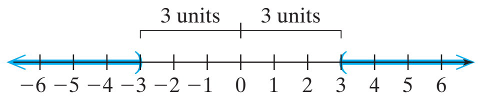
2. $|x| > 3$

$x < -3 \quad \text{or} \quad x > 3$

Solution:

The set of all points more than 3 units from zero

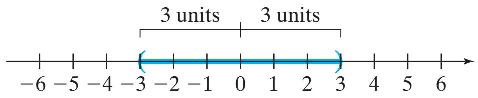


3. $|x| < 3$

$-3 < x < 3$

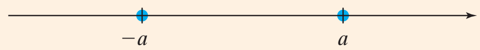
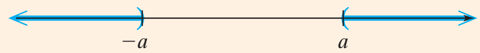
Solution:

The set of all points less than 3 units from zero



Absolute Value Equations and Inequalities

Let a be a real number such that $a > 0$. Then

Equation/ Inequality	Solution (Equivalent Form)	Graph
$\lvert x\rvert = a$	$x = -a$ or $x = a$	
$\lvert x\rvert > a$	$x < -a$ or $x > a$	
$\lvert x\rvert < a$	$-a < x < a$	

To solve an absolute value inequality, first isolate the absolute value and then rewrite the absolute value inequality in its equivalent form.

Example 1 Solving Absolute Value Inequalities

Solve the inequalities.

1a) **a.** $|3w + 1| - 4 < 7$ **b.** $3 \le 1 + \left|\frac{1}{2}t - 5\right|$

Solution:

a. $|3w + 1| - 4 < 7$

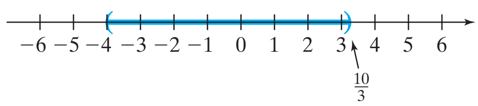
$|3w + 1| < 11$ ⟵ Isolate the absolute value first.

The inequality is in the form $|x| < a$, where $x = 3w + 1$.

$-11 < 3w + 1 < 11$ Rewrite in the equivalent form $-a < x < a$.

$-12 < 3w < 10$ Solve for w.

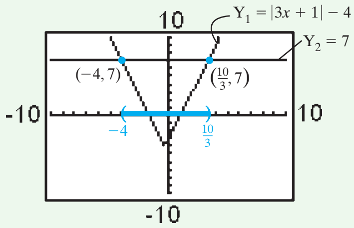
$-4 < w < \frac{10}{3}$



The solution is $\{w \mid -4 < w < \frac{10}{3}\}$ or, equivalently in interval notation, $(-4, \frac{10}{3})$.

Calculator Connections

Graph $Y_1 = \text{abs}(3x + 1) - 4$ and $Y_2 = 7$. On the given display window, $Y_1 < Y_2$ (Y_1 is below Y_2) for $-4 < x < \frac{10}{3}$.



b. $3 \le 1 + \left|\frac{1}{2}t - 5\right|$

$1 + \left|\frac{1}{2}t - 5\right| \ge 3$ Write the inequality with the absolute value on the left.

$\left|\frac{1}{2}t - 5\right| \ge 2$ Isolate the absolute value.

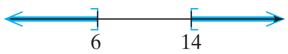
The inequality is in the form $|x| \ge a$, where $x = \frac{1}{2}t - 5$.

$\frac{1}{2}t - 5 \le -2$ or $\frac{1}{2}t - 5 \ge 2$ Rewrite in the equivalent form $x \le -a$ or $x \ge a$.

$\frac{1}{2}t \le 3$ or $\frac{1}{2}t \ge 7$ Solve the compound inequality.

$2\left(\frac{1}{2}t\right) \le 2(3)$ or $2\left(\frac{1}{2}t\right) \ge 2(7)$ Clear fractions.

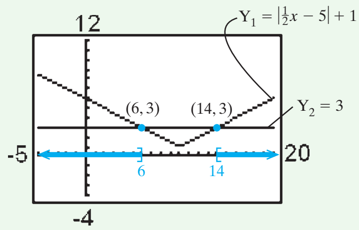
$t \le 6$ or $t \ge 14$



The solution is $\{t \mid t \le 6 \text{ or } t \ge 14\}$ or, equivalently in interval notation, $(-\infty, 6] \cup [14, \infty)$.

Calculator Connections

Graph $Y_1 = \text{abs}((1/2)x - 5) + 1$ and $Y_2 = 3$. On the given display window, $Y_1 \ge Y_2$ for $x \le 6$ or $x \ge 14$.



Skill Practice Solve the inequalities. Write the solutions in interval notation.

1. $|2t + 5| + 2 \le 11$ **2.** $5 < 1 + \left|\frac{1}{3}c - 1\right|$

By definition, the absolute value of a real number will always be nonnegative. Therefore, the absolute value of any expression will always be greater than

Skill Practice Answers

1. $[-7, 2]$
2. $(-\infty, -9) \cup (15, \infty)$

a negative number. Similarly, an absolute value can never be less than a negative number. Let a represent a positive real number. Then

- The solution to the inequality $|x| > -a$ is all real numbers, $(-\infty, \infty)$.
- There is no solution to the inequality $|x| < -a$.

Example 2 Solving Absolute Value Inequalities

Solve the inequalities.

a. $|3d - 5| + 7 < 4$ **b.** $|3d - 5| + 7 > 4$

Solution:

a. $|3d - 5| + 7 < 4$

$|3d - 5| < -3$

Isolate the absolute value. An absolute value expression cannot be less than a negative number. Therefore, there is no solution.

No solution

b. $|3d - 5| + 7 > 4$

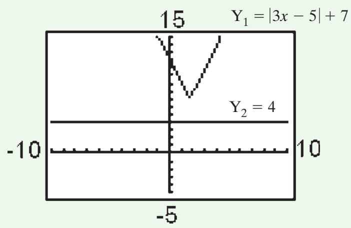
$|3d - 5| > -3$

Isolate the absolute value. The inequality is in the form $|x| > a$, where a is negative. An absolute value of any real number is greater than a negative number. Therefore, the solution is all real numbers.

All real numbers, $(-\infty, \infty)$

Calculator Connections

By graphing $Y_1 = \text{abs}(3x - 5) + 7$ and $Y_2 = 4$, we see that $Y_1 > Y_2$ (Y_1 is above Y_2) for all real numbers x on the given display window.



Skill Practice Solve the inequalities.

3. $|4p + 2| + 6 < 2$ **4.** $|4p + 2| + 6 > 2$

Example 3 Solving Absolute Value Inequalities

Solve the inequalities.

a. $|4x + 2| \geq 0$ **b.** $|4x + 2| > 0$

c)

Solution:

a. $|4x + 2| \geq 0$ ← The absolute value is already isolated.

The absolute value of any real number is nonnegative. Therefore, the solution is all real numbers, $(-\infty, \infty)$.

d)

b. $|4x + 2| > 0$

An absolute value will be greater than zero at all points *except where it is equal to zero*. That is, the point(s) for which $|4x + 2| = 0$ must be excluded from the solution set.

Skill Practice Answers

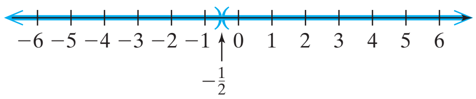
3. No solution
4. All real numbers; $(-\infty, \infty)$

$|4x + 2| = 0$

$4x + 2 = 0$ or $4x + 2 = -0$ The second equation is the same as the first.

$4x = -2$

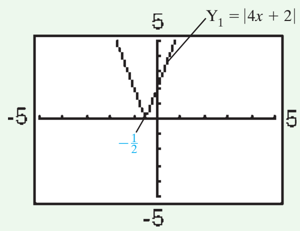
$x = -\dfrac{1}{2}$ Therefore, exclude $x = -\frac{1}{2}$ from the solution.



The solution is $\{x \mid x \neq -\frac{1}{2}\}$ or equivalently in interval notation, $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$.

Calculator Connections

Graph $Y_1 = \text{abs}(4x + 2)$. From the graph, $Y_1 = 0$ at $x = -\frac{1}{2}$ (the x-intercept). On the given display window, $Y_1 > 0$ for $x < -\frac{1}{2}$ or $x > -\frac{1}{2}$.



Skill Practice Solve the inequalities.

5. $|3x - 1| \geq 0$ **6.** $|3x - 1| > 0$

2. Solving Absolute Value Inequalities by the Test Point Method

For each problem in Example 1, the absolute value inequality was converted to an equivalent compound inequality. However, sometimes students have difficulty setting up the appropriate compound inequality. To avoid this problem, you may want to use the test point method to solve absolute value inequalities.

Solving Inequalities by Using the Test Point Method

1. Find the boundary points of the inequality. (Boundary points are the real solutions to the related equation and points where the inequality is undefined.)
2. Plot the boundary points on the number line. This divides the number line into regions.
3. Select a test point from each region and substitute it into the original inequality.
   - If a test point makes the original inequality true, then that region is part of the solution set.
4. Test the boundary points in the original inequality.
   - If a boundary point makes the original inequality true, then that point is part of the solution set.

Skill Practice Answers

5. $(-\infty, \infty)$

6. $\left\{x \mid x \neq \dfrac{1}{3}\right\}; \left(-\infty, \dfrac{1}{3}\right) \cup \left(\dfrac{1}{3}, \infty\right)$

B. Solving "Less Than" Absolute Value Inequalities

Absolute value *inequalities* can be solved using the basic concept underlying the property of absolute value equalities. Whereas the equation $|x| = 4$ asks for all numbers x whose distance from zero is *equal* to 4, the inequality $|x| < 4$ asks for all numbers x whose distance from zero is *less than* 4.

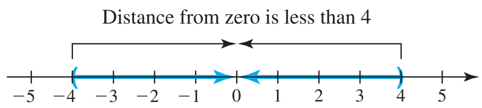


Figure 1.6

As Figure 1.6 illustrates, the solutions are $x > -4$ and $x < 4$, which can be written as the joint inequality $-4 < x < 4$. This idea can likewise be extended to include the absolute value of an algebraic expression X as follows.

WORTHY OF NOTE

Property I can also be applied when the "≤" symbol is used. Also notice that if $k < 0$, the solution is the empty set since the absolute value of any quantity is always positive or zero.

Property I: Absolute Value Inequalities

If X represents an algebraic expression and k is a positive real number,

then $|X| < k$

implies $-k < X < k$

EXAMPLE 4 ► **Solving "Less Than" Absolute Value Inequalities**

e)

Solve the inequalities:

a. $\dfrac{|3x + 2|}{4} \leq 1$ **b.** $|2x - 7| < -5$

Solution ►

a.
$$\frac{|3x + 2|}{4} \leq 1 \quad \text{original inequality}$$
$$|3x + 2| \leq 4 \quad \text{multiply by 4}$$
$$-4 \leq 3x + 2 \leq 4 \quad \text{apply Property I}$$
$$-6 \leq 3x \leq 2 \quad \text{subtract 2 from all three parts}$$
$$-2 \leq x \leq \frac{2}{3} \quad \text{divide all three parts by 3}$$

The solution interval is $[-2, \frac{2}{3}]$.

b. $|2x - 7| < -5$ original inequality

17)

Since the absolute value of any quantity is always positive or zero, the solution for this inequality is the empty set: { }.

WORTHY OF NOTE

As with the inequalities from Section 1.2, solutions to absolute value inequalities can be checked using a test value. For Example 4(a), substituting $x = 0$ from the solution interval yields:

$$\frac{1}{2} \leq 1 ✓$$

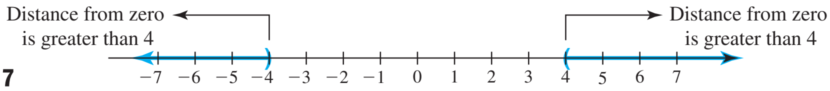
Now try Exercises 25 through 38 ►

☑ **B.** You've just learned how to solve less than absolute value inequalities

C. Solving "Greater Than" Absolute Value Inequalities

For "greater than" inequalities, consider $|x| > 4$. Now we're asked to find all numbers x whose distance from zero is *greater than* 4. As Figure 1.7 shows, solutions are found in the interval to the left of -4, or to the right of 4. The fact the intervals are disjoint (disconnected) is reflected in this graph, in the inequalities $x < -4$ **or** $x > 4$, as well as the interval notation $x \in (-\infty, -4) \cup (4, \infty)$.

Figure 1.7



As before, we can extend this idea to include algebraic expressions, as follows:

Property II: Absolute Value Inequalities

If X represents an algebraic expression and k is a positive real number,

then $|X| > k$
implies $X < -k$ or $X > k$

EXAMPLE 5 ▶ Solving "Greater Than" Absolute Value Inequalities

g)

Solve the inequalities:

a. $-\frac{1}{3}\left|3 + \frac{x}{2}\right| < -2$ **b.** $|5x + 2| \geq -\frac{3}{2}$

Solution ▶ **a.** Note the exercise is given as a *less than* inequality, but as we multiply both sides by -3, we must *reverse the inequality symbol*.

$$-\frac{1}{3}\left|3 + \frac{x}{2}\right| < -2 \quad \text{original inequality}$$

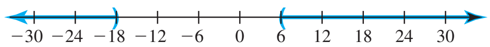
$$\left|3 + \frac{x}{2}\right| > 6 \quad \text{multiply by } -3\text{, reverse the symbol}$$

$$3 + \frac{x}{2} < -6 \quad \text{or} \quad 3 + \frac{x}{2} > 6 \quad \text{apply Property II}$$

$$\frac{x}{2} < -9 \quad \text{or} \quad \frac{x}{2} > 3 \quad \text{subtract 3}$$

$$x < -18 \quad \text{or} \quad x > 6 \quad \text{multiply by 2}$$

Property II yields the disjoint intervals $x \in (-\infty, -18) \cup (6, \infty)$ as the solution.



b. $|5x + 2| \geq -\frac{3}{2}$ original inequality

Since the absolute value of any quantity is always positive or zero, the solution for this inequality is all real numbers: $x \in \mathbb{R}$.

☑ **C.** You've just learned how to solve greater than absolute value inequalities

Now try Exercises 39 through 54 ▶

CAUTION ▶ Be sure you note the difference between the individual solutions of an absolute value equation, and the solution intervals that often result from solving absolute value inequalities. The solution $\{-2, 5\}$ indicates that both $x = -2$ and $x = 5$ are solutions, while the solution $[-2, 5)$ indicates that all numbers between -2 and 5, including -2, are solutions.

And we can describe the solution using interval and set-builder notation.

$$(-\infty, 1/2) \cup (1, \infty) = \{x : x < 1/2 \text{ or } x > 1\}$$

Again, let $a > 0$. As we did with $|x| \leq a$, we can take the union of the solutions of $|x| = a$ and $|x| > a$ to find the solution of $|x| \geq a$. This leads to the following property.

Property 19. If $a > 0$, then the inequality $|x| \geq a$ is equivalent to the inequality $x \leq -a$ or $x \geq a$.

► **Example 20.** *Solve the inequality* $3|1-x| - 4 \geq |1-x|$ *for* x.

2a)

Again, at first glance, the inequality

$$3|1-x| - 4 \geq |1-x|$$

looks unlike any inequality we've attempted to this point. However, if we subtract $|1-x|$ from both sides of the inequality, then add 4 to both sides of the inequality, we get

$$3|1-x| - |1-x| \geq 4.$$

On the left, we have like terms. Note that $3|1-x| - |1-x| = 3|1-x| - 1|1-x| = 2|1-x|$. Thus,

$$2|1-x| \geq 4.$$

Divide both sides of the last inequality by 2.

$$|1-x| \geq 2$$

We can now use **Property 19** to write

$$1 - x \leq -2 \qquad \text{or} \qquad 1 - x \geq 2.$$

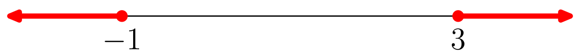
We can solve each of these inequalities independently. First, subtract 1 from both sides of each inequality, then multiply both sides of each resulting inequality by -1, reversing each inequality as you go.

$$\begin{aligned} 1 - x &\leq -2 \\ -x &\leq -3 \\ x &\geq 3 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 1 - x &\geq 2 \\ -x &\geq 1 \\ x &\leq -1 \end{aligned}$$

We prefer to write this in the order

$$x \leq -1 \qquad \text{or} \qquad x \geq 3.$$

We can sketch the solutions on a number line.



And we can describe the solutions using interval and set-builder notation.

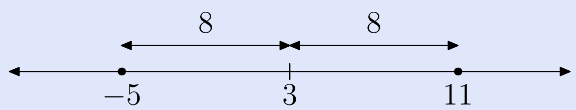
$$(-\infty, -1] \cup [3, \infty) = \{x : x \leq -1 \text{ or } x \geq 3\}$$

Revisiting Distance

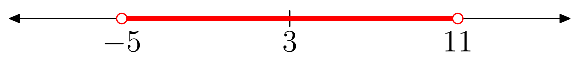
If a and b are any numbers on the real line, then the distance between a and b is found by taking the absolute value of their difference. That is, the distance d between a and b is calculated with $d = |a - b|$. More importantly, we've learned to pronounce the symbolism $|a - b|$ as "the distance between a and b." This pronunciation is far more useful than saying "the absolute value of a minus b."

▶ **Example 21.** *Solve the inequality* $|x - 3| < 8$ *for* x.

This inequality is pronounced "the distance between x and 3 is less than 8." Draw a number line, locate 3 on the line, then note two points that are 8 units away from 3.



Now, we need to shade the points that are *less than* 8 units from 3.



Hence, the solution of the inequality $|x - 3| < 8$ is

$$(-5, 11) = \{x : -5 < x < 11\}.$$

▶ **Example 22.** *Solve the inequality* $|x + 5| > 2$ *for* x.

First, write the inequality as a difference.

$$|x - (-5)| > 2$$

This last inequality is pronounced "the distance between x and -5 is greater than 2." Draw a number line, locate -5 on the number line, then note two points that are 2 units from -5.

Method 1.

b)

$$||x| + x| \leq 2$$

For $x < 0$, we have $|x| = -x$. Therefore:

$$|-x + x| \leq 2$$

which is always true.

For $x \geq 0$, we have $|x| = x$. Therefore:

$$\begin{aligned} |x + x| &\leq 2 \\ x &\leq 1 \end{aligned}$$

Combining both answers, we get $x \leq 1$.

https://gmatclub.com/forum/if-x-3-12-13-what-is-the-range-of-x-276038.html

Solution

Given:

We are given an absolute value inequality, $||x+3| - 12| < 13$

Note that we do not have any constraint on x. So, x can be an integer or a decimal

To find:

We need to find the range of values of x, that satisfies the given inequality

Approach and Working:

First, let us substitute the inner modulus function, $|x + 3|$, with t, which gives, $|t - 12| < 13$

We know that, the range of x, for $|x| < a$, is $a < x < -a$

So, the range of x, for which $|t - 12| < 13$, is

$-13 < t – 12 < 13$,

Adding 12 on all the sides, we get, $-1 < t < 25$

Now, substituting, back the value of t as $|x + 3|$, we get, $-1 < |x + 3| < 25$

Since, the value of $|x + 3|$ is always greater than or equal to 0

Hence, $0 \le |x + 3| < 25$

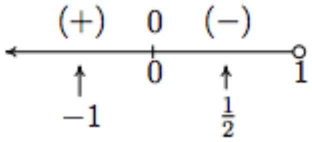
Now, solving the inequality, $|x + 3| < 25$, we get the range of x as (-28, 22)

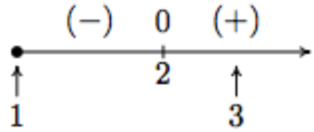
Therefore, the range of x, that satisfies the inequality, $||x+3| - 12| < 13$, is (-28, 22)

Hence, the correct answer is option C.

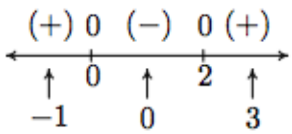
d)

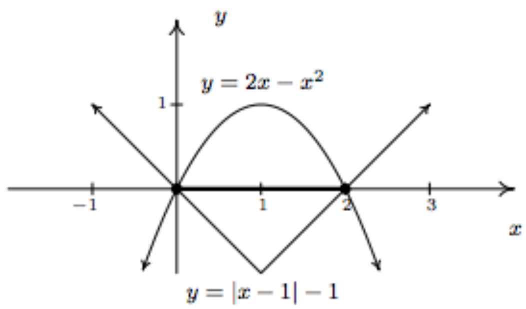
4. To solve our last inequality, $2x - x^2 \geq |x-1| - 1$, we re-write the absolute value using cases. For $x < 1$, $|x-1| = -(x-1) = 1-x$, so we get $2x - x^2 \geq 1 - x - 1$, or $x^2 - 3x \leq 0$. Finding the zeros of $f(x) = x^2 - 3x$, we get $x = 0$ and $x = 3$. However, we are only concerned with the portion of the number line where $x < 1$, so the only zero that we concern ourselves with is $x = 0$. This divides the interval $x < 1$ into two intervals: $(-\infty, 0)$ and $(0, 1)$. We choose $x = -1$ and $x = \frac{1}{2}$ as our test values. We find $f(-1) = 4$ and $f\left(\frac{1}{2}\right) = -\frac{5}{4}$. Hence, our solution to $x^2 - 3x \leq 0$ for $x < 1$ is $[0, 1)$. Next, we turn our attention to the case $x \geq 1$. Here, $|x-1| = x - 1$, so our original inequality becomes $2x - x^2 \geq x - 1 - 1$, or $x^2 - x - 2 \leq 0$. Setting $g(x) = x^2 - x - 2$, we find the zeros of g to be $x = -1$ and $x = 2$. Of these, only $x = 2$ lies in the region $x \geq 1$, so we ignore $x = -1$. Our test intervals are now $[1, 2)$ and $(2, \infty)$. We choose $x = 1$ and $x = 3$ as our test values and find $g(1) = -2$ and $g(3) = 4$. Hence, our solution to $g(x) = x^2 - x - 2 \leq 0$, in this region is $[1, 2)$.





Combining these into one sign diagram, we have that our solution is $[0, 2]$. Graphically, to check $2x - x^2 \geq |x-1| - 1$, we set $h(x) = 2x - x^2$ and $i(x) = |x-1| - 1$ and look for the x values where the graph of h is above the the graph of i (the solution of $h(x) > i(x)$) as well as the x-coordinates of the intersection points of both graphs (where $h(x) = i(x)$). The combined sign chart is given on the left and the graphs are on the right.

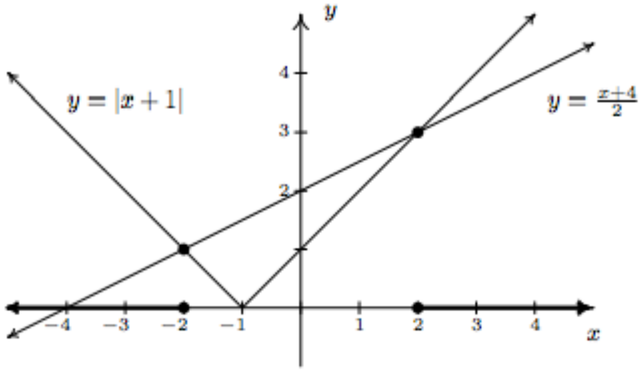




$\square$

e)

4. We need to exercise some special caution when solving $|x+1| \geq \frac{x+4}{2}$. As we saw in Example 2.2.1 in Section 2.2, when variables are both inside and outside of the absolute value, it's usually best to refer to the definition of absolute value, Definition 2.4, to remove the absolute values and proceed from there. To that end, we have $|x+1| = -(x+1)$ if $x < -1$ and $|x+1| = x+1$ if $x \geq -1$. We break the inequality into cases, the first case being when $x < -1$. For these values of x, our inequality becomes $-(x+1) \geq \frac{x+4}{2}$. Solving, we get $-2x-2 \geq x+4$, so that $-3x \geq 6$, which means $x \leq -2$. Since all of these solutions fall into the category $x < -1$, we keep them all. For the second case, we assume $x \geq -1$. Our inequality becomes $x+1 \geq \frac{x+4}{2}$, which gives $2x+2 \geq x+4$ or $x \geq 2$. Since all of these values of x are greater than or equal to -1, we accept all of these solutions as well. Our final answer is $(-\infty, -2] \cup [2, \infty)$.



https://math.stackexchange.com/questions/1342995/quadratic-absolute-value-inequality

f)

$$|x^2 - 3x + 1| < 1 \iff -1 < x^2 - 3x + 1 < 1$$

We have, using the lower bound,

$$\begin{aligned} -1 < x^2 - 3x + 1 \iff x^2 - 3x + 2 > 0 &\iff (x-2)(x-1) > 0 \\ &\iff x \in (-\infty, 1) \cup (2, \infty) \end{aligned}$$

Also, we have, using the upper bound,

$$x^2 - 3x + 1 < 1 \iff x(x-3) < 0 \iff x \in (0, 3)$$

For both upper and lower bounds to hold, we need the intersection of these solution sets which is $(0, 1) \cup (2, 3)$.

We need to solve the following system

$$\frac{x^2-5x+4}{x^2-4}\leq1$$

and

$$\frac{x^2-5x+4}{x^2-4}\geq-1$$

For the first we need to solve

$$\frac{x^2-5x+4}{x^2-4}-1\leq0$$

or

$$\frac{8-5x}{(x-2)(x+2)}\leq0,$$

which by the intervals method gives $x>2$ or $-2<x\leq1.6$.

The second inequality gives

$$\frac{x^2-5x+4}{x^2-4}+1\geq0$$

or

$$\frac{2x^2-5x}{x^2-4}\geq0$$

or

$$\frac{x(2x-5)}{(x-2)(x+2)}\geq0,$$

which by the intervals method again gives $x\geq2.5$ or $0\leq x<2$ or $x<-2$.

Thus, after solving of this system we'll get the answer:

$$[0,1.6]\cup[2.5,+\infty)$$

3a

- **Solve $\left|\left|\left|\left| x - 1 \right| - 1 \right| - 1 \right| - 1 \right| = 0$**

This *looks* awful, but nested absolute values aren't actually that bad. I just have to work in steps. I'll start by taking the outermost absolute-value bars off. Since the original equation is "equals zero", I don't even have to bother with considering the "plus" and "minus" cases:

$$\left|\left|\left| x - 1 \right| - 1 \right| - 1 \right| - 1 = 0$$

$$\left|\left|\left| x - 1 \right| - 1 \right| - 1 \right| = 1$$

Now I need to start considering cases. I'll start with the "minus" for the equation above.

$$\left|\left| x - 1 \right| - 1 \right| - 1 = -1$$

$$\left|\left| x - 1 \right| - 1 \right| = 0$$

This is another "equals zero", so I can go straight to the next set of bars:

$$\left| x - 1 \right| - 1 = 0$$

$$\left| x - 1 \right| = 1$$

Now I'll split into cases again; I'll start with the "minus" case:

$$x - 1 = -1$$

$$x = 0$$

Okay; that's one solution. Now I'll consider the "plus" case:

$$x - 1 = +1$$

$$x = 2$$

That's another solution.

I'd found these two solutions by taking the first "minus" case, and tracing it down to its end. That consideration started back at the purple "Now", above. So I need to start there, and consider the "plus" case.

$$\left|\left| x - 1 \right| - 1 \right| - 1 = +1$$

$$\left|\left| x - 1 \right| - 1 \right| = 2$$

I'll split this into two cases; I'll consider the "minus" case first:

$$\left| x - 1 \right| - 1 = -2$$

$$\left| x - 1 \right| = -1$$

This case doesn't work, because it's telling me that an absolute value is equal to a negative number. That can't be true. So I discard this branch and try the "plus" case:

$$\left| x - 1 \right| - 1 = +2$$

$$\left| x - 1 \right| = 3$$

This works, so I can proceed to the two branches for this equation. I'll do the "minus" case first:

$$x - 1 = -3$$

$$x = -2$$

Now I'll do the "plus" case:

$x - 1 = +3$

$x = 4$

I've considered every case, and have arrived at four solutions. My complete answer is:

$\boldsymbol{x = -2, 0, 2, 4}$

3b

We have

$$|x|+|x+2|=\frac{4-3p}{1-p} \tag{1}$$

Case 1 : For $x>0$,

$$(1)\iff x+(x+2)=\frac{4-3p}{1-p}\iff x=\frac{2-p}{2(1-p)}$$

Here, $\frac{2-p}{2(1-p)}>0\iff p<1\quad\text{or}\quad p>2.$

Case 2 : For $-2<x\le 0$,

$$(1)\iff -x+(x+2)=\frac{4-3p}{1-p}\iff p=2$$

Case 3 : For $x\le -2$,

$$(1)\iff -x-(x+2)=\frac{4-3p}{1-p}\iff x=\frac{6-5p}{2(p-1)}$$

Here, $\frac{6-5p}{2(p-1)}\le -2\iff p<1\quad\text{or}\quad p\ge 2.$

So, the answer is the followings :

- For $p<1$ or $p>2$, $x=\frac{2-p}{2(1-p)},\frac{6-5p}{2(p-1)}$.
- For $p=2$, $-2\le x\le 0$.
- For $1\le p<2$, there is no such x.

3. Translating to an Absolute Value Expression

Absolute value expressions can be used to describe distances. The distance between c and d is given by $|c - d|$. For example, the distance between -2 and 3 on the number line is $|(-2) - 3| = 5$ as expected.

Example 6 Expressing Distances with Absolute Value

Write an absolute value inequality to represent the following phrases.

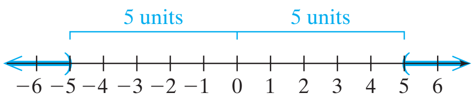
a. All real numbers x, whose distance from zero is greater than 5 units

b. All real numbers x, whose distance from -7 is less than 3 units

Solution:

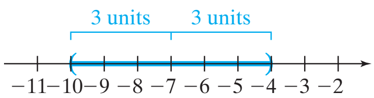
a. All real numbers x, whose distance from zero is greater than 5 units

$|x - 0| > 5$ or simply $|x| > 5$



b. All real numbers x, whose distance from -7 is less than 3 units

$|x - (-7)| < 3$ or simply $|x + 7| < 3$



Skill Practice Write an absolute value inequality to represent the following phrases.

11. All real numbers whose distance from zero is greater than 10 units

12. All real numbers whose distance from 4 is less than 6

Absolute value expressions can also be used to describe boundaries for measurement error.

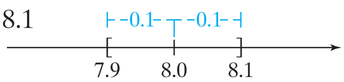
Example 7 Expressing Measurement Error with Absolute Value

Latoya measured a certain compound on a scale in the chemistry lab at school. She measured 8 g of the compound, but the scale is only accurate to ± 0.1 g. Write an absolute value inequality to express an interval for the true mass, x, of the compound she measured.

Solution:

Because the scale is only accurate to ± 0.1 g, the true mass, x, of the compound may deviate by as much as 0.1 g above or below 8 g. This may be expressed as an absolute value inequality:

$|x - 8.0| \leq 0.1$ or equivalently $7.9 \leq x \leq 8.1$



Skill Practice Answers

11. $|x| > 10$

12. $|x - 4| < 6$