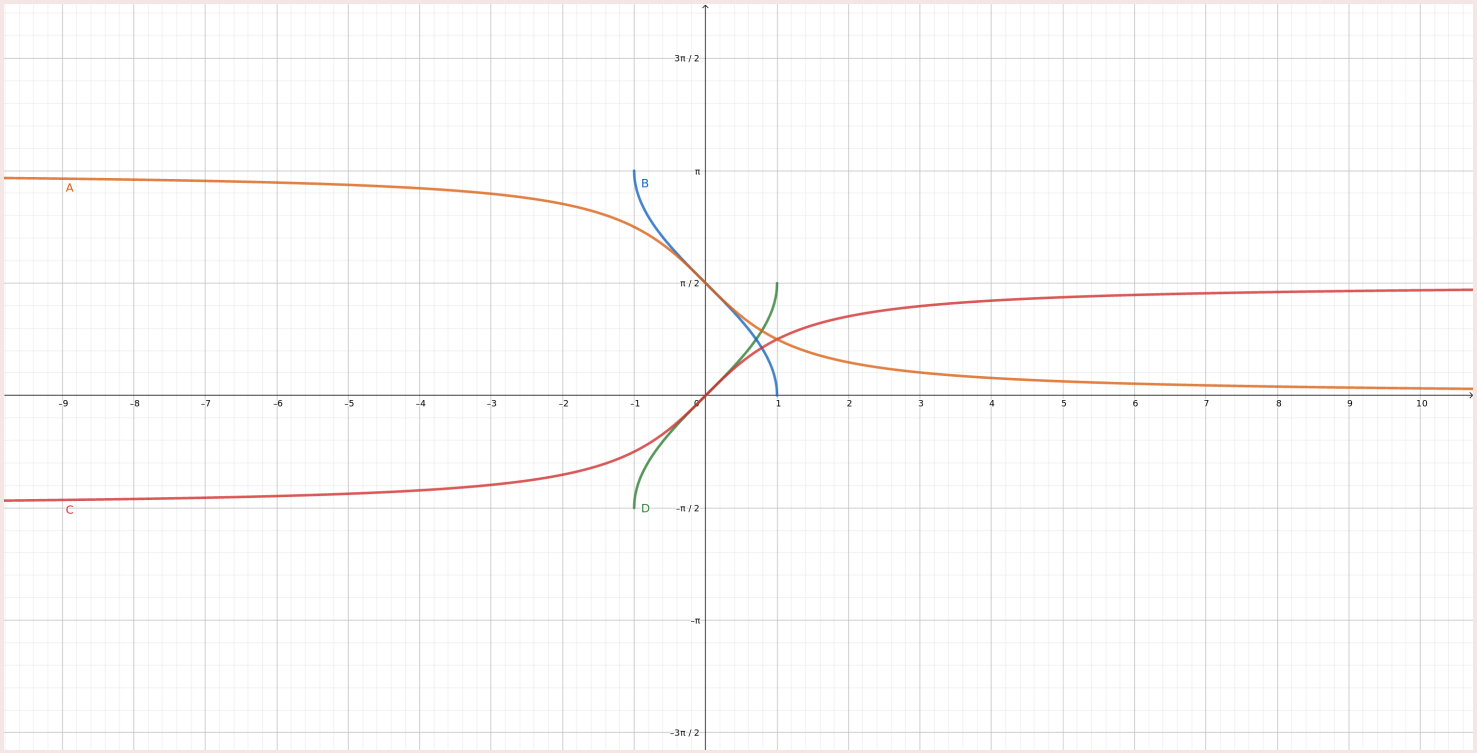
1. arcsin x
2. arccos x
3. arctan x
4. arccot x



Question

Find graphs of

1. arcsin x
2. arccos x
3. arctan x
4. arccot x



D (green), B (blue), C (red), A (yellow)

Question

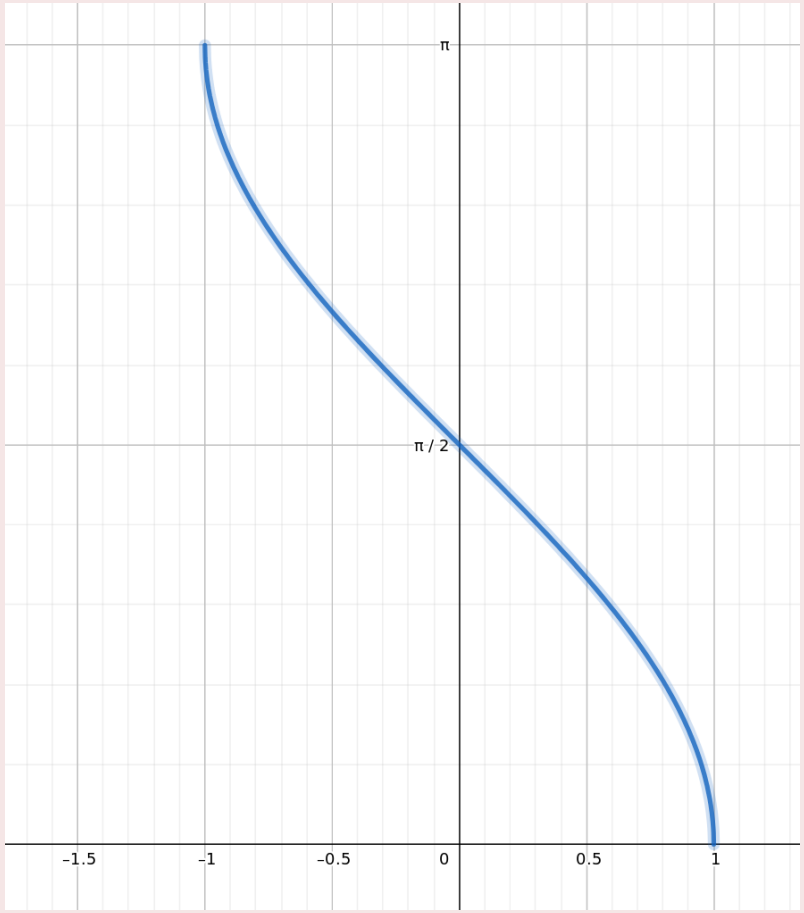
Find the prescription

A $\arccos x$

B $|\arccos x|$

C $\frac{\pi}{2} - \arcsin x$

D $\pi - \arccos(-x)$



Question

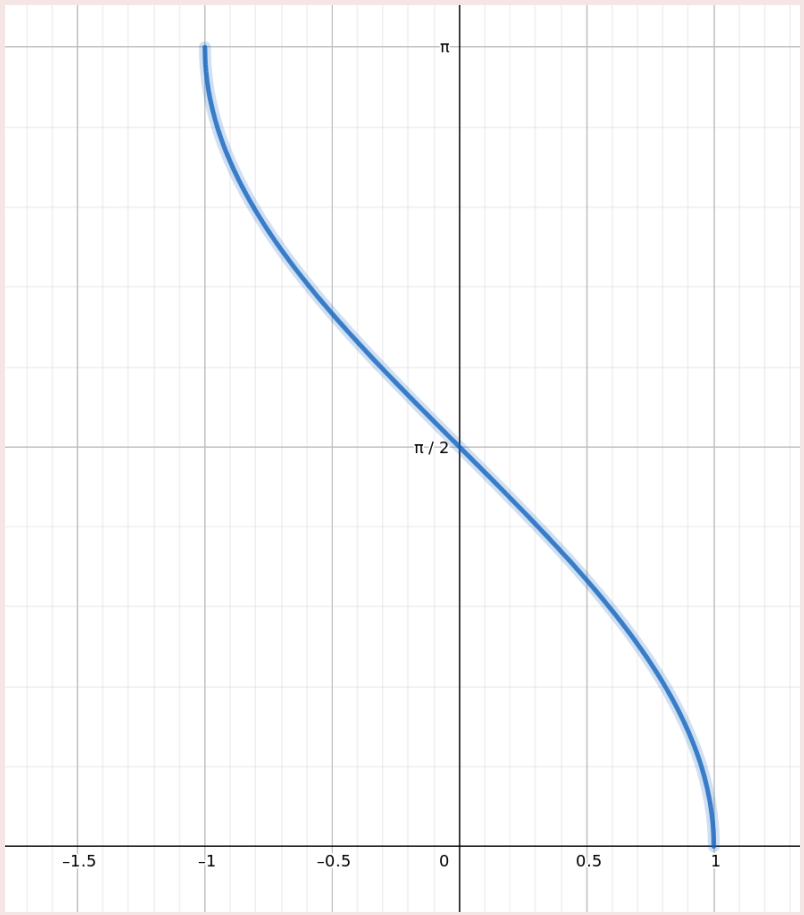
Find the prescription

A $\arccos x$

B $|\arccos x|$

C $\frac{\pi}{2} - \arcsin x$

D $\pi - \arccos(-x)$



A, B, C, D

Function compositions

Question (True-False)

A $\arcsin\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$

B $\sin(\arcsin\frac{\pi}{6}) = \frac{\pi}{6}$

C $\arcsin\left(\sin\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

D $\sin(\arcsin\frac{\pi}{3}) = \frac{\pi}{3}$

Function compositions

Question (True-False)

A $\arcsin\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$

B $\sin(\arcsin\frac{\pi}{6}) = \frac{\pi}{6}$

C $\arcsin\left(\sin\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

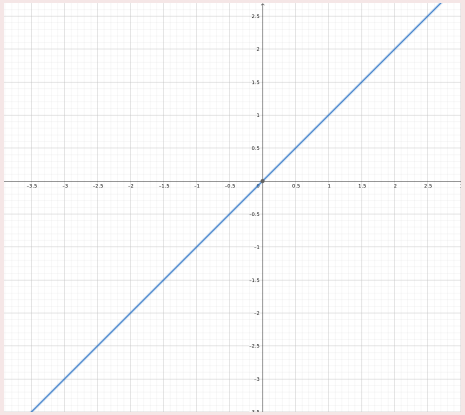
D $\sin(\arcsin\frac{\pi}{3}) = \frac{\pi}{3}$

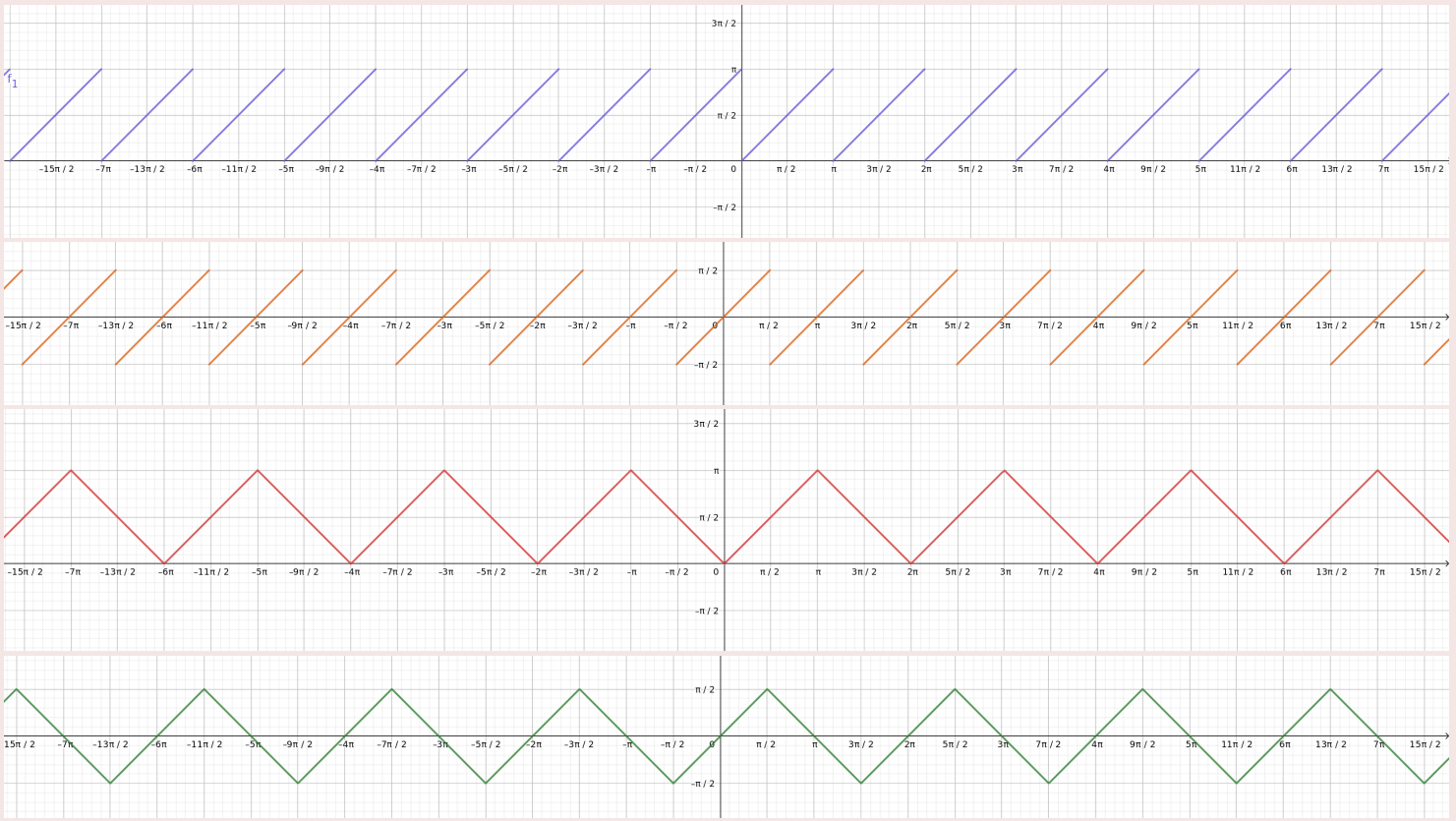
A, B

C is not true and D is not defined.

Question

Find the graph of $\arcsin(\sin x)$





Question

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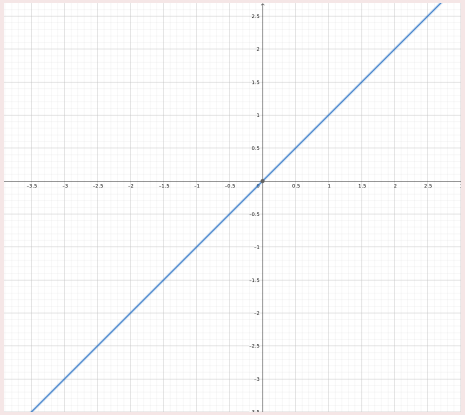
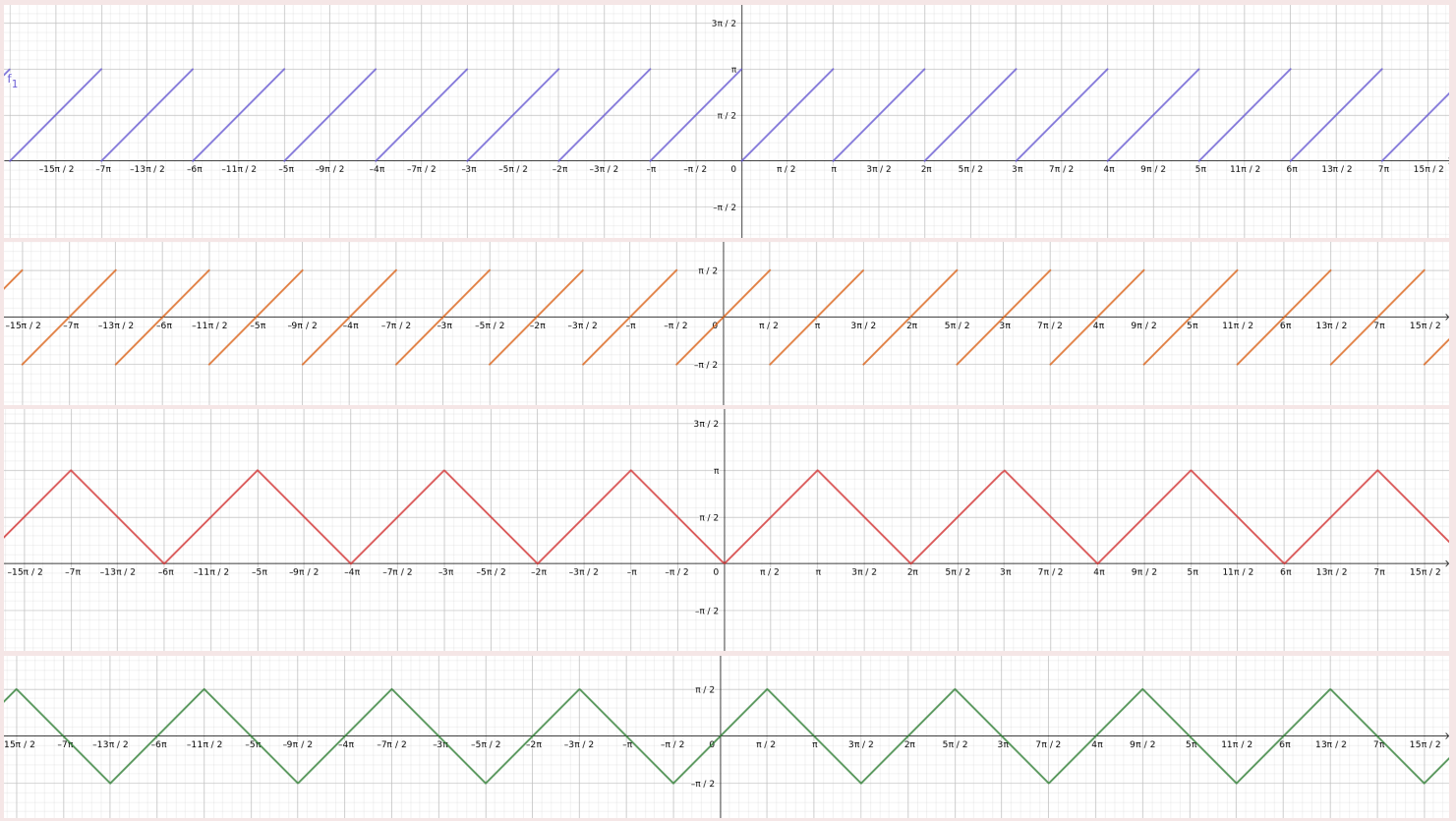
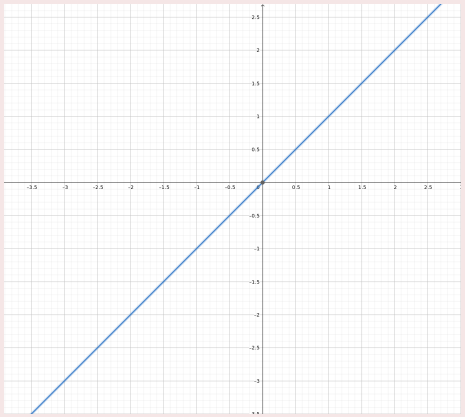


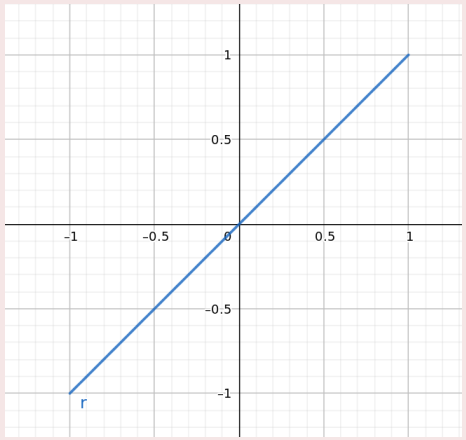


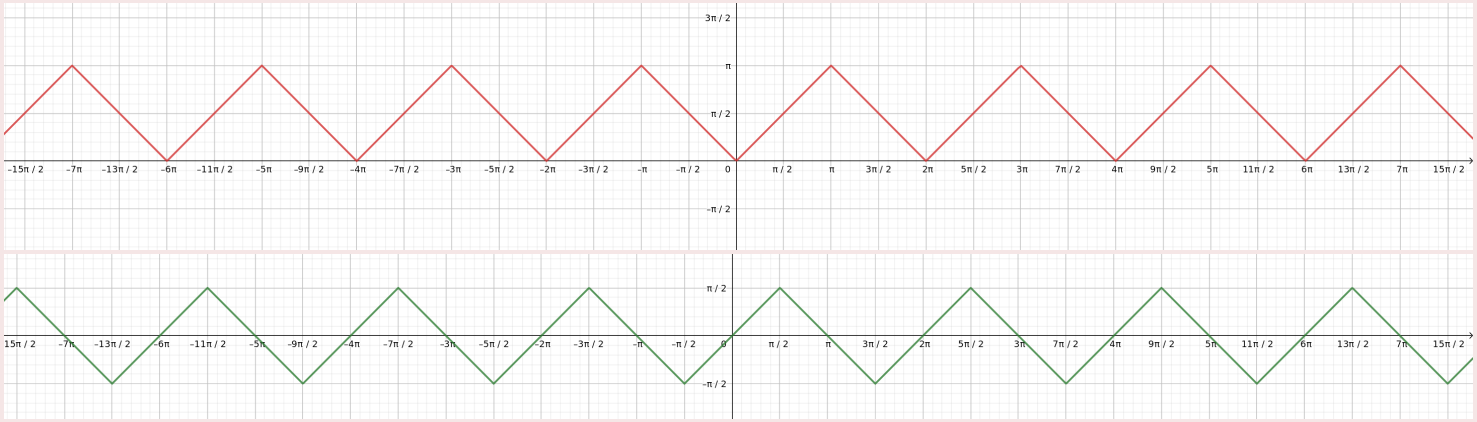
E

Question

Find the graph of $\sin(\arcsin x)$







Question

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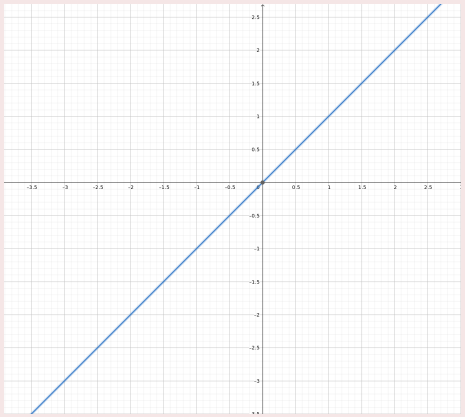
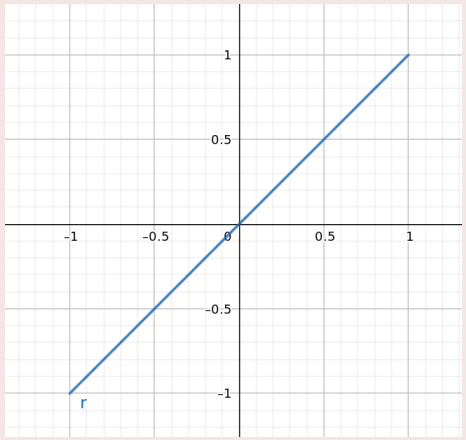
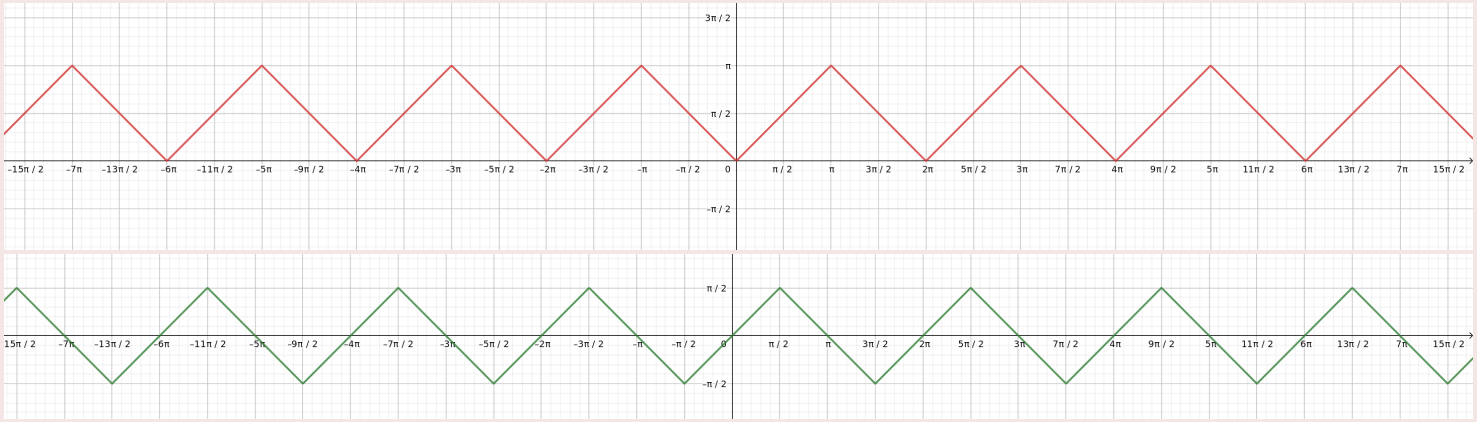






B

Question

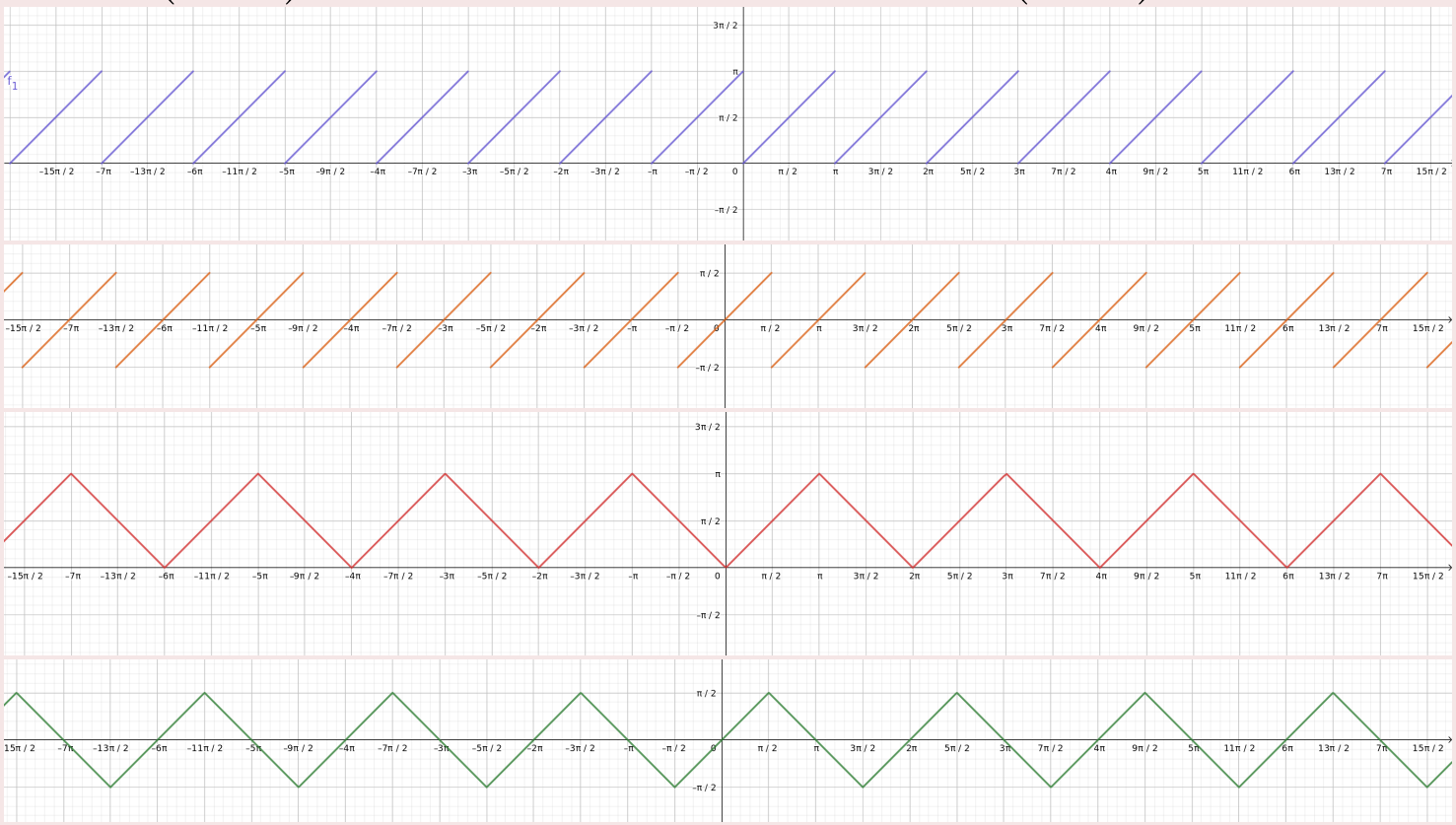
Assign the graphs

A $\arcsin(\sin x)$

B $\arccos(\cos x)$

C $\arctan(\tan x)$

D $\operatorname{arccot}(\cot x)$



Question

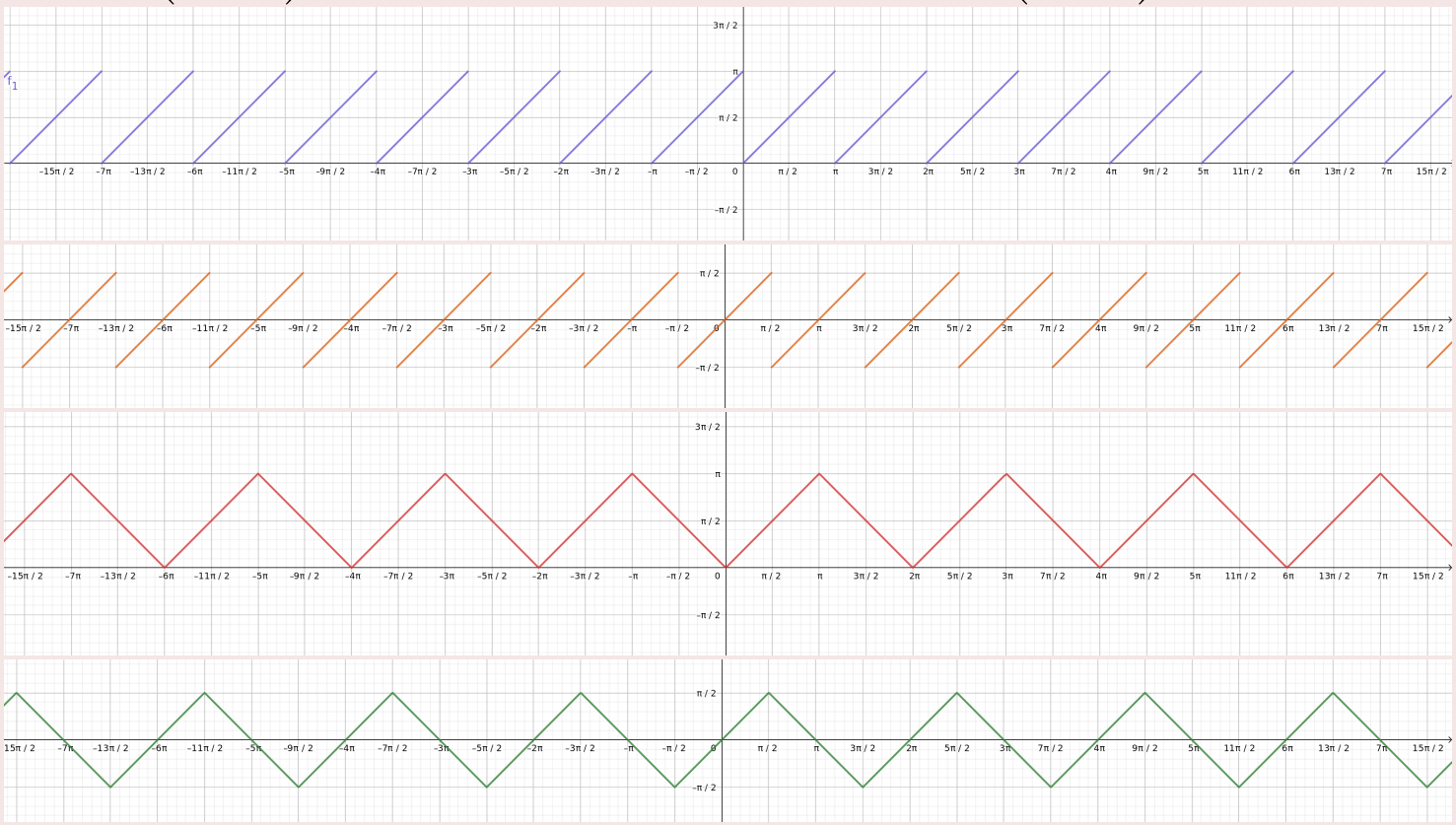
Assign the graphs

A $\arcsin(\sin x)$

B $\arccos(\cos x)$

C $\arctan(\tan x)$

D $\operatorname{arccot}(\cot x)$



D, C, B, A

Function compositions

Question

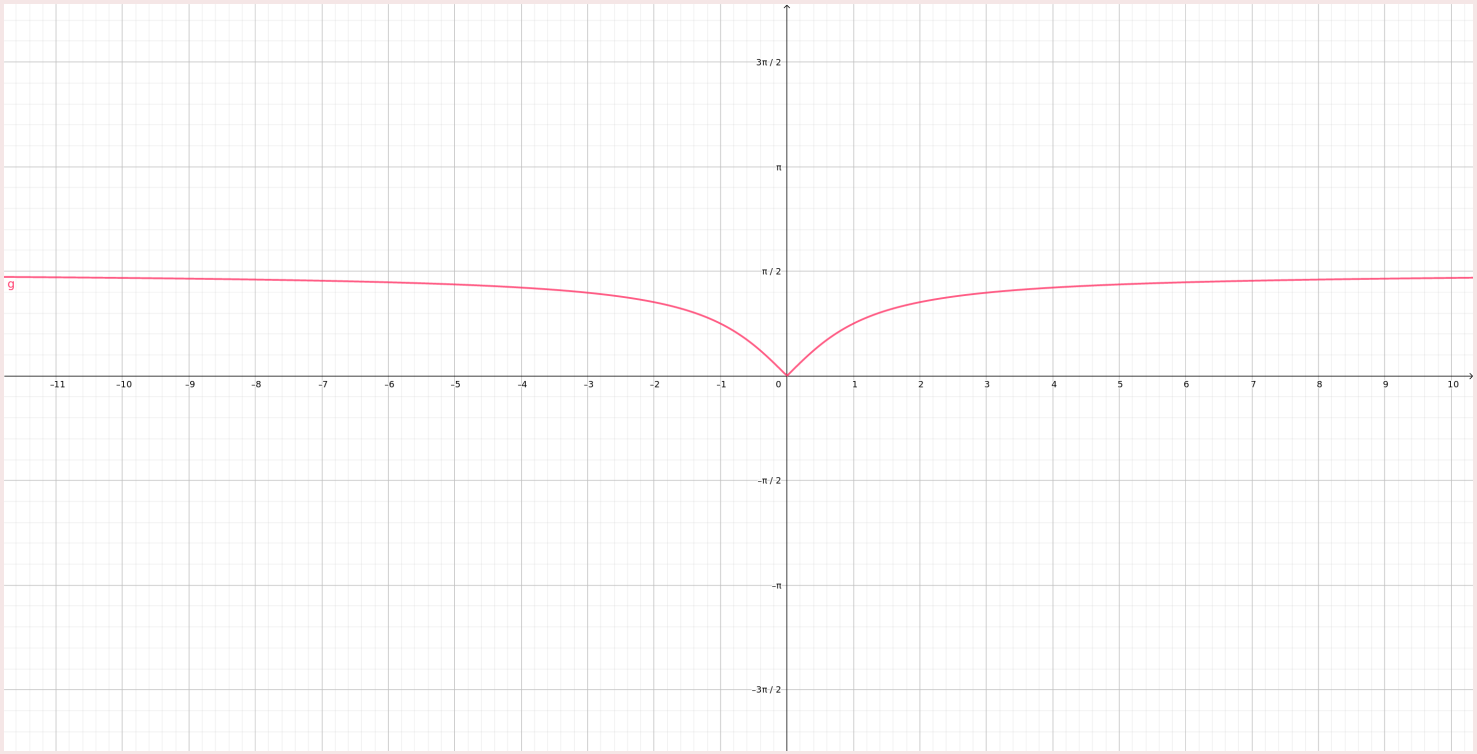
Find the prescription.

A $\arctan |x|$

B $\arctan - |x|$

C $|\arctan x|$

D $|\arctan (-x)|$



Question

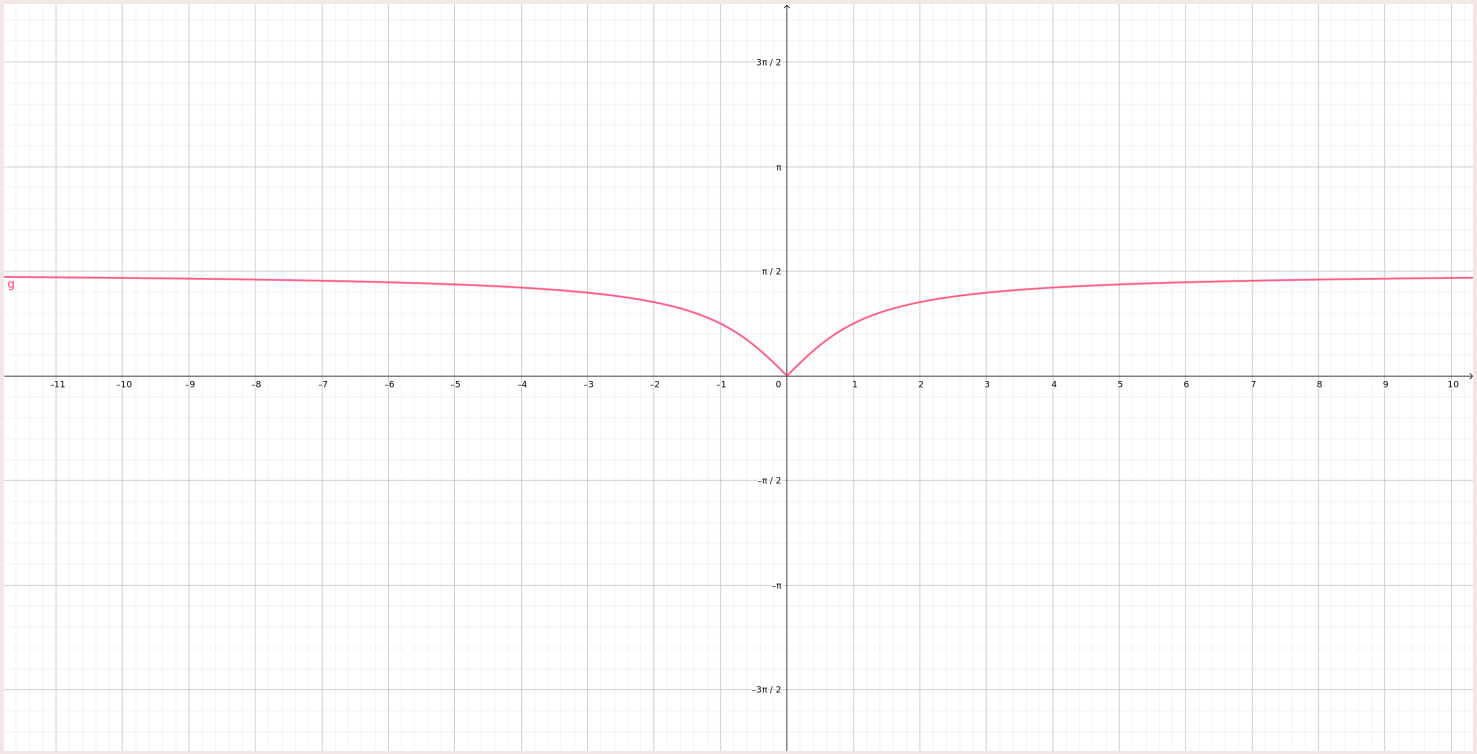
Find the prescription.

A $\arctan |x|$

B $\arctan\ -|x|$

C $|\arctan x|$

D $|\arctan(-x)|$



A, C, D

Function compositions

Question

Sketch the graph of $f(x) = |-\pi + 2\text{arccot}\,(x-3)|$

Function compositions

Question

Sketch the graph of $f(x) = |-\pi + 2\text{arccot}\,(x-3)|$