

## 6th lesson

<https://www2.karlin.mff.cuni.cz/~kuncova/en/teachIM.php>, [kunck6am@natur.cuni.cz](mailto:kunck6am@natur.cuni.cz)

### Theory

**Věta 1.** Let  $f(x) = a_n x^n + \dots + a_1 x + a_0$  be a function with  $a_i \in \mathbb{Z}$ ,  $i = 0, \dots, n$ . Then every rational root of  $f$  is of the following form

$$\frac{p}{q} = \pm \frac{\text{factor of } a_0}{\text{factor of } a_n}.$$

### Exercises

1. Find all roots and factor polynomials

(a)  $x^3 + 2x^2 - 11x - 12$

(d)  $x^4 - 3x^3 - 5x^2 + 3x + 4$

(b)  $10x^4 - 3x^3 - 29x^2 + 5x + 12$

(e)  $2x^4 - 7x^3 - 2x^2 + 28x - 24$

(c)  $2x^3 - 13x^2 + 3x + 18$

(f)  $8x^5 + 36x^4 + 46x^3 + 7x^2 - 12x - 4$

*Main source:* <http://mathquest.carroll.edu/libraries/PRE.student.00.40.pdf>

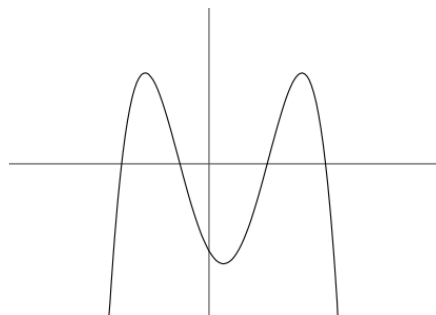
2. The sketched polynomial is of:

(a) odd degree, lead coefficient negative

(b) odd degree, lead coefficient positive

(c) even degree, lead coefficient negative

(d) even degree, lead coefficient positive



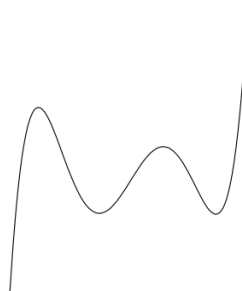
3. The sketched polynomial is of:

(a) odd degree, lead coefficient negative

(b) odd degree, lead coefficient positive

(c) even degree, lead coefficient negative

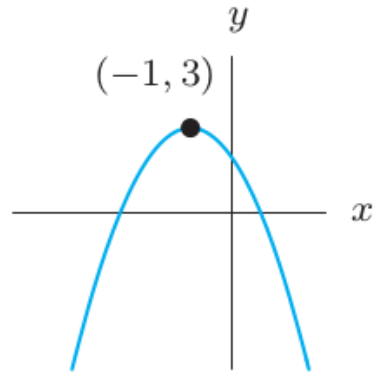
(d) even degree, lead coefficient positive



*Source for 4,5: Calculus: Single and Multivariable, 6th Edition, Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum*

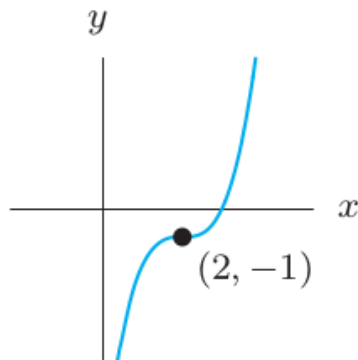
4. The sketched polynomial is:

- (a)  $(x - 1)^2 + 3$
- (b)  $-(x + 3)^2 - 1$
- (c)  $(x - 3)^2 + 1$
- (d)  $(x + 3)^2 - 1$
- (e)  $-(x + 1)^2 + 3$



5. The sketched polynomial is:

- (a)  $(x - 2)^3 - 1$
- (b)  $(x + 2)^3 - 1$
- (c)  $(x + 2)^3 + 1$
- (d)  $(x - 2)^3 + 1$
- (e)  $-(2 - x)^3 - 1$

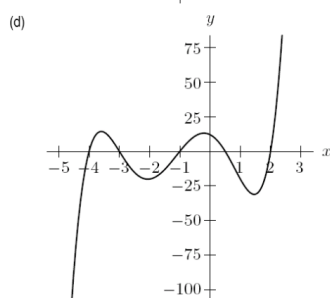
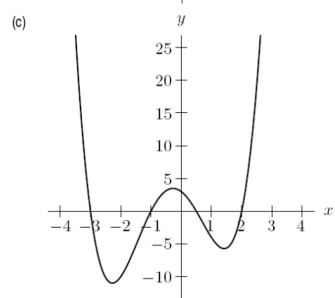
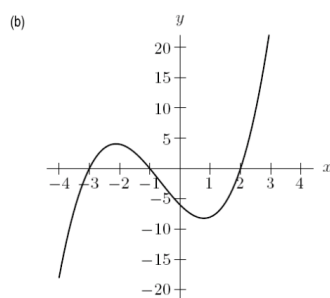
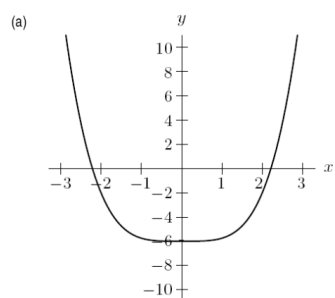


6. What is the degree of the polynomial  $y = x(2x + 1)^3(x - 4)^2(5 - x)^5$ ?

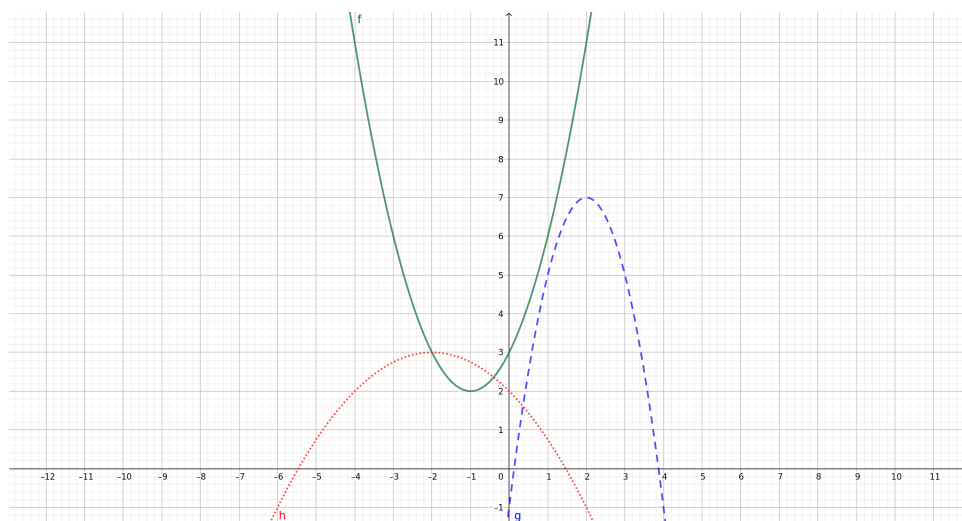
7. Find the polynomial with the smallest possible degree, with zeros at  $x = 1$ ,  $x = 2$  and  $x = 3$  such that  $f(5) = 8$ .

- (a)  $(x - 1)(x - 2)(x - 3)$
- (b)  $(x - 1)(x - 2)(x - 3)(x - 5)$
- (c)  $8(x - 1)(x - 2)(x - 3)$
- (d)  $8(x - 1)(x - 2)(x - 3)(x - 5)$
- (e)  $\frac{1}{3}(x - 1)(x - 2)(x - 3)$
- (f)  $\frac{1}{42}(x - 1)(x - 2)(x - 3)$

8. Find the graph of the function  $y = x^3 + 2x^2 - 5x - 6$



9. Find the formula for the quadratic functions:



10. Decide

TRUE - FALSE If  $f(x)$  is a polynomial such that  $f(c) = 0$  for  $c \in \mathbb{R}$ , then  $f(x)$  can be written as  $(x - c)g(x)$  for some polynomial  $g(x)$ .

TRUE - FALSE A polynomial function may have a horizontal asymptote.

TRUE - FALSE A polynomial function may have a vertical asymptote.

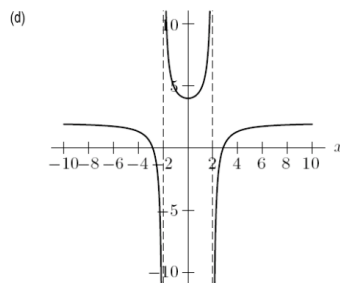
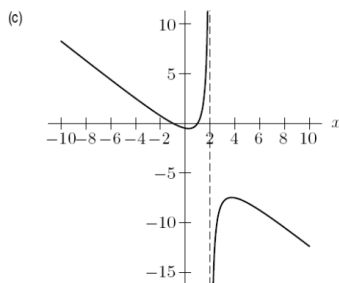
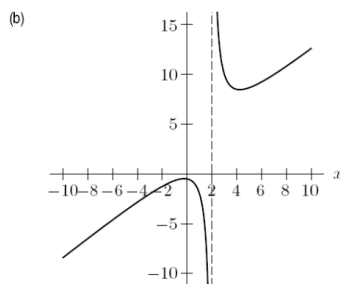
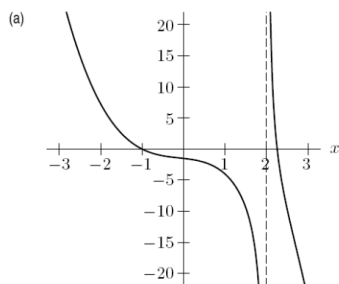
TRUE - FALSE For  $x \in \mathbb{R}$  we have:  $x \leq x^2$ .

TRUE - FALSE Every polynomial of even degree is an odd function and every polynomial of odd degree is even function.

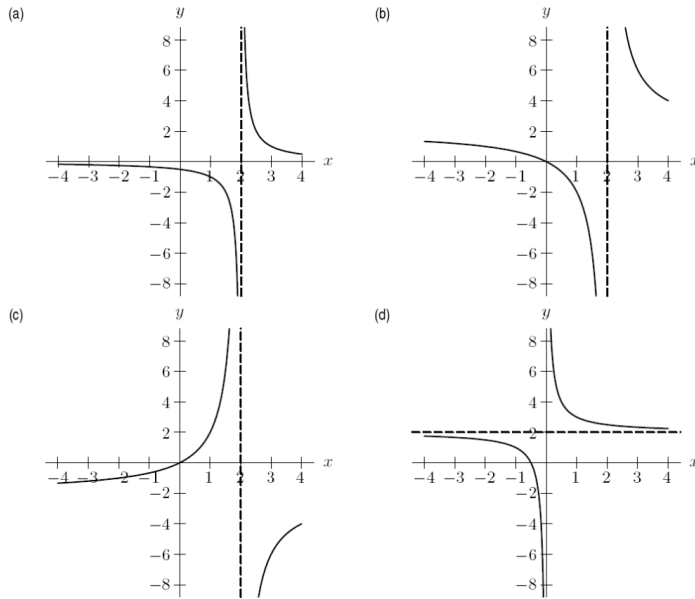
TRUE - FALSE Every polynomial of even degree is an even function and every polynomial of odd degree is odd function.

TRUE - FALSE Let  $f(x) = \frac{x^2-1}{x+1}$ ,  $g(x) = x - 1$ . Then  $f(x) = g(x)$ .

11. Find the graph of the function  $y = \frac{1-x^2}{x-2}$



12. Find the graph of the function  $y = \frac{2x}{x-2}$



Source: <http://www.opentextbookstore.com/precalc/2/Precalc3-7.pdf>

13. Find the possible formulas for graphed functions.

