

$$y' = 3x^2 y^2$$

$$y(0) = -1 - \frac{1}{2}$$

$$x \in \mathbb{R} = I$$

$$g(y) = 3y^2 \quad h(x) = x^2$$

$$y \equiv 0 \quad x \in \mathbb{R}$$

$$J_1 = (-\infty, 0), \quad J_2 = (0, \infty)$$

$$\frac{y'}{y^2} = 3x^2$$

$$\int \frac{1}{y^2} dy = \int 3x^2 dx$$

$$\rightarrow \left| -\frac{1}{y} = x^3 + c \right| \leftarrow H(x) + c$$

$G(y)$

$$\bullet (a) \quad x \in \mathbb{R}, \quad y \in J_1 = (-\infty, 0)$$

$$G(J_1) = (0, \infty)$$

$\forall x \in \mathbb{R}$

$$x^3 + c > 0 \quad x^3 > -c \quad \left| x^3 > \sqrt[3]{-c} \right|$$

$$\left| \frac{-1}{x^3 + c} = y \right|$$

$$\bullet (b) \quad x \in \mathbb{R} \quad y \in J_2 = (0, \infty)$$

$$G(J_2) = (-\infty, 0)$$

$\forall x \in \mathbb{R}$

$$x^3 + c < 0 \quad x < \sqrt[3]{-c}$$

$$y = \frac{-1}{x^3 + c}$$

Poc. podm. $y \equiv 0$ nelež

$$y(0) = -\frac{1}{2} = \frac{-1}{0^3 + c} \rightarrow c = 2$$

$$\underline{y = \frac{-1}{x^3 + 2} \quad x \in (-\sqrt[3]{2}, \infty)}$$

Záver:

$$y_1 \equiv 0 \quad x \in \mathbb{R}$$

$$y_2 = \frac{-1}{x^3 + 2}$$

$$x \in (-\infty, -\sqrt[3]{2})$$

$$x \in (-\sqrt[3]{2}, \infty)$$