3rd lesson

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Algorithm

1. Find conditions (dividing by zero).
2. Consider two cases. What happen if the interior of the absolute value is positive (or zero) OR if the interior is negative.
   (a) At first, consider the interior is ≥ 0 (you obtain an inequality) and then cancel the absolute value (you obtain the second inequality).
   (b) Solve both inequalities. Sketch a picture, where the first AND the second inequality is valid.
   (c) Second, consider the interior is < 0 (you obtain the third inequality) and then change the signs inside the absolute value and then cancel it (you obtain the fourth inequality).
   (d) Solve both inequalities. Sketch a picture, where the third AND the fourth inequality is valid.
3. Take both the pictures and make their UNION.
4. Be careful about the ending points of all intervals and dividing by zero.

Exercises

Solve

1. (a) $|3x+1|-4<7$
   (b) $3 \leq 1 + \left|\frac{1}{2}x - 5\right|$
   (c) $|4x+2| \geq 0$
   (d) $|4x+2| > 0$
   (e) $\frac{|3x+2|}{4} \leq 1$
   (f) $|2x-7| < -5$
   (g) $-\frac{1}{3}\left|3+\frac{x}{2}\right| < -2$
2. (a) $3|1-x|-4 \geq |1-x|$
   (b) $||x|+x| \leq 2$
   (c) $||x+3|-12| < 13$
   (d) $2x - x^2 \geq |x-1| - 1$
   (e) $|x+1| \geq \frac{x+4}{2}$
   (f) $|x^2-3x+1| < 1$
   (g) $\left|\frac{x^2-5x+4}{x^2-4}\right| \leq 1$

Bonus

3. (a) $||||x–1|–1|–1|–1| = 0$
   (b) $(1-p)(|x+2|+|x|) = 4-3p$, $p \in \mathbb{R}$

4. Express using the absolute value:
    (a) All real numbers x, whose distance from zero is greater than 5 unit
    (b) All real numbers x, whose distance from 7 is less than 3 units
5. The pictures represents solutions of inequations with absolute value. Find the inequation. (You are looking for expressions similar to $|x + 5| < 4$, just change the numbers and the inequality sign.)