2nd lesson

https://www2.karlin.mff.cuni.cz/∼kuncova/en/teachIM.php
kunck6am@natur.cuni.cz

Algorithm

1. Find conditions (dividing by zero).
2. Move every expressions and numbers to the left side. On the right side should stay only zero.
3. Decompose the numerator and denominator. Find critical points, where expressions are equal to 0 or points, where expressions are not defined.
4. Separate intervals and make a table/sketch of signs. Check conditions again.
5. Write down the solution. (Check conditions;))

Warning

If it is possible, do not multiply (or divide) by expressions with x. This operation can change the sign in the equation or You can multiply by zero expression. Be careful.

Hints

For any quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, there is a quadratic formula

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quadratic equation then can be factored into:

$$a(x - x_1)(x - x_2) = 0.$$

(This operation sometimes involved complex roots.)

Exercises

1. Under what condition is the graph of the quadratic function described by $f(x) = ax^2 + bx + c$ concave down?
   (a) $a < 0$
   (b) $b < 0$
   (c) $c < 0$
   (d) More than one of the above.
   (e) None of the above.

2. If $f(x) = ax^2 + bx + c$ is a quadratic function, then the lowest point on the graph of $f(x)$ occurs at $x = -b/2a$.

   (a) True

   (b) False

   From:

   http://mathquest.carroll.edu/libraries/PRE.student.01.06.pdf

3. Solve inequations

   (a) $x^2 + 4x \geq 21$

   (b) $4x^2 \leq 15 - 17x$

   (c) $x^2 + 34 > 12x$

   (d) $x^2 - 2x + 1 \leq 0$

4. Solve inequations

   (a) $\dfrac{x^2 + 5x - 6}{x - 3} \geq 0$

   (b) $\dfrac{2x}{2x^2 + 5x + 2} > \dfrac{1}{x + 1}$

   (c) $\dfrac{x^2 + 2x - 3}{x + 1} \geq 0$

   (d) $\dfrac{x^2 - 9}{x + 2} \geq 0$

   (e) $\dfrac{4x^2 + 5x - 9}{x^2 - x - 6} \geq 0$

   (f) $\dfrac{(x + 4)^2(x + 6)}{(x^2 + 7)(x - 2)^3} < 0$

   (g) $\dfrac{x^3 + 1}{x^2 - 9} < 0$

   (h) $\dfrac{x + 1}{(x - 2)(x + 3)} \leq 1 - \dfrac{2}{x - 2}$

   (i) $\dfrac{5}{x^2 - 2x - 15} > 0$

   (j) $\dfrac{1}{3} - \dfrac{2}{x^2} < \dfrac{5}{3x}$

Bonus

5. Find a quadratic inequation (like in the 3rd exercise) such that its solution is

   (a) $x \in (-\infty, -3] \cup [2, \infty)$

   (b) $x \in (-1, 5)$

   (c) $x = -6$

   (d) $\emptyset$

6. Find the values of a parameter $c \in \mathbb{R}$ such that the range of the function

$$f(x) = \frac{x^2 + 2x + c}{x^2 + 4x + 3c}$$

   is equal to $\mathbb{R}$.