

ONLY with FODU & const. coeff

$$y''' - 6y'' + 9y' = 0$$

$$\lambda^3 - 6\lambda^2 + 9\lambda = 0 \quad (\lambda \neq 0)$$

$$\lambda(\lambda - 3)^2 = 0$$

$$\lambda_1 = 0$$

1-vals

$$\lambda_{2,3} = 3$$

2-vals.

$$x \in \mathbb{R}$$

$$y = c_1 e^{0x} + c_2 e^{3x} + c_3 x e^{3x}$$

$$c_{1,2,3} \in \mathbb{R}$$

$$FS_{\mathbb{R}} = \{1, e^{3x}, x e^{3x}\}$$

$$y^{(4)} + 8y'' + 16y = 0$$

$$\lambda^4 + 8\lambda^2 + 16 = 0$$

$$(\lambda^2 + 4)^2 = 0$$

$$\lambda = 2i = 0 + 2i$$

2-vals

$$\lambda = -2i = 0 - 2i$$

2-vals

$$\sqrt{-1 \pm 16} = \pm 4i$$

$$\lambda_{1,2} = \frac{0 \pm \sqrt{0 - 16}}{2} = \pm \frac{4i}{2}$$

$$FS_{\mathbb{R}} = \left\{ \begin{matrix} e^{0x} \cos(2x) \\ x e^{0x} \cos(2x) \end{matrix} , \begin{matrix} e^{0x} \sin(2x) \\ x e^{0x} \sin(2x) \end{matrix} \right\}$$

$$y = c_1 \cos 2x + c_2 \sin 2x + c_3 x \cos 2x + c_4 x \sin 2x$$

$$c_{1,2,3,4} \in \mathbb{R}$$

$$x \in \mathbb{R}$$

082 n. 1000 Fädeln 5 PS

$$f = e^{ux} (P(x) \cos(vx) + Q(x) \sin(vx))$$

$$y'' + ay = 8 \cos x$$

$$y(0) = 2$$

$$y'(0) = 0$$

$$y' + ay = 0$$

$$\lambda^2 + a = 0$$

$$\lambda = 0 + 3i$$

$$\lambda = 0 - 3i$$

$$y_h = c_1 e^{\alpha} \cos(3x) + c_2 e^{\alpha} \sin(3x)$$

$$x \in \mathbb{R} \quad c_1, c_2 \in \mathbb{R}$$

$$8 \cos x = e^{0x} (8 \cos(1x) + 0 \sin(1x))$$

$$S + P = 0 = S + Q$$

$$y_p = x^0 e^{0x} (A \cos(1x) + B \sin(1x))$$

$$(u + iv) = 0 + 1i = i$$

$$1 + 3i \quad m = 1$$

$$i \text{ ist Eigen?} \rightarrow m = 0$$

$$y_p = A \cos x + B \sin x$$

$$y_p' = -A \sin x + B \cos x$$

$$y'' + ay = 8 \cos x$$

$$y_p'' = -A \cos x - B \sin x$$

Einsetzen

$$-A \cos x - B \sin x + A \cos x + B \sin x = 8 \cos x + 0 \sin x$$

$$\cos x: -A + A = 8$$

$$\sin x: -B + B = 0$$

$$A = 1$$

$$B = 0$$

$$y_p = 1 \cos x (+ 0 \sin x)$$

Daher

$$y = y_h + y_p = c_1 \cos(3x) + c_2 \sin(3x) + \cos x$$

$$c_1, c_2 \in \mathbb{R}$$

$$f.p. \quad y(0) = 2 \quad y'(0) = 0$$

$$y' = c_1 (-\sin(3x)) \cdot 3 + c_2 \cos(3x) \cdot 3 - \sin x$$

$$2 = y(0) = c_1 \cdot 1 + c_2 \cdot 0 + 1$$

$$0 = y'(0) = c_1 \cdot 0 + c_2 \cdot 3 - 0$$

$$2 = c_1 + 1$$

$$0 = 3c_2$$

$$\underline{c_1 = 1 \quad c_2 = 0}$$

Resonant S.P.P.

$$\underline{y = 1 \cos 3x + \cos x} \quad x \in \mathbb{R}$$

Progn.

$$y' + y = x + \sin x$$

$$y'' + y = 0$$

$$y'' + y = x$$

$$y'' + y = \sin x$$

$$y_h$$

$$y_{p1}$$

$$y_{p2}$$

$$y = y_h + y_{p1} + y_{p2}$$