1st lesson

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Algorithm

1. Find conditions (dividing by zero).
2. Move every expressions and numbers to the left side. On the right side should stay only zero.
3. Decompose the numerator and denominator. Find critical points, where expressions are equal to 0 or points, where expressions are not defined.
4. Separate intervals and make a table/sketch of signs. Check conditions again.
5. Write down the solution. (Check conditions;))

Warning

If it is possible, do not multiply (or divide) by expressions with x. This operation can change the sign in the equation or You can multiply by zero expression. Be careful.

Hints

$$(A^2 - B^2) = (A - B)(A + B)$$

Exercises

1. Solve inequations

   (a) $(x+6)(x+2) < 0$

   (b) $(x+5)(4x-3) \leq 0$

   (c) $x(x+3)^2(x-4) < 0$

   (d) $2x^4 > 3x^3 + 9x^2$

2. Solve inequations

   (a) $\dfrac{x-7}{x-2} \leq 0$

   (b) $\dfrac{x+1}{-x-6} > 0$

   (c) $\dfrac{(x-4)(x+1)}{x-3} \leq 0$

   (d) $\dfrac{4x+5}{x+2} \geq 3$

   (e) $\dfrac{7}{x+3} < 2$

   (f) $\dfrac{3x+8}{x-1} < -2$

   (g) $\dfrac{7-x}{x+1} > \dfrac{4-x}{x+3}$

   (h) $\dfrac{x^3-6x^2}{x-2} > 0$

   (i) $\dfrac{x-7}{x+5} \geq -3$

   (j) $\dfrac{(x-4)(x+2)}{x-1} \geq 0$

   (k) $\dfrac{1}{x^2-4} \leq \dfrac{1}{2-x}$

Bonus

3. Let $C(x)$ denotes the cost function - returns the total cost of producing x items. Further, let $\bar{C}(x)$ denotes the average cost function: $\bar{C}(x) = \frac{C(x)}{x}$ computes the cost per item for x items produced.

   Let us suppose, that the cost function for producing x fairy-tale books is defined as $C(x) = 80x + 150$, $x \geq 0$.

   (a) Express the average cost function $\bar{C}(x)$. Do not forget define its domain.

   (b) Solve $\bar{C}(x) < 100$ and interpret.

   (c) Sketch the graph of $\bar{C}(x)$ and estimate its behaviour, when $x \to \infty$.

4. A student solved an inequation $\dfrac{x+4}{x-3} \leq 0$. (S)he multiplied both sides of the inequation by $x - 3$, then obtained $x + 4 \leq 0$. Finally (s)he said that the solution is $x \in (-\infty, -4]$. Is it true? Why? Why not?

5. Solve inequations by graphing $f(x)$. (Use some software, if You wish.)

   (a) $(x+3)(x-1)^2 > 0$, $f(x) = (x+3)(x-1)^2$

   (b) $\dfrac{x-1}{x^2-4} \geq 0$, $f(x) = \dfrac{x-1}{x^2-4}$